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Study on Magnetohydrostatic Problems and Force Free Fields

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ABSTRACT

In this paper, the firmness of a layered fluid flowing between the parallel planes in a permeable medium under the effect of suspended particles have been considered. Then we have proved that the both the rotations as well as vertical oscillations have a stabilizing effect on the firmness of a layered fluid, while the suspended particles can have stabilizing or destabilizing effect which depends upon the parameter in the absence of porous medium.

Keywords: parallel planes, stabilizing or destabilizing effect.

1. INTRODUCTION

Taylor was the first to predict the stability of motion between two concentric rotating cylinders because of its some significant applications in the field of engineering and technology. Later on, Di Prima[5] studied the problem of Taylor, by applying different methods to solve the eigen value problem. Weigelmann *et al*.[8] described a method which can determine models for global solar corona using observed data. Many researchers have studied that radial and inward flow inside the porous cylinders stabilizes the flow under the effect of magnetic field, whereas radial outward flow destabilizes the motion for weak flow and stabilizes the motion for strong flow. Physics of space plasma activity was studied by K. Schindler[6]. Orientation of magnetic fields for constant-α magnetohydrostatic models has been studied by Aulanier *et al.*[2]. The stability of heterogeneous fluid was investigated by Rayleigh. Sutherland *et al.*[7] discussed the amplitude of gravity waves generated by oscillating vertical cylinder for different frequencies and amplitudes. Aly J. J.[1] investigated x-invariant magnetostatic equilibrium which clearly describes the ideal evolutions with conditions imposed on boundary.

Recently, Kirti and Seema[3] have discussed Raleigh-Taylor instability of an incompressible and homogeneous fluid which is conducting in nature in a permeable medium under the steady rotation and action of some suspended particles.

In this paper, we have made an effort to study the stability of a stratified fluid flowing in a permeable medium in the presence of some suspended particles by considering rotation and vertical oscillations.

Force Free Fields:

When the density of magnetic energy exceeds not only the kinetic energy but also the internal energy, we have an "equilibrium of low ζ" , { where ζ is the ratio of typical thermodynamic pressure to a typical magnetic pressure} in which the gas pressure is small compared with the magnetic pressure, so that for equilibrium $J \times B_0 = 0$ to a close approximation. If $J \times B_0 = 0$, the electric currents flow along the magnetic field lines, and

Curl B_0 = KJ = r ζ where r is the function of position.

The fields such that $J \times B_0 = 0$ in any region are called force free fields.

The three basic types of force free field can readily be obtained. One is a field in a doughnut shaped surface, confined by external pressures and other two types of field are introduced in applications to the sun's atmosphere.

Mathematical discussions of force free fields have tended to concentrate on the case when is constant everywhere. In this case

Curl Curl $B_0 = r^2 B_0$

Where $(\nabla^2 + r^2)\, B_0 = 0$

is an equation whose solutions are well known. However, these solutions hardly corresponds to conditions actually met in nature.

2. FORMULATION OF THE PROBLEM

Here, we have assumed the following notations:

V = Velocity of suspended particles

ω = Uniform angular velocity

N_0 = Constant number density of the particles

r = Particle radius

k = Permeability of isotropic porous medium

Here, it is assumed that the fluid and planes rotate with uniform angular velocity ω about z axis and have vertical oscillations with acceleration $c\theta^2\cos\theta t$ in the homogeneous incompressible media consisting of fluid and particles which is bounded by two rigid planes at z = 0 and z = h. The two particles are considered to be at a large distance in comparison to their diameters so that interparticle reactions are neglected and the body force per unit volume is $K_1 N_0(V-u)$ (where in $K_1 = 6\pi\rho r v$) is added to the momentum transfer equation for the fluid in the presence of suspended particles within the fluid.

In the above frame of reference, the fluid situated between z = 0 and z = h is at rest before the disturbance and is subjected to gravitational acceleration $a(t) = -a_0 - c\theta\cos\theta t$ along z axis. The density of fluid before the disturbances is given by

$$\rho = f\, e^{z/\alpha}$$

If u (u_x, u_y, u_z), p_0, ρ_0 denotes the perturbations in velocity, pressure and density respectively.

In view of above facts, the equations of perturbation can be expressed in the form

$$\frac{\partial u}{\partial t} = -\frac{\nabla P}{\rho} + 2u\omega x + \frac{K_1 N_0 (V-u)}{\rho} - \frac{\nu u}{k} + \frac{a(t) e_z \rho_0}{\rho} \quad (1)$$

$$\left(\tau \frac{\partial}{\partial t} + 1\right) V = u \quad (2)$$

$$\frac{\partial \rho_0}{\partial t} + u.\nabla P = 0 \quad (3)$$

and $\nabla . u = 0$ (4)

The stability of the system is studied against small disturbances which are given by

$$\eta_1 = \widehat{\eta_1}\,(z,t)\, e^{i(K_1 x + K_2 y)} \quad (5)$$

wherein $K_1^{\,2} + K_2^{\,2} = K'^2$

K_1 and K_2 be the horizontal wave numbers and the $\widehat{\eta_1}$ (z,t) be the some function of z and t.

3. SOLUTION OF THE PROBLEM

Let us assume

$$\frac{K_1 u_x + K_2 u_y}{K'} = u' \quad (6)$$

Then equation (4) becomes

$$iK' u' + \frac{\partial u_z}{\partial z} = 0 \quad (7)$$

Taking the curl of equation (1), we obtain

$$\left[\frac{\nu}{k} + (1+\beta)\frac{\partial}{\partial t}\right] \text{curl } u + \frac{\nabla\rho}{\rho}\left[\left(\frac{\nu}{k} + (1+\beta)\frac{\partial}{\partial t}\right) u\right] x + 2\text{curl}(u\ \omega x) + 2\ \frac{\nabla\rho}{\rho}\ (\nabla u x) x = \frac{a(t)\nabla\rho_0 e_z x}{\rho} \quad (8)$$

wherein

$$\frac{K_1 N_0 \tau}{\rho} v = \beta u \quad (9)$$

$\frac{K_1 N_0 \tau}{\rho}$ gives the mass concentration of suspended particles.

Equation (9) manifests that the flow of suspended particles be either in the direction of fluid flow or in opposite direction, depending on the sign of β, being positive or negative.

On equating, z component of equation (8), we have

$$\frac{\partial v'}{\partial t} = -2\ \omega\ \frac{(K_1 u_x + K_2 u_y)}{K'} \quad (10)$$

wherein $v' = \frac{(K_1 u_y - K_2 u_x)}{K'}$ (10a)

From equations (6) and (10), we get

$$\frac{\partial v'}{\partial t} = -2\omega u' \quad (11)$$

On adding K_1 times of x component and K_2 times of y component of equation (8), we get

$$\left[\left\{\frac{\nu}{k} + (1+\beta)\frac{\partial}{\partial t}\right\}\frac{\partial v'}{\partial z} + 2\omega\frac{\partial u'}{\partial t}\right] + \frac{1}{\alpha}\left[\left\{\frac{\nu}{k} + (1+\beta)\frac{\partial}{\partial z}\right\} v' + 2\omega u'\right] = 0 \quad (12)$$

Now subtract K_2 times of x component from K_1 times of y component of equation (8), we obtain

$$\left[\frac{\nu}{k} + (1+\beta)\frac{\partial}{\partial t}\right]\left[\frac{\partial u'}{\partial z} - iK' u_z + \frac{u'}{\alpha}\right] = 2\ \omega \frac{\partial v'}{\partial z} + \frac{2\omega v'}{\alpha} - ia\frac{\rho_0}{\rho} K' \quad (13)$$

From equation (3), we have

$$u_z = -\alpha\,\frac{\partial}{\partial t}\left(\frac{\rho_0}{\rho}\right) \quad (14)$$

As a consequence of equation (7), (11), (13)and (14), we obtain

$$\left[\frac{\nu}{k} + (1+\beta)\frac{\partial}{\partial t}\right]\left[\frac{\partial^3}{\partial z^2\,\partial t} + \frac{1}{\alpha}\frac{\partial^2}{\partial z\,\partial t} - K'^2\frac{\partial}{\partial t}\right]\frac{\rho_0}{\rho} + 4\ \omega^2\left(\frac{\partial^2}{\partial z^2} + \frac{1}{\alpha}\frac{\partial}{\partial z}\right)\frac{\rho_0}{\rho} - K'^2\,\frac{a(t)}{\alpha}\frac{\rho_0}{\rho} = 0 \quad (15)$$

For the solution of the equation (15), we consider variable separable method in the form

$$\frac{\rho_0}{\rho} = T(t)\ Z(z) \quad (16)$$

Let us assume

$$\gamma = \left(Z'' + \frac{Z'}{\alpha}\right) Z^{-1} \quad (17)$$

In view of equations (16) and (17), equation (15) takes the form

$$T'' + \frac{\nu}{k(1+\beta)}\,T' - \left[\frac{K'^2 a(t) - 4\,\alpha\omega^2\gamma}{\alpha(1+\beta)(\gamma - K'^2)}\right] T = 0 \quad (18)$$

Substituting

$T(t) = \delta(t)e^{-\frac{vt}{2k(1+\beta)}}$ into equation (18), we get (19)

$$\delta'' - \left[\frac{v^2}{2k^2(1+\beta)^2} + \frac{K'^2 a(t) - 4\alpha\omega^2\gamma}{\alpha(1+\beta)(\gamma - K'^2)}\right]\delta = 0 \quad (20)$$

Let us define the following parameter

$$\varphi = \theta t \quad (21)$$

$$\mu = \frac{(K'^2 a_0 + 4\alpha\omega^2\gamma)}{l\theta^2} \quad (22)$$

wherein

$$l = \alpha(1+\beta)(K'^2 - \gamma) \quad (23)$$

$$\mu_l = \frac{\mu}{l} \quad (24)$$

$$\xi = \frac{v^2}{2k^2\theta^2(1+\beta)^2} \quad (25)$$

Using the parameters which defined by equations (21), (22), (23), (24) and (25) in equation (20), we obtain

$$\delta'' - (\xi + \mu + \mu_l \cos\varphi)\,\delta = 0 \quad (26)$$

4. RESULTS AND DISCUSSION

Here, we have discussed the following three cases:

A. In absence of Porous media:

Since on the rigid planes, the normal component of fluid velocity vanishes. Solution of equation (17) yields

$$\gamma = -\left[\frac{1}{4\alpha^2} + \frac{n^2\pi^2}{h^2}\right] \quad (27)$$

where n is an integer, known as vertical wave number.

B. In absence of suspended particle:

If we define the parameters ξ, η_2 and μ_l in such a way

$\xi = \frac{K'^2 a_0}{\eta_2\,\theta^2} + \frac{v^2}{2k\,\theta^2}$ wherein $\eta_2 = \alpha\,(K'^2 - \gamma)$

And

$$\mu_l = \frac{c}{\eta_2}$$

Then the equation (26) reduces to in the form

$$\delta'' - (\xi + \mu_l \cos\varphi)\,\delta = 0 \quad (28)$$

Remark (1):

If $v \to 0$, then equation (28) reduced to equation (3) , which describes dynamic stabilization on Rayleigh- Taylor stability in the absence of finite Lamar radius effect and relation between ξ and μ_l is given by

$$\mu_l = \frac{c\,\theta^2}{a_0}\,\xi - \frac{c\,v^2}{2k^2 a_0} \quad (29)$$

Remark (2):

If $v \neq 0$, then equation (28) gives the relation between ξ and μ_l through a line which always passes through the unstable region, (i.e.,) porous media has a destabilizing effect.

Remark (3):

If $v = 0$, then for fixed values of $\frac{c\,\theta^2}{a_0}$ the possible values of ξ and μ_l lie on a straight line.

C. Effect of suspended particles:

The suspended particle does not affect the region of stability and they have stabilizing and destabilizing effect, according as β is positive and negative.

Since l > or $<\eta_2$ as β is negative or positive

wherein $\eta_2 = \alpha\,(K'^2 - \gamma)$.

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