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Topsis Application to Fuzzy Game Problem

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ABSTRACT

Multicriteria decision making problems (MCDM) is a problem about how to find the best optimal solution from all the feasible alternatives on the basis of two or more attributes. In many real life MCDM problems, the data are often vague. If the uncertainties occur in 12 different points, the dodecagonal fuzzy numbers are used. In this paper, two person zero sum game has been considered with imprecise values in payoff matrix. All the imprecise values are assumed to be dodecagonal fuzzy numbers. The fuzzy payoffs are converted to crisp values using ranking methods based on "centroid of centroids" method. There are many methods to solve MCDM problems. Out of these approaches "TOPSIS" method has gained popularity in the field of MCDM because of its simplicity and practicality. The importance is expressed by attributing weight to each criterion. Nowadays purchasing of an automobile, especially cars in the market is very tough task to the customers due to day to day changes in various technical and operational parameter specifications. To overcome this confusion, "TOPSIS" procedure is one of the best selection procedure which is adapted to solve two person zero sum game.

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Keywords: Two person games, ranking of fuzzy number, positive and negative ideal solution, relative closeness.

1. INTRODUCTION

Two person zero sum game is one of the basic problems in game theory. John Von Newman's paper published in 1928 laid the foundation for the theory of two person zero game. The two person zero sum games are games with only two players where a gain of one player equals a loss to the other. The result can be represented in the form of a matrix called the payoff matrix of a game. The two players are referred as player I and player II. Player I (whose strategies are represented by rows) selects the strategy which maximizes the minimum gain; the minimum is taken over all the strategies of player II. Player II selects his strategy which minimizes his maximum losses.

TOPSIS is a multicriteria decision analysis method which was originally developed by Hwang and Yoon in 1981, with further development by Yoon in 1987 and Hwang, Lai & Liu in 1993. It is a method of compensatory aggregation that compare a set of alternatives by identifying weights for each criterion, normalizing scores for each criterion and calculating the distance between each alternative and the ideal alternative which is the best score in each criterion. TOPSIS allows tradeoff between criteria where a poor result in one criterion can be negated by a good result in another criterion. In this paper we have considered 2 competitors (called players) in the field of car manufacturers, Player I and Player II. These 2 players have manufactured cars with many criteria as like i) Features ii) Styling iii) Colour iv) Capacity v) Technology vi) Fuel economy vii) Safety viii) Pricing etc. But we have selected only 3 important criteria for our convenience say i) Safety ii) Fuel economy iii) Specification.

Various attempts have been made in the literature to study fuzzy game theory. L. A. Zadeh[6] introduced the concept of fuzzy sets. Jatinder Pal Singh *et al*.[7] introduced dodecagonal fuzzy numbers. Car selection using TOPSIS method was proposed by Srikrishna *et al*. in[2]. Lavanya *et al*.[1] used topsis method to solve a 2 person zero sum game using octagonal fuzzy numbers.

In this paper we have used TOPSIS method to determine the closeness distance of the strategies for player I and Player II and observed that player I is identified as the best among these two players which has the best relative closeness value from the positive ideal solution.

2. PRELIMINARIES

Definition 2.1: A Fuzzy set $\tilde{A}$ in X (Set of real numbers) is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$ $\mu_{\tilde{A}}(x)$ is called membership function of x in $\tilde{A}$ which maps X to [0, 1].

Definition 2.2: A fuzzy set $\tilde{A}$ is defined on the set of real numbers R is said to be a fuzzy number of its membership function $\mu_{\tilde{A}} : R \rightarrow [0, 1]$ has the following characteristics.

i) $\tilde{A}$ is normal. It means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$.

ii) $\tilde{A}$ is convex. It means that for every $x_1, x_2 \in R$,

$\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda) x_2] \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0, 1]$

iii) $\mu_{\tilde{A}}$ is upper semi-continuous

iv) Supp $(\tilde{A})$ is bounded in R.

Definition 2.3: The parametric form of Dodecagonal Fuzzy numbers is defined as $\tilde{A}\omega = (f_1(r), g_1(s), h_1(t), h_2(t), g_2(s), f_2(r))$ for $r \in [0, k_1]$ and $s \in [k_1, k_2]$ and $t \in [k_2, \omega]$ where $f_1(r)$, $g_1(s)$ and $h_1(t)$ are bounded left continuous non decreasing functions over $[0, \omega_1]$ $[k_1, \omega_2]$ and $[k_2, \omega_3]$ respectively, $f_2(r)$, $g_2(s)$ and $h_2(t)$ are bounded left continuous non increasing function over $[0, \omega_1]$, $[k_1, \omega_2]$ and $[k_2, \omega_3]$ respectively and $0 \leq \omega_1 \leq k_1$, $k_1 \leq \omega_2 \leq k_2$ and $k_2 \leq \omega_3 \leq \omega$.

Here $\tilde{A}\omega$ represents a fuzzy number in which "ω" is the maximum membership value that a fuzzy number takes on. Whenever a normal fuzzy number is meant, the fuzzy number is shown by $\tilde{A}$, for convenience. The Dodecagonal fuzzy number becomes normal Dodecagonal fuzzy number if ω = 1.

Definition 2.4: The membership function for the Dodecagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ is defined as follows.

$$\mu_{\tilde{D}}(x) = \begin{cases} 0 & x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k_1 & a_2 \le x \le a_3 \\ k_1 + (k_2 - k_1)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ k_2 & a_4 \le x \le a_5 \\ k_2 + (1 - k_2)\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ 1 & a_6 \le x \le a_7 \\ k_2 + (1 - k_2)\left(\frac{a_8-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ k_2 & a_8 \le x \le a_9 \\ k_1 + (k_2 - k_1)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ k_1 & a_{10} \le x \le a_{11} \\ k_1\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \le x \le a_{12} \\ 0 & a_{12} \le x \quad \text{where } 0 < k_1 < k_2 < 1 \end{cases}$$

Remark 2.1: If $0 < k_1 = k_2 < 1$, the dodecagonal fuzzy number reduces to the octagonal fuzzy number $(a_1, a_2, a_5, a_6, a_7, a_8, a_{11}, a_{12})$ and if $k_1 = k_2 = 1$, it reduces to the trapezoidal fuzzy number $(a_1, a_2, a_{11}, a_{12})$.

Remark 2.2: The collection of all dodecagonal fuzzy real numbers from R to I is denoted as $R_\omega(I)$ and if ω=1, then the collection of normal dodecagonal fuzzy numbers is denoted by R(I).

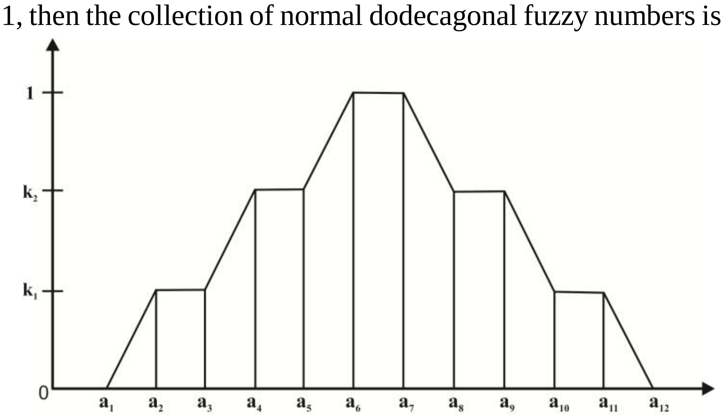


Figure – 1: Graphical representation of Dodecagonal Fuzzy Number

Definition 2.5: For $\alpha \in [0, 1]$ the α -cut of dodecagonal fuzzy numbers, $\tilde{A}$= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}) is defined as follows.

$$[\tilde{A}]_\alpha = \begin{cases} a_1 + \left(\frac{\alpha}{k_1}\right)(a_2 - a_1), a_{12} - \left(\frac{\alpha}{k_1}\right)(a_{12} - a_{11}) & \text{for } \alpha \in [0, k_1] \\ a_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3), a_{10} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{10} - a_9) & \text{for } \alpha \in (k_1, k_2] \\ a_5 + \left(\frac{\alpha - k_2}{1 - k_2}\right)(a_6 - a_5), a_8 - \left(\frac{\alpha - k_2}{1 - k_2}\right)(a_8 - a_7) & \text{for } \alpha \in (k_2, 1] \end{cases}$$

Remark 2.3: The α-cuts of dodecagonal fuzzy number are convex sets and so the dodecagonal fuzzy number is convex.

3. CENTROID OF CENTROIDS – RANKING METHOD

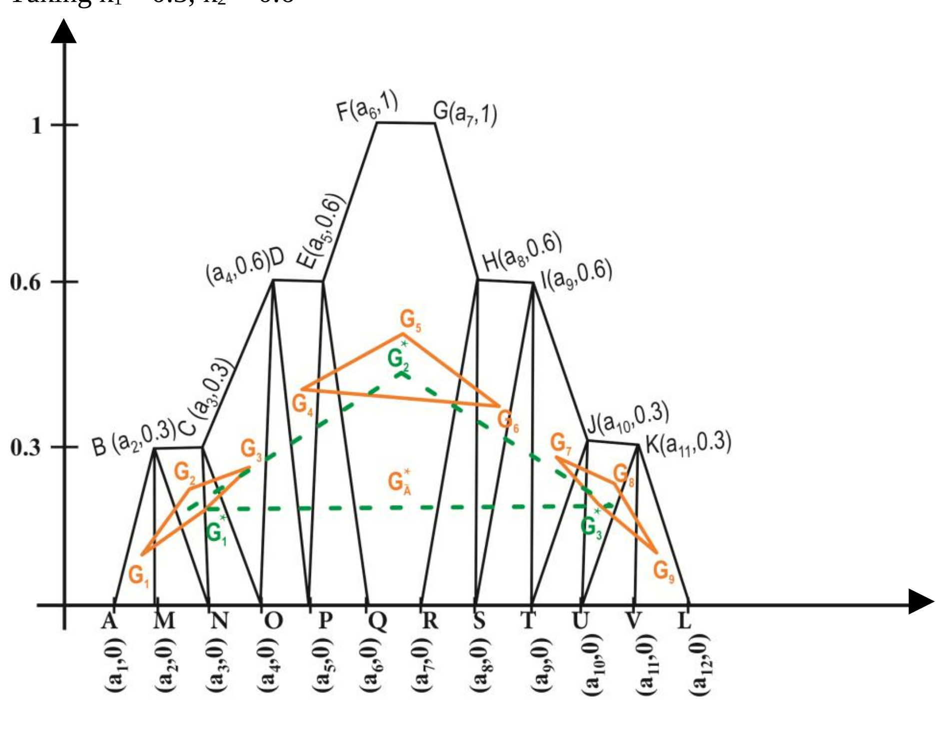
The ranking function of the dodecagonal fuzzy numbers $\tilde{A}$= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}) which maps the set of all fuzzy numbers to set of all real numbers is defined as

$R(\tilde{A}) = [2(a_1+a_6+a_7+a_{12}) + 6(a_2+a_3+a_4+a_9+a_{10}+a_{11}+5\,(a_5 + a_8)] \text{ X } 25/4374$

$= 0.0057\,[2(a_1+a_6+a_7+a_{12}) + 6(a_2+a_3+a_4+a_9+a_{10}+a_{11}) + 5\,(a_5 + a_8)].$

3.1 Grapical Representation of Centroid of Centroids

Taking $k_1 = 0.3$, $k_2 = 0.6$



PROCEDURE TO FIND THE RANKING FUNCTION R($\tilde{A}$)

Step 1: First find the centroids of the eight triangles and one hexagon say Δ^{le} ABM (G_1), Δ^{le} BNC (G_2), Δ^{le} COE (G_3), Δ^{le} DPE (G_4), hexagon EFGHRQ (G_5), Δ^{le} HSI (G_6), Δ^{le} ITJ (G_7), Δ^{le} JUK (G_8), Δ^{le} KVL (G_9).

Step 2: Then find the centroid of the 3 triangles Δ^{le} $G_1G_2G_3$, Δ^{le} $G_4G_5G_6$ and Δ^{le} $G_7G_8G_9$. It is represented as G_1^*, G_2^* ,G_3^* respectively where $G_1^* = (G_1+G_2+G_3)/3$, $G_2^* = (G_4+G_5+G_6)/3$, $G_3^*= (G_7+G_8+G_9)/3$,

Step 3: Join these three 3 centroids to get a new triangle $G_1^*G_2^*G_3^*$. Again find the centroid of this Δ^{le} say $G_{\tilde{A}}^*$, where $G_{\tilde{A}}^*= (G_1^*+ G_2^* + G_3^*)/3$.

Step 4: The ranking function say R($\tilde{A}$) is determined by the formula

$$R(\tilde{A}) = [2(a_1+a_6+a_7+a_{12}) + 6(a_2+a_3+a_4+a_9 +a_{10} +a_{11})+5 (a_5 + a_8)] \times 25/4374.$$

4. TOPSIS METHOD

Consider a fuzzy game problem whose payoff matrix is expressed as follows:

$$\mathcal{M} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13}\cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23}\cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \vdots \;\cdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{m3}\cdots & \tilde{a}_{mn} \end{bmatrix}_{m\times n}$$

where each $\tilde{a}_{ij}$ is a fuzzy number that may be triangular, trapezoidal or octagonal.

Step 1: Calculate the Normalized Payoff matrix. The normalized value $\tilde{n}_{ij}$ is calculated as $\tilde{n}_{ij} = \frac{\tilde{a}_{ij}}{\sqrt{\sum_{i=1}^{m}(s(\tilde{a}_{ij},0))^2}}$, $j = 1,2,\ldots n$ where $s(\tilde{a}_{ij},0)$ is the distance of the fuzzy number from the crisp number 0.

Step 2: Calculate the Weighted Normalized pay off matrix$[\tilde{v}_{ij}]$.
If w is a crisp value, $\tilde{v}_{ij} = w_j \times \tilde{n}_{ij}, i = 1,2,\ldots,m, j = 1,2,\ldots,n$ Where w_j is the weight of the strategies of players and $\sum_{j=1}^{n} w_j = 1$.
If w is a fuzzy value, $\tilde{v}_{ij} = s(\tilde{w}_j,0) \times \tilde{n}_{ij}, i = 1,2,\ldots,m, j = 1,2,\ldots,n$.

Step 3: Find the crisp value matrix A corresponding to $\tilde{A}$ using $S(\tilde{v}_{ij},0)$.
Determine the Positive Ideal and Negative Ideal Solutions of Player I and Player II.

Step 4: The Positive Ideal and Negative Ideal Solutions of Player I respectively are
$A_I^+ = \{v_1^+, v_2^+ \ldots v_m^+\}$ where $v_i^+ = \{Max_j\{S(\tilde{v}_{ij},0)\}, i = 1,2,\ldots m$
$A_I^- = \{v_1^-, v_2^- \ldots v_m^-\}$ where $v_i^- = \{Min_j\{S(\tilde{v}_{ij},0), i = 1,2,..m\}$
The Positive Ideal and Negative Ideal Solutions of Player II respectively are
$A_{II}^+ = \{v'_1{}^+, v'_2{}^+ \ldots v'_n{}^+\}$ where $v'_j{}^+ = \{Max_i\{S(\tilde{v}_{ij},0), j = 1,2,..n\}$

$A_{II}^{-} = \{v'_1{}^{-}, v'_2{}^{-} \dots v'_n{}^{-}\}$where $v'_j{}^{-} = \{Min_i\{S(\tilde{v}_{ij}, 0), j = 1,2, .. n\}$

Step 5: Calculate the Separation Measures using Euclidean distance.
The Separation of each strategy of player I from the Positive-ideal solution is

$$\boldsymbol{d^{+}_{i}} = \sqrt{\sum_{j=1}^{n} (v_i{}^{+} - s(\tilde{v}_{ij}, 0))^2}, i = 1, 2, \dots, m$$

The Separation of each strategy of player I from the Negative -ideal solution is

$$\boldsymbol{d^{-}_{i}} = \sqrt{\sum_{j=1}^{n} (v_i{}^{-} - s(\tilde{v}_{ij}, 0))^2}, i = 1,2, \dots, m$$

The Separation of each strategy of player II from the Positive-ideal solution is

$$\boldsymbol{d^{+}_{j}} = \sqrt{\sum_{i=1}^{m} (v_j{}^{+} - s(\tilde{v}_{ij}, 0))^2}, j = 1, 2, \dots, n$$

The Separation of each strategy of player II from the Negative -ideal solution is

$$\boldsymbol{d^{-}_{j}} = \sqrt{\sum_{i=1}^{m} (v_j{}^{-} - s(\tilde{v}_{ij}, 0))^2}, j = 1,2, \dots, n$$

Step 6: Calculate the relative closeness to ideal solution.
The relative closeness distance of the strategies A_i with respect to maximum gain is defined as

$$cl^{+}_{i} = \frac{d^{+}_{i}}{d^{+}_{i} + d^{-}_{i}}, i = 1,2, \dots, m$$

The relative closeness distance of the strategiesB_j with respect to minimum loss^{-}is defined as

$$cl^{-}_{j} = \frac{d^{-}_{j}}{d^{+}_{j} + d^{-}_{j}}, j = 1,2, \dots, n$$

Step 7: The strategy for which the closeness distance is least will be the best strategy.

4.1 NUMERICAL EXAMPLE

Consider two car manufacturers **Player I and Player II**. Both players have manufactured cars with 3 important criteria namely i) Safety ii) Fuel economy iii) Technology. The following pay off matrix gives gain for player I when they choose their different criteria along with the player II.

PAY OFF MATRIIX WITH PAYOFFS AS DODECAGONAL FUZZY NUMBERS

Player II

Player I			
	(2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 15, 16)	(-5, -4, -3, -2, -1, 0, 2, 3, 4, 5, 6, 7)	(-2, -1, 0,1, 2, 3, 5, 6, 7, 8, 9, 10)
	(-6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6)	(0, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12)	(-3, -2, -1, 0, 1, 2, 4, 5, 6, 7, 8, 9)
	(-2, -1, 0, 1, 3, 4, 6, 7, 9, 10, 11, 12)	(-5, -4, -3, -1, 0, 1, 3, 4, 5, 7, 8, 9)	(2, 3,4, 5, 6, 7,9,10,11,12, 13, 14)

……(∗)

In this problem, the three types of criteria are not given equal importance. The weights assigned to them are i) Safety - 0.5
ii) Fuel economy – 0.4
iii) Technology – 0.1

Step 1: The normalized decision matrix is determined by using the formula $\tilde{n}_{ij} = \frac{\tilde{a}_{ij}}{\sqrt{\sum_{i=1}^{m}(s(\tilde{a}_{ij},0))^2}}$, $j = 1,2,\dots n$ where $S(\widetilde{a_{ij}},0) = [2(a_1+a_6+a_7+a_{12}) + 6(a_2+a_3+a_4+a_9+a_{10}+a_{11}) + 5(a_5+a_8)] \times 25/4374$.

Player II

Player I			
	(0.6,0.9,1.6,1.9,2.2,2.5,3.2,3.5,3.8,4.1,4.7,5)	(-2.4,-1.9,-1.4,-0.9,-0.5,0,0.9,1.4,1.9,2.4,2.9,3.4)	(-0.7,-0.3, 0, 0.3, 0.7, 1, 1.7,2, 2.4,2.7, 3.1, 3.4)
	(-1.9,-1.6,-1.3, -0.9,-0.6, -0.3, 0, 0.3,0.6, 0.9, 1.3,1.6)	(0,0.5, 1,1.5, 2, 2.5,3.5, 4, 4.5, 5,5.5,6)	(-1.0,-0.7,-0.3,0,0.3,0.7,1.4,1.7 , 2.1, 2. 4, 2.7, 3.1)
	(-0.6,-0.3,0, 0.3, 0.9, 1.3, 1.9, 2. 2, 2.8, 3.2, 3.5, 3.8)	(-2.5,-2, -1.5, -0.5, 0, 0.5,1.5, 2,2.5, 3.5,4, 4. 5)	(0.7, 1,1.4,1.7,2.1,2.4,3.1,3.4, 3.8, 4.1, 4.5, 4.8)

Step 2: Weighted Normalized pay off matrix with w = (0.5, 0.4, 0.1) is

(0.3,0.45,0.8,0.95,1.1,1.25,1.6,1.75,1.9,2.05,2.35,2.5)	(-0.96,-0.76,-0.56,-0.36,-0.2,0,0.36,0.56, 0.76,0.96,1.16,1.36)	(-0.07, -0.03, 0, 0.03, 0.07, 0.1,0.17,0. 2,0. 24, 0.27, 0.31, 0.34)
(-0.95, -0.8, -0.65, -0.45, -0.3, -0.15, 0, 0.15,0.3,0.45, 0.65,0.8)	(0,0.2, 0.4, 0.6,0.8, 1,1.4, 1.6,1.8, 2, 2.2, 2.4)	(-0.1, -0.07, -0.03, 0, 0.03, 0.07, 0.14, 0.17 , 0.21,0. 24,0. 27,0. 31)
(-0.3, -0.15, 0, 0.15, 0.45, 0.65, 0.95, 1.1, 1.4, 1.6, 1.75, 1.9)	(-1, -0.8, -0.6, -0.2, 0, 0.2, 0.6, 0.8, 1, 1.4, 1.6, 1.8)	(0.07, 0.1, 0.14, 0.17, 0.21, 0.24, 0.31, 0.34, 0.38 ,0.41,0.45, 0.48)

Step 3: Resultant crisp value matrix is

Player II

$$\text{Player I} \begin{pmatrix} 0.44 & 0.06 & 0.04 \\ -0.03 & 0.36 & 0.03 \\ 0.24 & 0.12 & 0.08 \end{pmatrix}$$

Positive Ideal of Player I $A_I^+ = (v_1^+, v_2^+, v_3^+)$ = (0.44, 0.36, 0.24)

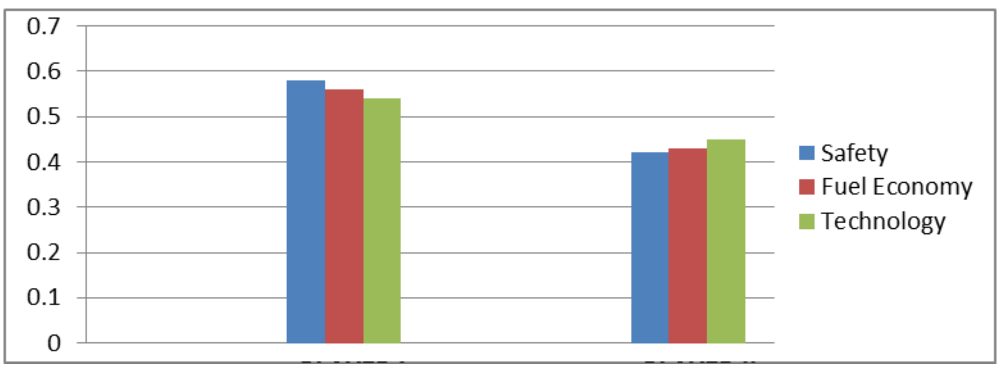
Positive Ideal of Player II $A_{II}^{+} = (v_1'^{+},\ v_2'^{+},\ v_3'^{+}) = (0.44, 0.36, 0.08)$
Negative Ideal of Player I $A_I^{-} = (v_1^{-}, v_2^{-}, v_3^{-}) = (0.04, -0.03, 0.08)$
Negative Ideal of Player II $A_{II}^{-} = (v_1'^{-}, v_2'^{-}, v_3'^{-}) = (-0.03, 0.06, 0.03)$
Calculate the Separation measures for Player I and Player II

PLAYER I		PLAYER II	
d_1^+ = 0.55173	d_1^- = 0.4005	d_1^+ = 0.5108	d_1^- = 0.5420
d_2^+ = 0.51088	d_2^- = 0.3946	d_2^+ = 0.3842	d_2^- = 0.3059
d_3^+ = 0.2	d_3^- = 0.1649	d_3^+ = 0.0640	d_3^- = 0.0509

Calculate the relative closeness distance of strategies with maximum gain for Player I and minimum loss of player II

Strategies of player I	Relative closeness distance of strategies with maximum gain for Player I	Strategies of player II	Relative closeness distance of strategies with minimum loss for Player II
A_1	$cl^+{}_1$ = 0.5794	B_1	$cl^-{}_1$ = 0.4206
A_2	$cl^+{}_2$ = 0.5642	B_2	$cl^-{}_2$ = 0.4358
A_3	$cl^+{}_3$ = 0.5481	B_3	$cl^-{}_3$ = 0.4519

Since the closeness coefficient for A_3 and B_1 are least, the best strategies of player I and II are A_3 and B_1 respectively.



5. CONCLUSION

In this chapter we have considered two person zero sum game with pay offs as dodecagonal Fuzzy Numbers. The fuzzy payoffs are converted to crisp values using ranking based on "centriod of centroids" method. TOPSIS (Technique for order preference by similarity to ideal solution) procedure is proposed when the relative importance of strategies are not the same that is weights are assigned to the strategies. We explain the proposed method through a numerical example for Dodecagonal fuzzy number and the weights for strategies are taken as 0.5, 0.4 and 0.1. From the table value and the Bar diagram the closeness distance of the strategies for player I is higher than player II from the positive ideal solution. It is observed that player I is identified as the best car among these two players which has the best relative closeness value.

REFERENCES

1. S. Lavanya, K. Selva Kumara, Topsis Method for Solving Fuzzy Game Problem using Octagonal Fuzzy Numbers, *International Journal of Emerging Technology & Advanced Engneering*, 7, 10 (2017).
2. Sri Krishna, S. Sreenivasulu Reddy, A. Vani, New Car Selection in the Market using TOPSIS Technique, *International Journal of Engineering Research and General Science*, 2, 4, 177-181 (2014).
3. M. Clement Joe Anand, A. Victor Devadoss, Application of Topsis Method to Analyze Causes of Suicide Thought in Domestic Violence, *International Journal of Computing Algorithm*, 02, 354-362 (October 2013).
4. Mahmut Yavuz, *Earth and Environmental Science*, 44, 042040 (2016).
5. George J. Klir, Bo yuan, Fuzzy Sets and Fuzzy Logic, Theory and Applications, Prentice Hall of India (2010).
6. Zadeh L.A., Fuzzy Sets, *Information and Control*, 8, 338-353 (1965).
7. Jatinder Pal Singh, Neha Ishesh Takur, An Approach for Solving Fuzzy Transportation Problem using Dodecagonal Fuzzy Number. *International Journal of Mathematical Archive*. 6(4), 105–112 (2015).
8. Jatinder Pal Singh, Neha Ishesh Takur: Ranking of Generalized Dodecagonal fuzzy Numbers using Centroid of Centroids, *Annals of pure and Applied Mathematics*. 10(2), 191–198 (2015).
9. Kauffmaan, A., Introduction to Theory of Fuzzy Subsets, Accademic Press, New York, Vol. I (1975).