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$\mathcal{H}$ −hyperconnected spaces

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ABSTRACT

The aim of this Study is to analysis the $\mathcal{H}-hyperconnected$ spaces and in $\mathcal{H}-hyperconnected$ spaces properties of various sets are established. $\mathcal{H}-submaximal$ spaces and $T_{\mathcal{H}}-$ spaces are defined. $\mathcal{H}-submaximal$ spaces and strong $\beta-\mathcal{H}-open$ sets are characterized.

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1. INTRODUCTION

Let X be a nonempty set. A non empty subfamily κ of $\wp(X)$ is called a generalized topology on X [1] if $\emptyset \in \kappa$ and κ is closed under arbitrary union. The pair (X, κ) is called generalized topological space. Elements of κ are called κ −open sets and the complement of a κ −open set is called a κ −closed set. The largest κ −open set contained in a subset A of X is denoted by $\text{int}_\kappa(A)$[3] and is called the κ-interior of A. The smallest κ-closed set containing A is called the κ - closure of A and is denoted by $\text{cl}_\kappa(A)$[3].

Throughout the paper, by a space, we always mean a generalized topological space (X,κ). A subset A is said to be κ-dense if $\text{cl}_\kappa(A) = X$. A generalized topology κ is said to be a quasi topology[4] on X if M, N $\in \kappa$ implies $M \cap N \in \kappa$. A hereditary class $\mathcal{H}$ is a non empty family of subset of X such that $A \subset B$, $B \in \mathcal{H}$ implies $A \in \mathcal{H}$[2]. For each subset A of X, a subset $A^*(\mathcal{H})$ or simply A^* of X is defined by $A^*(\mathcal{H}) = \{x \in X \mid M \cap A \notin \mathcal{H}$ for every $M \in \kappa$ containing $x\}$[2]. If $\text{int}_\kappa(\text{cl}_\kappa(A))=\phi$, then A is said to be κ-rare[2]. Where N_κ is the family of all κ-rare sets in (X,κ), then $A^*(N_\kappa) = \text{cl}_\kappa(\text{int}_\kappa(\text{cl}_\kappa(A)))$. A subset A of a space (X,κ) is said be κ^*-closed (res κ^*-dense in itself,κ^*-perfect) if $A^* \subset A$(resp.,$A \subset A^*$,$A = A^*$). A subset A of a space (X,κ) is said to ba κ-preopen$C5$ if $A \subset int_\kappa(cl_\kappa(A))$. The family of all κ-preopen$C5$ sets denoted by π_κ. $\mathcal{H}$ is said to be κ-codense if $\kappa \cap \mathcal{H} = \{\phi\}$[2] and is said to be strongly κ -codense[2] if M, N $\in \kappa$ and M $\cap$ N $\in \mathcal{H}$, then M $\cap$ N = ϕ. If $\pi(x) \cap \mathcal{H} = \emptyset$ then $\mathcal{H}$ is called completely κ-codense. Every strongly κ-codense hereditary class is κ-codense but the converse is not true[2]. If $cl_\kappa^*(A) = A \cap A^*$for every subset A of X, with respect to κ and a hereditary class $\mathcal{H}$ of subsets of X then $\kappa^* = \{ A \subset X / cl_\kappa^*(X - A) = X - A\}$ is a generalized topology[2]. Elements of κ^* are called κ^*-open sets and the complement of a κ^*-open set is called a κ^*-closed set. $int_\kappa^*(A)$ is the interior of A in (X, κ^*). Let (X, κ) be a generalized topological space and $\mathcal{H}$ be a hereditary class of subsets of X. If $cl_\kappa^*(A) = X$, then A is called κ^*-dense. Let (X,κ) be a generalized topology. A subset A of X is said to be $\kappa - regularclosed$ [2] if $cl_\kappa(int_\kappa(A)) = A$, κ-*semi open*[2] if $A \subset cl_\kappa(int_\kappa(A))$, κ-$\alpha - open$ [2] if $A \subset int_\kappa(cl_\kappa(int_\kappa(A)))$, κ-$\beta - open$[2] if $A \subset cl_\kappa(int_\kappa(cl_\kappa(A)))$, $\beta - \mathcal{H} - open$[3] if $A \subset cl_\kappa(int_\kappa(cl_\kappa^*(A)))$, $\alpha - \mathcal{H} - open$[3] if $A \subset int_\kappa(cl_\kappa^*(int_\kappa(A)))$, $pre - \mathcal{H} - open$[3] if $A \subset int_\kappa(cl_\kappa^*(A))$, $semi - \mathcal{H} - open$[3] if $A \subset cl_\kappa^*(int_\kappa(A))$. The family of all $\kappa -$semiopen ($\kappa - \beta -$open, α-$\mathcal{H}$-open) sets is denoted by $\sigma_\kappa(\beta_\kappa, \alpha\mathcal{H}o(X))$. If $\mu \in \{\kappa, \sigma, \beta.\pi\}$, then int_μ and cl_μ are , respectively, the interior and closure operators with respect to the generalized topology μ. Let (X,κ) be a quasi topology. A subset A of X is said to be almost strong $\mathcal{H}$-open[7] if $A \subset cl_\kappa^*(int_\kappa(A^*))$, almost $\mathcal{H}$-open[7] if $A \subset cl_\kappa(int_\kappa(A^*))$, strong -$\beta$-$\mathcal{H}$-open[7] if $A \subset cl_\kappa^*(int_\kappa(cl_\kappa^*(A))$, β -$\mathcal{H}$-open[7] if $A \subset cl_\kappa(int_\kappa(cl_\kappa^*(A)))$, weakly semi-$\mathcal{H}$-open[7] if $A \subset cl_\kappa^*(int_\kappa(cl_\kappa(A)))$, $\mathcal{H}$ locally closed[7] if $A = G \cap V$, where G and V are * $-$perfect. A subset A of a generalized topological spaces (X,κ) is said to be a $\kappa -$locally closed set[2] if $A = U \cap V$ where U is $\kappa -$open and V is $\kappa -$closed. The family of all $\kappa -$locally closed sets denoted by $\mathcal{H}_{LC}(X)$. A subset A of a generalized topological spaces is said to be an $\mathcal{H}_1 -$ set [4] if $A = U \cap V$ where U is $\kappa -$open and V is $\kappa -$regular closed. The family of all $\mathcal{H}_1 -$ sets is denoted by $\mathcal{H}_1(X)$. A is said to be a $\mathcal{H}_2 -$ set [4] if $A = U \cap V$ where U is $\kappa -$open and V is $\kappa -$semiclosed. The family of all $\mathcal{H}_2 -$ sets is denoted by $\mathcal{H}_2(X)$. A is said to be an $\mathcal{H}_3 -$ set[4] if $A = U \cap V$ where U is $\kappa -$open and V is $\kappa -$pre closed. The family of all $\mathcal{H}_3 -$ sets is denoted by $\mathcal{H}_3(X)$. Clearly, X is $\kappa -$regular closed and so every $\kappa -$open set is an $\mathcal{H}_1 -$ set. Clearly $\mathcal{H}_1(X) \subset \mathcal{H}_{LC}(X) \subset \mathcal{H}_2(X) \subset \mathcal{H}_3(X)$. A subset A of a generalized topological spaces is said to be $\kappa^* -$ regular closed[2] if $cl_\kappa^*(int_\kappa^*(A)) = A$.

2. $\mathcal{H}$ −HYPERCONNECTED SPACE

Definition 2.1 Let (X, κ) be a generalized topological space and $\mathcal{H}$ be a hereditary class of subsets of X is called $\mathcal{H}$ −connected if X can not be written as the union of nonempty and disjoint on $\mathcal{H} -$ open set of X.

Let (X, κ) be a generalized topological space, $\mathcal{H}$ be a hereditary class of subsets of X and $A \subset X$. Then A is called $\mathcal{H} -$ open if $A \subset int_\kappa(A^*)$. If $A^* = X$, then A is called $\mathcal{H} -$ dense. (X, κ) is said to be an $\mathcal{H} -$ hyperconnected space , if every non empty open set is $\mathcal{H} -$ dense.

Clearly, every subset with nonempty interior, in every $\mathcal{H}-$hyperconnected space, is $\mathcal{H}-$dense.
The following Theorem 2.2 shows that in $\mathcal{H}-$hyperconnected spaces, $\mathcal{H}-$open sets are $\mathcal{H}-$dense sets.

Theorem 2.2 Let (X, κ) be an $\mathcal{H}-$ hyperconnected space and $A \subset X$. Then the following are equivalent.
(a) A is $\mathcal{H}-$ dense.
(b) A is $\mathcal{H}-$ open.

Proof. (a) Let $A \subset X$. Then A is $\mathcal{H}-$ dense and so $A^\star = X$ which implies that $\mathrm{int}_\kappa(A^\star) = X$. Hence A is $\mathcal{H}-$ open. Conversely, suppose that A is $\mathcal{H}-$ open. Then $A \subset \mathrm{int}_\kappa(A^\star)$. Hence $A^\star \subset (\mathrm{int}_\kappa(A^\star))^\star$ which implies that $X \subset ((A^\star))^\star \subset A^\star$. Therefore, $A^\star = X$. Hence A is $\mathcal{H}-$ dense.

The following Theorem 2.3 gives properties of various sets in an $\mathcal{H}-$ hyperconnected space.

Theorem 2.3 Let (X, κ) be an $\mathcal{H}-$hyperconnected space. Then the following statements hold:
(a) Every nonempty subset of X is a $f_{\mathcal{H}}-$ set.
(b) Every subset of X is a $\mathcal{H}-$ open set.
(c) κ coincides with $\mathcal{H}_A-$ sets.
(d) If A is a nonempty subset $\mathcal{H}-$ open set and B is a $f_{\mathcal{H}}-$ set such that $A \cap B = \emptyset$, then $B = \emptyset$.
(e) $\mathrm{int}_\kappa(A^\star) = X$ for every nonempty $\mathcal{H}-$ open set A of X.
(f) Every nonempty $\mathcal{H}-$ open set is $\mathcal{H}-$ dense.

Proof. (a) Let $A \subset X$ be a nonempty subset of X. Therefore, $A \subset X = (\mathrm{int}_\kappa(A))^\star$ which implies that $A \subset (\mathrm{int}_\kappa(A))^\star$. Hence every nonempty subset of X is an $f_{\mathcal{H}}-$ set.
(b) Let A be any subset of X. Therefore, $A \subset (\mathrm{int}_\kappa(A))^\star = X$ implies that $A^\star \subset (\mathrm{int}_\kappa(A))^\star)^\star \subset A^\star$. Hence $A^\star = (\mathrm{int}_\kappa(A))^\star = X$. since (X, κ) is an $\mathcal{H}-$hyperconnected space. Therefore A is $\mathcal{H}-$ dense. By Theorem 2.2, A is an $\mathcal{H}-$ open set for every subset of X.
(c) Let $A \subset X$ be an $\kappa-$open subset of X. Then $A = A \cap X$. Since $A \in \kappa, X = (\mathrm{int}_\kappa(X))^\star = X^\star$, since (X, κ) is an $\mathcal{H}-$ hyperconnected space. Hence A is an $\mathcal{H}_A-$ set. Conversely, if A is an $\mathcal{H}_A-$ set, then $A = U \cap V, U \in \kappa, V = (\mathrm{int}_\kappa(V))^\star = X$, which implies that $A = U \cap X = U$ which shows that A is an $\kappa-$open set. Hence κ is the family of all $\mathcal{H}_A-$ set.
(d) Let A be a nonempty $\mathcal{H}-$open set and B be an $f_{\mathcal{H}}-$ set such that $A \cap B = \emptyset$. Then $A \subset \mathrm{int}_\kappa(A^\star), B \subset (\mathrm{int}_\kappa(B))^\star$. Since $A \cap B = \emptyset$, $A \cap \mathrm{int}_\kappa(B) = \emptyset$ which implies that $(A \cap \mathrm{int}_\kappa(B))^\star = \emptyset^\star = \emptyset$. So $A^\star \cap \mathrm{int}_\kappa(B) = \emptyset$ and $X \cap \mathrm{int}_\kappa(B) = \emptyset$ which implies that $\mathrm{int}_\kappa(B) = \emptyset$ which in turn implies that $B = \emptyset^\star = \emptyset$.
(e) Let A be a nonempty $\mathcal{H}-$ open subset. Then $A \subset \mathrm{int}_\kappa(A^\star)$ and so $A^\star \subset (\mathrm{int}_\kappa(A^\star))^\star \subset$

$(A^\star)^\star$. Hence $A^\star \subset X \subset (A^\star)^\star \subset A^\star$ which implies that $A^\star = X$. Thus A is an $\mathcal{H}-$ dense set. Hence $\text{int}_\kappa(A^\star) = X$ for every non empty $\mathcal{H}$ −open subset A of X.

The following Theorem 2.4 gives a characterization of non empty $\kappa-$open sets in $\mathcal{H}-$ hyperconnected spaces.

Theorem 2.4 Let (X, κ) be an $\mathcal{H}-$ hyperconnected space. Then the following are equivalent.
(a) $A \subset X$ is $\kappa-$open, if A is nonempty.
(b) $A \subset X$ is an $\mathcal{H}-$ open set and an $\mathcal{H}-$locally closed set.

Proof. (a)⇒(b). Suppose that (a) holds. Let $A \subset X$ be an $\kappa-$open set. Since (X, κ) is an $\mathcal{H}-$ hyperconnected space, A is $\mathcal{H}-$ dense and every $\mathcal{H}-$ dense set is $\mathcal{H}-$ open, A is $\mathcal{H}-$ open. Since A is $\kappa-$open, $A = A \cap X, A \in \kappa, X = X^\star$, A is an $\mathcal{H}-$locally closed.
(b)⇒(a). Suppose that A is an $\mathcal{H}-$ open set and an $\mathcal{H}-$locally closed. set. Then $A \subset \text{int}_\kappa(A^\star)$ and $A = U \cap V, U \in \kappa, V = V^\star$. Since A is $\mathcal{H}-$ open, $A^\star \subset ((\text{int}_\kappa(A^\star))^\star \subset (A^\star)^\star \subset A^\star$ which implies that $A^\star = X$. Therefore A is $\mathcal{H}-$ dense. Now $A = U \cap V, U \in \kappa, V = V^\star, A \subset V, A^\star \subset V^\star = V \Rightarrow X \subset V$ which implies that $X = V$. Hence $A = U \cap X = U$ which shows that A is a $\kappa-$open set.
A generalized topological space (X, κ) with a hereditary class $\mathcal{H}$ is said to be an $\mathcal{H}-$ submaximal space, if every $\kappa^\star-$ dense set is $\kappa-$open.
The following Theorem 2.5 gives a characterization of $\mathcal{H}-$ submaximal spaces in $\mathcal{H}-$ hyperconnected space.

Theorem 2.5 Let (X, κ) be an $\mathcal{H}-$ hyperconnected space. Then the following are equivalent:
(a) (X, κ) is an $\mathcal{H}-$ submaximal space.
(b) $A^\star - A$ is $\kappa-$closed for every $A \subset X$.

Proof. (a) ⇒ (b). Consider $(X-(A^\star - A))^\star = ((X-A^\star) \cup A))^\star = (X-A^\star)^\star \cup A^\star \supset (X^\star - (A^\star)^\star) \cup A^\star \supset (X-A^\star) \cup A^\star = X$. Hence $(X-(A^\star - A))^\star = X$. Hence $X-(A^\star - A)$ is $\mathcal{H}-$dense which is $\kappa-$open by hypothesis. Hence $A^\star - A$ is $\kappa-$closed for every $A \subset X$.
(a) ⇒ (b). Let A be $\mathcal{H}-$ dense. Then $A^\star = X$. Since , $A^\star - A$ is $\kappa-$closed, $X - A$ is $\kappa-$closed . Thus A is $\kappa-$open.
The following Theorem 2.6 gives a characterization of strong $\beta-\mathcal{H}-$ open sets in $\mathcal{H}-$ hyperconnected spaces.

Theorem 2.6 Let (X, κ) be an $\mathcal{H}-$ hyperconnected space. Then a nonempty set A is a strong $\beta-\mathcal{H}-$ open set if and only if $\text{cl}_\kappa^\star(A)$ contains a nonempty $\kappa-$open set.

Proof. Let A be a nonempty strongly $\beta-\mathcal{H}-$ open set of X. Then $A \subset \text{cl}_\kappa^\star(\text{int}_\kappa(\text{cl}_\kappa^\star(A)))$ implies that $\text{cl}_\kappa^\star(\text{int}_\kappa(\text{cl}_\kappa^\star(A))) \neq \emptyset$ which implies that $\text{int}_\kappa(\text{cl}_\kappa^\star(A)) \neq \emptyset$. Hence $\text{cl}_\kappa^\star(A)$ contains a nonempty $\kappa-$open set.
Conversely, if A is a nonempty set such that $\text{cl}_\kappa^\star(A)$ contains a nonempty $\kappa-$open set G. Therefore $G \subset \text{cl}_\kappa^\star(A), G \subset \text{int}_\kappa(\text{cl}_\kappa^\star(A))$ which implies that $G^\star \subset (\text{int}(\text{cl}_\kappa^\star(A)))^\star \subset \text{cl}_\kappa^\star(\text{int}_\kappa(\text{cl}_\kappa^\star(A))), X \subset \text{cl}_\kappa^\star(\text{int}_\kappa(\text{cl}_\kappa^\star(A)))$ which implies that $A \subset \text{cl}_\kappa^\star(\text{int}_\kappa(\text{cl}_\kappa^\star(A)))$. Hence A is a strong $\beta-\mathcal{H}-$ open set.

The following Theorem 2.7 gives a characterization of $\mathcal{H}-$ hyperconnected spaces.

Theorem 2.7 Let (X,κ) be a generalized topological space. Then the following are equivalent:
(a) (X,κ) is an $\mathcal{H}-$ hyperconnected space.
(b) Every nonempty $\beta-\mathcal{H}-$ open set is $\mathcal{H}-$ dense.

Proof. (a) $\Rightarrow$ (b). Let A be a nonempty $\beta-\mathcal{H}-$ open set of X. Then $A\subset cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A)))$ and $int_{\kappa}(cl^{\star}_{\kappa}(A)\neq\emptyset$. If U is any nonempty $\kappa-$open set such that $U\cap int_{\kappa}(cl^{\star}_{\kappa}(A))\notin\mathcal{H}$ implies that $(U\cap int_{\kappa}(cl^{\star}_{\kappa}(A)))^{\star}\notin\mathcal{H}$ and so $int_{\kappa}(U\cap cl^{\star}_{\kappa}(A))\notin\mathcal{H}$ which implies that $U\cap cl^{\star}_{\kappa}(A)\notin\mathcal{H}$ which in turn implies that $(U\cap A)\cup(U\cap A^{\star})\notin\mathcal{H}$. If $U\cap A\in\mathcal{H}$, $(U\cap A)^{\star}=\emptyset$ which implies that $U\cap A^{\star}\in\mathcal{H}$ which is a contradiction. Therefore, $U\cap A\notin\mathcal{H}$ and hence $A^{\star}=X$.
(b) $\Rightarrow$ (a). If U is a nonempty $\kappa-$open set, then U is a nonempty $\beta-\mathcal{H}-$ open set which is $\mathcal{H}-$ dense. Hence (X,κ) is an $\mathcal{H}-$ hyperconnected space.

The following Theorem 2.8 gives a characterization of $\mathcal{H}-$ hyperconnected spaces if the hereditary class $\mathcal{H}$ is $\kappa-$codense.

Theorem 2.8 Let (X,κ) be a generalized topological space with a $\kappa-$codense hereditary class $\mathcal{H}$. Then the following are equivalent:
(a) (X,κ) is an $\mathcal{H}-$ hyperconnected space.
(b) $A\cup cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A)))=X$ for every nonempty strong $\beta-\mathcal{H}-$ open set of X.
(c) Every nonempty strong $\beta-\mathcal{H}-$ open set is $\kappa^{\star}-$ dense.
(d) If A is a nonempty strong $\beta-\mathcal{H}-$ open set of X and B is a semi-$\mathcal{H}$- open set such that $A\cap B=\emptyset$, then $B=\emptyset$.

Proof. (a) $\Rightarrow$ (b). Suppose that A is a nonempty strong $\beta-\mathcal{H}-$open set. Then $int_{\kappa}(cl^{\star}_{\kappa}(A))\neq\emptyset$ which implies that $int_{\kappa}(cl^{\star}_{\kappa}(A))$ is $\mathcal{H}-$ dense. Hence $cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A)))=X$. Since A is a strong $\beta-\mathcal{H}-$open set of X, $A\subset cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A)))=X\Rightarrow A\cup cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A)))=X$.
(b) $\Rightarrow$ (c). If A is a strong $\beta-\mathcal{H}-$ open set of X such that $A\cup cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A)))=X$. By (b), $cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A)))=X$ and so $int_{\kappa}(cl^{\star}_{\kappa}(int_{\kappa}(cl^{\star}_{\kappa}(A))))=X$ which implies that $int_{\kappa}(cl^{\star}_{\kappa}(A))=X$ which implies that $cl^{\star}_{\kappa}(A)=X$. Hence A is $\kappa^{\star}-$ dense.
(c) $\Rightarrow$ (d). If A is a nonempty strong $\beta-\mathcal{H}-$open set of X and B is a semi-$\mathcal{H}$- open set such that $A\cap B=\emptyset$. Now $A\cap B=\emptyset$ implies that $A\cap int_{\kappa}(B)=\emptyset$ which in turn implies that $cl^{\star}_{\kappa}(A)\cap int_{\kappa}(B)=\emptyset$ and so $X\cap int_{\kappa}(B)=\emptyset$ which implies that $int_{\kappa}(B)=\emptyset$. Since B is semi $-\mathcal{H}-$ open, $B\subset cl^{\star}_{\kappa}(int_{\kappa}(B))$. Hence $B=\emptyset$.
(d) $\Rightarrow$ (a). Suppose that A and B are nonempty $\kappa-$open sets such that $A\cap B\in\mathcal{H}$. Since $\mathcal{H}$ is $\kappa-$codense, $A\cap B=\emptyset$. by hypothesis,$B=\emptyset$. But B is a nonempty $_{\kappa}-$open set which is a contradiction. Hence $A\cap B\notin\mathcal{H}$. Therefore (X,κ) is an $\mathcal{H}-$hyperconnected space.

The following Theorem 2.9 gives another characterization of $\mathcal{H}-$ hyperconnected spaces.

Theorem 2.9 Let (X,κ) be an generalized topological space with a hereditary class $\mathcal{H}$. Then the following are equivalent.

(a) (X, κ) is $\mathcal{H}-$ hyperconnected space.
(b) For every nonempty strong $\beta-\mathcal{H}-$ open sets U and V of X, $U \cap V \notin \mathcal{H}$.

Proof. (a) $\Rightarrow$ (b).Let U and V be non empty strong $\beta-\mathcal{H}-$ open subsets of X. Since (X, κ) is $\mathcal{H}-$hyperconnected, $cl_\kappa^\star(U)$ and $cl_\kappa^\star(V)$ contains a nonempty $\kappa-$open sets U' and V'. Then $U' \subset cl^\star(U), V' \subset cl_\kappa^\star(V)$. Now $X = U'^\star \subset (cl_\kappa^\star(U))^\star = U^\star$ which implies that $X \subset U^\star$ which implies that $U^\star = X$. Hence U is $\mathcal{H}-$ dense which implies that $U \cap V \notin \mathcal{H}$.
(b) $\Rightarrow$ (a). Suppose that a nonempty strong $\beta-\mathcal{H}-$ open set U is not $\mathcal{H}-$ dense. Then $U^\star \neq X$. Let $x \in X$ and $x \notin U^\star$. Then there exists an $\kappa-$open set V such that $U \cap V \in \mathcal{H}$ which is a contradiction. Hence U is $\mathcal{H}-$ dense. Thus (X, κ) is an $\mathcal{H}-$ hyperconnected space.
Recall that a generalized topological space (X, κ) with a hereditary class $\mathcal{H}$ is a $T_{\mathcal{H}}-$space if every singleton set is a $\kappa-$closed set. The following Theorem 2.10 gives a characterization of $T_{\mathcal{H}}-$ spaces.

Theorem 2.10 Let (X, κ) be an $\mathcal{H}-$ hyperconnected and an $\mathcal{H}-$ submaximal space. Then $X-\{x\}$ is κ^*- dense for every $x \in X$ if and only if (X, κ) is a $T_{\mathcal{H}}-$ space.

Proof. Let $X-\{x\}$ be κ^*- dense for every $x \in X$. Since (X, κ) is an $\mathcal{H}-$ submaximal space, $X-\{x\}$ is $\kappa-$open. Therefore $\{x\}$ is $\kappa-$closed for every $x \in X$. Hence (X, κ) is a $T_{\mathcal{H}}-$ space. Conversely, suppose that (X, κ) is a $T_{\mathcal{H}}-$ space. Then $\{x\}$ is $\kappa-$closed for every $x \in X$. Hence $X-\{x\}$ is $\kappa-$open which implies that $X-\{x\}$ is κ^*- dense, since (X, κ) is an $\mathcal{H}-$ hyperconnected space.

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