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Connected Two Out Degree Equitable Domination Number of Semi Total-point Graph

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ABSTRACT

Let G= (V, E) be a simple graph. Let D A dominating set in a graph G is called connected two out degree equitable dominating set if for any two vertices u,v∈ D, $|od_D(u) - od_D(v)| \leq 2,$ and the induced sub graph <D> is connected The minimum cardinality of a connected two-out degree equitable dominating set is called connected two-out degree equitable domination number, and is denoted by γ_{nc2oe}(G). In this paper connected two-out degree equitable domination number for semi total-point graph.

AMS Classification: 05C9.

Keywords: two-out degree, equitable, Domination number, semi total-point graph.

INTRODUCTION

By a graph G=(V,E). we mean a finite, undirected with neither loops nor multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Chartand and Lenisk[4]. Let G=(V,E) be a graph For any vertex v∈ V then open neighborhood of v is the set N(v)= {u∈ V; uv ∈ $E(G)$} and closed neighborhood of v is the set N[v]= N(v) ∪ v. A set D⊆ V of vertices in a graph G= (V,E) is a dominating set if For every vertex v ∈ $V - D$, there exists a vertex u ∈ Dsuch that v is adjacent to u. . The minimum cardinality of dominating set is the domination number is denoted by $\gamma(G)$. Sampath Kumar and Waliker[3] introduced the concept of connected domination in graph. A dominating set D of G is called a connected dominating set if the induced sub graph <D> is connected. The out degree of v with respect to D denoted by $od_D(v)$, is defined as $od_D(v) = |N(\text{v}) \cap (V - D)|$. Ali Sahal and Veena Mathad[2] are define two out degree equitable dominating set as a dominating set D in a graph G is called a two-out degree equitable dominating set if for any two vertices u,v∈ D, $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of a two-out degree equitable dominating set is called the two-out degree equitable domination number of G, and is denoted by γ_{2oe}(G). M.S.Mahesh and P.Namasivayam[5] are define connected two out degree equitable domination number.

2. CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION NUMBER

Definition: 2.1

Let G= (V,E) be a connected graph. A non empty set D of G is called connected two-out degree equitable dominating set if D is dominating set, then for any $u, v \in D, |od_D(u) - od_D(v)| \leq 2$ the and induced sub graph <D> is connected . The minimum cardinality of a connected two-out degree equitable domination number of G and is denoted by γ_{c2oe}(G).

Example: 2.2

From the below graph, we can find connected two out degree equitable domination number.

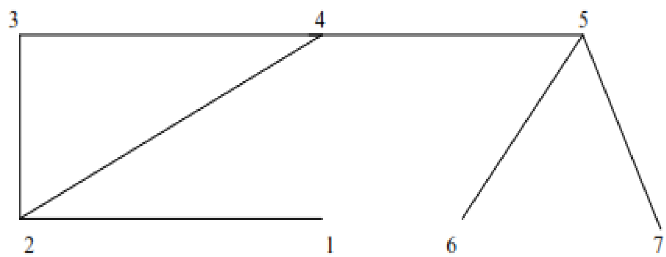


Fig.1

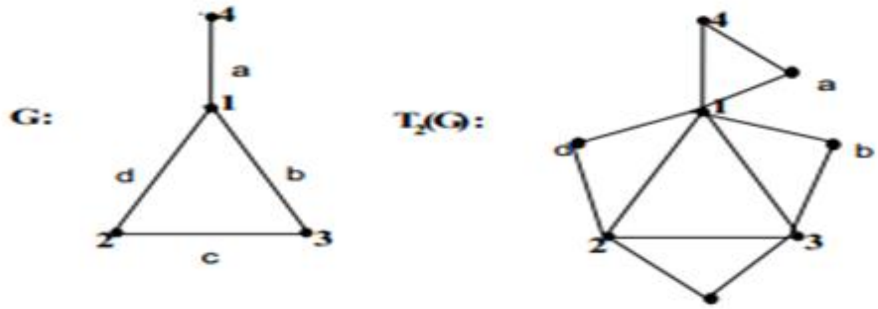
From the above {2, 3, 5}and {2,3,4,5} is connected two-out degree equitable dominating set and {2,3,5} is connected two-out degree equitable dominating set with minimum cardinality so $\gamma_{c2oe}(G)$ =3.

3. CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION NUMBER OF SEMI TOTAL POINT GRAPH

Definition:3.1

For any graph G=(V,E), the semi total point graph $T_2(G)$=H is the graph whose vertex set is the union of vertices and edges in which two vertices are adjacent if and only if they are adjacent vertices of G or one is a vertex and other is an edge of G incident with it. The concept of semi total point graph was introduced in[5]. B. Basavanagoud and S. H. Hosaman introduced the connected domination number in semi total point graph and obtained. Here the connected two out degree equitable domination number in semi total point graph and obtain some results in terms of element of G.

Example : 3.2



In the above graph V(G)=p=4 and E(G)=q=4
In $T_2(G)$, $V(T_2(G)) = p + q$ and $E(T_2(G)) = 3q$

The minimal connected two out degree equitable dominating set in G is $\{v_1, v_2\}$. There fore $\gamma_{c2oe}(G)$ =2.The minimal connected two out degree equitable dominating set semi total-point graph $T_2(G)$ is $\{v_1, v_2, v_3\}$. There fore $\gamma_{c2oe}(T_2(G))$ =3

Observation: 3.3

For any connected graph G, $\gamma_{c2oe}(G) \leq \gamma_{c2oe}(T_2(G))$
By the above example, $\gamma_{c2oe}(G) = 2$ and $\gamma_{c2oe}(T_2(G)) = 3$

Observation: 3.4

Let H be a sub graph of G then $\gamma_{c2oe}(T_2(H)) \leq \gamma_{c2oe}(T_2(G))$
We first calculate $\gamma_{c2oe}(G)$ for semitotal-point graph of some standard class graphs

Theorem: 3.5

For any cycle C_p; p≥ 3, $\gamma_{c2oe}\left(T_2(C_p)\right) = p - 1$

Proof:

Let V$(C_p) = \{v_1, v_2, - - - - - v_p\}$ and E$(C_p) = \{e_1, e_2, - - - - - e_p\}$
We know that $\gamma_c\left(T_2(C_p)\right) = p - 1$
Let us take D= $\{v_1, v_2, - - - - - v_{p-1}\}$ be minimal connected dominating set in $T_2(C_p)$
Clearly <D> is a path and $od_D(v_i) = 3$ for $i = 1, p - 1$ and $od_D(v_i) = 2$ for $i \neq 1, p - 1$
Clearly $|od_D(v_i) - od_D(v_j)| \leq 2$ for all v_i, v_j
Then D is a minimal connected two out degree equitable dominating set in $T_2(C_p)$.
Then $\gamma_{c2oe}\left(T_2(C_p)\right) = p - 1$

Theorem: 3.6

For any complete graph K_p; p≥ 3, $\gamma_{c2oe}\left(T_2(K_p)\right) = p - 1$

Proof :

Let V$(K_p) = \{v_1, v_2, - - - - - v_p\}$ and E$(K_p) = \{e_1, e_2, - - - - - e_p\}$

We know that $\gamma_c\left(T_2(K_p)\right) = p - 1$

Let us take D= $\{v_1, v_2, - - - - - v_{p-1}\}$ be minimal connected dominating set in $T_2(K_p)$

Clearly <D> is a path and $od_D(v_i) = p, for\ i = 1, p - 1$

So $\left|od_D(v_i) - od_D(v_j)\right| \leq |p - p| = 0\ \ for\ all\ v_i, v_j$

$\left|od_D(v_i) - od_D(v_j)\right| \leq 2$

Then D is a minimal connected two out degree equitable dominating set in $T_2(K_p)$.

Then $\gamma_{c2oe}\left(T_2(K_p)\right) = p - 1$

Theorem: **3.7**

For any star $K_{1,p}$: p≥ 1 , $\gamma_{c2oe}\left(T_2(K_{1,p})\right) = 2p - 1$

Proof:

Let $\{v, v_1, v_2, v_3 - - - - - - - - v_p\}$ be a vertex set of $K_{1,p}$ and $\{e_1, e_2, e_3 - - - - - - - - e_{p-1}\}$ be a edge set of $K_{1,p}$

$V\left(T_2(K_{1,p})\right) = \{v, v_1, v_2, v_3 - - - - - - - - v_{p,} e_1, e_2, e_3 - - - - - - - - e_p\}$be the vertices of $T_2(K_{1,p})$

Let us consider D = $\{v_1, v_2, v_3 - - - - - - - - v_{p,} e_1, e_2, e_3 - - - - - - - - e_{p-2}\}$

Clearly D is connected dominating set Then $od_D(v) = 2$

If $e_p \in V - D$ is an edge between v and $v_i \in D$ then $od_D(v_i) = 1$

If $e_p \in V - D$ is an not edge between v and $v_p \in D$ then $od_D(v_p) = 0$

There for any $u, v \in D,\ |od_D(u) - od_D(v)| \leq 2$

Hence $\gamma_{c2oe}\left(T_2(K_{1,p})\right) = 2p - 1$ for any p≥ 1

Theorem: 3.8

For any path $P_p; p \geq 2, \gamma_{c2oe}\left(T_2(P_p)\right) = p - 1$

Proof:

Let $\{v, v_1, v_2, v_3 - - - - - - - - v_p\}$ be a vertex set of P_p and $\{e_1, e_2, e_3 - - - - - - - - e_{p-1}\}$ be a edge set of P_p

$V\left(T_2(P_p)\right) = \{v_1, v_2, v_3 - - - - - - - - v_{p,} e_1, e_2, e_3 - - - - - - - - e_{p-1}\}$be the vertices of $T_2(P_p)$

Let us consider D = $\{v_1, v_2, v_3 - - - - - - - - v_{p-1}\}$ and $V - D = \{v_P, e_1, e_2 - - - - - - - e_{p-1}\}$
Clearly D is minimal connected dominating set
Now $od_D(v_1) = |N(v_1) \cap D| = 1$
$od_D(v_I) = |N(v_I) \cap V - D| = 2$ for all i such that $2 \leq i \leq p - 2$
$od_D(v_{p-1}) = |N(v_{p-1}) \cap V - D| = 3$
Hence for any $v_i, v_j \in D$, $|od_D(u) - od_D(v)| \leq 2$
So D is minimal connected two out degree equitable dominating set
Hence $\gamma_{c2oe}\left(T_2(P_p)\right) = p - 1$ for $p \geq 3$

Theorem: 3.9

For any connected graph G, $\gamma_{c2oe}(T_2(G)) \leq p - 1$. The equality holds for G$\cong P_p$or K_p or C_p

Proof:

Let G be a connected (p,q) graph
Let $\{v_1, v_2, - - - - - - - v_p\}$ and $\{e_1, e_2, - - - - - - - e_p\}$ be the set of vertices and edges of G.
Let $\{v'_{1,} v'_{2,} - - - - - - - - - - - v'_{p,}\}$ and $\{e'_{1,} e'_{2,} - - - - - - - - - - - e'_{p,}\}$ be the corresponding point vertices and line vertices in $T_2(G)$
Each line vertex $e'_{i,}$; $1 \leq i \leq q$ will form a cycle in $T_2(G)$
Since G is connected therefore G must have a spanning tree T,
By the definition G$\subset$ $T_2(G)$ and therefore T is also a subset of $T_2(G)$
Then we have $\gamma_{c2oe}(T_2(G)) \leq p - 1$
Equality follows from the above theorem

Corollary: 3.10

For any connected graph G, $\gamma_{c2oe}(T_2(G)) \leq \gamma_{c2oe}(G) + 1$. The equality holds for G $is\ ismorphic\ P_p$or K_p or C_p

Proof:

We know that $\gamma_{c2oe}(G) \leq p - 2$
By the above theorem $\gamma_{c2oe}(T_2(G)) \leq p - 1 = p - 2 + 1$
Then $\gamma_{c2oe}(T_2(G)) \leq \gamma_{c2oe}(G) + 1$
Equality follows from the above theorem

REFERENCES

1. A. Anitha, S. Arumugam and M. Chellali, Equitable domination in graphs, *Discrete Math. Algorithms Appl*., 3, 311-321 (2011).
2. A. Sahal and V. Mathad "Two-out degree equitable Domination in graphs" *Transactions on Combinatorics*, Vol. 2 No. 3, pp. 13-19 (2013).
3. E. Sampathkumar and H.B. Walikar," The connected domination number of a graph", *J. Math. Phys. Sci*., 13, 607-613 (1979).
4. G. Chartrand and L. Lesniak, Graphs and Digraphs, Chapman and Hall. CRC, 4th edition, (2005).
5. M.S. Mahesh and P.Namasivayam" Connected two out degree equitable domination number in different graphs" *International Journal of Mathematical Archive*: Vol 5 No.1, 67-74 (2014).
6. T. W. Haynes, Stephen T. Hedetniemi and Peter J. Slater," Domination in graphs, Advanced topics," Marcel Dekker, New York. (1998).
7. T.W. Haynes, S.T. Hedetniemi and P.J. Slater, "Fundamentals of Domination in Graphs" Marcel Dekker Inc., (1998).
8. M.S. Mahesh and P. Namasivayam" Connected two out degree equitable domination polynomial of a graph " *International Journal of Mathematical Archive:* Vol 7 No.1 (2016), 110-115, ISSN 2229-5046, Impact Factor: 4.908, IC Value : 5.09
9. M.S. Mahesh and P. Namasivayam" Connected two out degree equitable domination number for trees" *Bulletin of Mathematics and Statistics Research*: Vol 4 No. 3 (2016) ISBN, Impact Factor.4.495
10. M.S. Mahesh and P. Namasivayam "Bounds of connected two out degree equitable domination number." Proceedings of UGC sponsored Two- Day "*National Seminar on Recent Developments in Functional Analysis, Topology and Graph theory*" organized by the PG and research department of mathematics, The M.D.T Hindu college, Tirunelveli 26th & 27th Match (2015), 189-202, ISBN 978-81-931128