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Some Mellin-type Definite Integrals Involving Sine and Cosine Functions

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ABSTRACT

In this paper, we derive some Mellin-type definite integrals associated with Sine and Cosine functions, using Laplace and inverse Laplace transforms technique. Making suitable adjustment of parameters and values of trigonometrical ratios of some typical angles, we evaluate some interesting integrals in terms of Gamma functions and irrational numbers.

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1. INTRODUCTION AND PRELIMINARIES

The following results will be required in our present investigation.

Suppose given function $F(t)$ is well defined for all real values of $t>0$, then Laplace transform of the function $F(t)$ is given by

$$\mathrm{L}\{F(t);\ p\} = \int_0^\infty e^{-pt}\, F(t)\,\mathrm{d}t = f(p) \tag{1.1}$$

$$\left(\mathrm{Re}(p) > 0\right)$$

$$\mathrm{L}^{-1}\{f(p);\,t\} = \frac{1}{2\text{®}\,i}\int_{h-i\infty}^{h+i\infty} e^{pt}\, f(p)\,\mathrm{d}p = F(t) \qquad (1.2)$$

$$\left(\mathrm{Re}(p) > 0 \text{ and } h > 0\right)$$

provided that above integrals (1.1) and (1.2) exist and p is a real or complex number.

$$\mathrm{L}\{t^{\sim -1};\ p\} = \int_0^\infty e^{-pt}\, t^{\sim -1}\,\mathrm{d}t = \frac{\Gamma(\sim)}{p^{\sim}} \qquad (1.3)$$

$$\left(\mathrm{Re}(p) > 0 \text{ and } \mathrm{Re}(\sim) > 0\right)$$

Hankel's Contour Integral

$$\mathrm{L}^{-1}\left\{\frac{1}{p^{\sim}};\, t\right\} = \frac{1}{2\text{®}\,i}\int_{h-i\infty}^{h+i\infty} e^{pt}\, p^{-\sim}\,\mathrm{d}p = \frac{t^{\sim -1}}{\Gamma(\sim)} \qquad (1.4)$$

$$\left(\mathrm{Re}(p) > 0,\ \mathrm{Re}(\sim) > 0 \text{ and } h > 0\right)$$

$$\int_0^\infty \exp\left(\frac{-p}{t}\right) t^{\sim -1}\,\mathrm{d}t = p^{\sim}\,\Gamma(-\sim) \qquad (1.5)$$

$$\left(\mathrm{Re}(p) > 0 \text{ and } \mathrm{Re}(\sim) < 0\right)$$

$$\int_0^\infty (e^{-t} - 1)\, t^{\sim -1}\,\mathrm{d}t = \Gamma(\sim) \qquad (1.6)$$

$$\left(-1 < \mathrm{Re}(\sim) < 0\right)$$

$$\mathrm{L}\{\sin(ct);\ p\} = \int_0^\infty e^{-pt}\sin(ct)\,\mathrm{d}t = \frac{c}{p^2 + c^2} \qquad (1.7)$$

$$\left(\mathrm{Re}(p) > |\,\mathrm{Im}(c)\,|\right)$$

$$\mathrm{L}\{\cos(ct);\ p\} = \int_0^\infty e^{-pt}\cos(ct)\,\mathrm{d}t = \frac{p}{p^2 + c^2} \qquad (1.8)$$

$$\left(\mathrm{Re}(p) > |\,\mathrm{Im}(c)\,|\right)$$

$$\int_0^{\frac{\text{®}}{2}} \sin^M \text{¯}\, \cos^N \text{¯}\ \mathrm{d}\text{¯} = \frac{\Gamma\left(\frac{M+1}{2}\right)\Gamma\left(\frac{N+1}{2}\right)}{2\Gamma\left(\frac{M+N+2}{2}\right)} \qquad (1.9)$$

$$\left(\min\{\mathrm{Re}(M),\ \mathrm{Re}(N)\} > -1\right)$$

Ramanujan stated his Master theorem [1;2; see also 10;11;12] as:
Suppose that, in some neighbourhood of $x = 0$,

$$F(x) = \sum_{n=0}^{\infty} \frac{\Phi(n)\,(-x)^n}{n!} \qquad (1.10)$$

then, Mellin transform of $F(x)$ is given by

$$\mathrm{M}\{F(x): x \to s\} = \int_0^\infty x^{s-1} F(x)\,\mathrm{d}x = \Gamma(s)\,\Phi(-s) \tag{1.11}$$

where s is not necessarily a positive integer and $F(x)$ is called inverse Mellin transform of $\Gamma(s)\,\Phi(-s)$ and it is given by

$$\mathrm{M}^{-1}\{\Gamma(s)\,\Phi(-s): s \to x\} = F(x) = \frac{1}{2\pi\, i}\int_{h-i\infty}^{h+i\infty} x^{-s}\,\Gamma(s)\,\Phi(-s)\,\mathrm{d}s \tag{1.12}$$

$$\left(0 < h < \delta,\ 0 < \delta < 1;\ x > 0\right)$$

provided that above integrals exist.

Motivated by the work on direct Mellin transforms and inverse Mellin transforms scattered in the literature see [3;4;5;6;7;8;9], we derive and evaluate some Mellin type integrals in next sections.

2. MELLIN-TYPE MAIN INTEGRALS

Any value of parameters and variables leading to the results which do not make sense are tacitly excluded, then

$$\int_0^\infty x^{\alpha}\sin(b\,x^{\beta})\,\mathrm{d}x = b^{-\left(\frac{\alpha+1}{\beta}\right)}\frac{1}{\beta}\Gamma\left(\frac{\alpha+1}{\beta}\right)\sin\left\{\pi\left(\frac{\alpha+1}{2\beta}\right)\right\}, \tag{2.1}$$

$$\left(\mathrm{Re}(\beta) > 0;\ b > 0;\ -1 < \mathrm{Re}\left(\frac{\alpha+1}{\beta}\right) < 1\right).$$

$$\int_0^\infty x^{\alpha}\cos(b\,x^{\beta})\,\mathrm{d}x = b^{-\left(\frac{\alpha+1}{\beta}\right)}\frac{1}{\beta}\Gamma\left(\frac{\alpha+1}{\beta}\right)\cos\left\{\pi\left(\frac{\alpha+1}{2\beta}\right)\right\}, \tag{2.2}$$

$$\left(\mathrm{Re}(\beta) > 0;\ b > 0;\ 0 < \mathrm{Re}\left(\frac{\alpha+1}{\beta}\right) < 1\right).$$

$$\int_0^\infty x^{\alpha}\sin\left(\frac{b}{x^{\beta}}\right)\mathrm{d}x = -b^{\left(\frac{\alpha+1}{\beta}\right)}\frac{1}{\beta}\Gamma\left(\frac{-\alpha-1}{\beta}\right)\sin\left\{\pi\left(\frac{\alpha+1}{2\beta}\right)\right\}, \tag{2.3}$$

$$\left(\mathrm{Re}(\beta) > 0;\ b > 0;\ -1 < \mathrm{Re}\left(\frac{\alpha+1}{\beta}\right) < 1\right).$$

$$\int_0^\infty x^{\alpha}\cos\left(\frac{b}{x^{\beta}}\right)\mathrm{d}x = b^{\left(\frac{\alpha+1}{\beta}\right)}\frac{1}{\beta}\Gamma\left(\frac{-\alpha-1}{\beta}\right)\cos\left\{\pi\left(\frac{\alpha+1}{2\beta}\right)\right\}, \tag{2.4}$$

$$\left(\mathrm{Re}(«) > 0;\, b > 0;\, -1 < \mathrm{Re}\left(\frac{ª+1}{«}\right) < 0\right).$$

$$\int_0^\infty x^{ª} \sin(b\,x^{«} + c)\,\mathrm{d}x = b^{-\left(\frac{ª+1}{«}\right)} \frac{1}{«}\Gamma\left(\frac{ª+1}{«}\right)\sin\left\{®\left(\frac{ª+1}{2«}\right) + c\right\}, \qquad (2.5)$$

$$\left(\mathrm{Re}(«) > 0;\, b > 0;\, 0 < \mathrm{Re}\left(\frac{ª+1}{«}\right) < 1\right).$$

$$\int_0^\infty x^{ª} \cos(b\,x^{«} + c)\,\mathrm{d}x = b^{-\left(\frac{ª+1}{«}\right)} \frac{1}{«}\Gamma\left(\frac{ª+1}{«}\right)\cos\left\{®\left(\frac{ª+1}{2«}\right) + c\right\}, \qquad (2.6)$$

$$\left(\mathrm{Re}(«) > 0;\, b > 0;\, 0 < \mathrm{Re}\left(\frac{ª+1}{«}\right) < 1\right).$$

$$\int_0^\infty x^{ª} \sin\left(\frac{b}{x^{«}} + c\right)\mathrm{d}x = -b^{\left(\frac{ª+1}{«}\right)} \frac{1}{«}\Gamma\left(\frac{-ª-1}{«}\right)\sin\left\{®\left(\frac{ª+1}{2«}\right) - c\right\}, \qquad (2.7)$$

$$\left(\mathrm{Re}(«) > 0;\, b > 0;\, -1 < \mathrm{Re}\left(\frac{ª+1}{«}\right) < 0\right).$$

$$\int_0^\infty x^{ª} \cos\left(\frac{b}{x^{«}} + c\right)\mathrm{d}x = b^{\left(\frac{ª+1}{«}\right)} \frac{1}{«}\Gamma\left(\frac{-ª-1}{«}\right)\cos\left\{®\left(\frac{ª+1}{2«}\right) - c\right\}, \qquad (2.8)$$

$$\left(\mathrm{Re}(«) > 0;\, b > 0;\, -1 < \mathrm{Re}\left(\frac{ª+1}{«}\right) < 0\right).$$

3. DERIVATIONS

Suppose $\sim,\ ,¥,¢$ are real numbers and $ª,«$ are complex numbers such that $ª = \sim + i$ and $« = ¥ + i¢$, where $i = \sqrt{-1}.$

To evaluate the integral (2.1), we shall consider the following integral:

$$F(t) = \int_0^\infty x^{ª} \sin(x^{«}\,t)\,\mathrm{d}x. \qquad (3.1)$$

Multiply both sides of equation (3.1) by e^{-pt} and integrate with respect to t over the interval $(0,\infty)$, we get

$$\mathrm{L}\{F(t);\ p\} = \int_0^\infty e^{-pt}\,F(t)\,\mathrm{d}t$$

$$= \int_0^\infty e^{-pt} \left\{ \int_0^\infty x^{\alpha} \sin(x^{\beta} t)\, \mathrm{d}x \right\} \mathrm{d}t \qquad (3.2)$$

$$= f(p), \text{ where } \mathrm{Re}(p) > 0. \qquad (3.3)$$

Since all four limits in double integral of equation (3.2) are constants, therefore we can change the order of integration in double integral easily.

$$f(p) = \int_0^\infty x^{\alpha} \left\{ \int_0^\infty e^{-pt} \sin(x^{\beta} t)\, \mathrm{d}t \right\} \mathrm{d}x$$

$$= \int_0^\infty \left(\frac{x^{\alpha+\beta}}{p^2 + x^{2\beta}} \right) \mathrm{d}x, \qquad (3.4)$$

$$\left(\mathrm{Re}(p) > |\, \mathrm{Im}(x^{\beta})\, | \text{ or } \mathrm{Re}(p) > |\, \exp(¥ \ln x) \sin(¢ \ln x)\, |\ \forall x > 0, ¥ < 0 \right).$$

Put $x^{2\beta} = p^2 \tan^2\theta$, where $\mathrm{Re}(\beta) > 0, \mathrm{Re}(p) > 0$ after simplification we get

$$f(p) = \frac{1}{\beta} p^{\frac{\alpha-\beta+1}{\beta}} \int_0^{\frac{\pi}{2}} (\tan\theta)^{\frac{\alpha+1}{\beta}}\, \mathrm{d}\theta$$

$$= \frac{1}{2\beta} p^{\frac{\alpha-\beta+1}{\beta}} \Gamma\left(\frac{\alpha+\beta+1}{2\beta} \right) \Gamma\left(\frac{-\alpha+\beta-1}{2\beta} \right), \qquad (3.5)$$

$$\left(\min\left\{ \mathrm{Re}\left(\frac{\alpha+1}{\beta} \right), \mathrm{Re}\left(\frac{-\alpha-1}{\beta} \right) \right\} > -1 \right).$$

Therefore,

$$f(p) = \frac{1}{2\beta} p^{\frac{\alpha-\beta+1}{\beta}} \frac{\pi}{\sin\left\{ \pi \left(\frac{\alpha+\beta+1}{2\beta} \right) \right\}} = \frac{\pi}{2\beta \cos\left\{ \pi \left(\frac{\alpha+1}{2\beta} \right) \right\}} \frac{1}{p^{\frac{\beta-\alpha-1}{\beta}}}, \qquad (3.6)$$

Now taking inverse Laplace transforms of $f(p)$ given by (3.6), we get

$$F(t) = \mathrm{L}^{-1}\left\{ \frac{\pi}{2\beta \cos\left\{ \pi \left(\frac{\alpha+1}{2\beta} \right) \right\}} \frac{1}{p^{\frac{\beta-\alpha-1}{\beta}}}; t \right\} = \frac{\pi}{2\beta \cos\left\{ \pi \left(\frac{\alpha+1}{2\beta} \right) \right\}} \mathrm{L}^{-1}\left\{ \frac{1}{p^{\frac{\beta-\alpha-1}{\beta}}}; t \right\}$$

$$= \frac{\pi}{2\beta \cos\left\{ \pi \left(\frac{\alpha+1}{2\beta} \right) \right\}} \frac{t^{-\frac{\alpha+1}{\beta}}}{\Gamma\left(1 - \frac{\alpha+1}{\beta} \right)}$$

So,

$$\int_0^\infty x^{ª}\sin(x^{«}t)\,\mathrm{d}x=\frac{\Gamma\left(\frac{ª+1}{«}\right)\sin\left\{®\left(\frac{ª+1}{2«}\right)\right\}}{«\,t^{\frac{ª+1}{«}}}, \tag{3.7}$$

Similarly, we can derive the integrals (2.2) to (2.8).

4. APPLICATIONS

If we substitute the different values of $ª, «$ in equations (2.1), (2.2), (2.3) and (2.4) with valid conditions, we can find the values of some interesting integrals:

In equation (2.3) put $ª=-4, «=4, b=1$ and value of $\cos\left(\frac{®}{8}\right)$, we get

$$\int_0^\infty \frac{\sin(x^{-4})}{x^4}\,\mathrm{d}x=\frac{\sqrt{(2+\sqrt{2})}}{8}\Gamma\left(\frac{3}{4}\right) \tag{4.1}$$

In equation (2.3) put $ª=-3, «=4, b=1$ and value of $\sin\left(\frac{®}{4}\right)$, we obtain

$$\int_0^\infty \frac{\sin(x^{-4})}{x^3}\,\mathrm{d}x=\frac{1}{4}\sqrt{\frac{®}{2}} \tag{4.2}$$

In equation (2.3) put $ª=-2, «=4, b=1$ and value of $\sin\left(\frac{®}{8}\right)$, we have

$$\int_0^\infty \frac{\sin(x^{-4})}{x^2}\,\mathrm{d}x=\frac{\sqrt{(2-\sqrt{2})}}{8}\Gamma\left(\frac{1}{4}\right) \tag{4.3}$$

In equation (2.3) put $ª=0, «=4, b=1$ and value of $\sin\left(\frac{®}{8}\right)$, we get

$$\int_0^\infty \sin(x^{-4})\,\mathrm{d}x=\frac{\sqrt{(2-\sqrt{2})}}{2}\Gamma\left(\frac{3}{4}\right) \tag{4.4}$$

In equation (2.3) put $ª=1, «=4, b=1$ and value of $\sin\left(\frac{®}{4}\right)$, we obtain

$$\int_0^\infty x\sin(x^{-4})\,\mathrm{d}x=\frac{1}{2}\sqrt{\frac{®}{2}} \tag{4.5}$$

In equation (2.3) put $\alpha = 2, \beta = 4, b = 1$ and value of $\cos\left(\frac{\pi}{8}\right)$, we have

$$\int_0^\infty x^2 \sin(x^{-4})\,\mathrm{d}x = \frac{\sqrt{(2+\sqrt{2})}}{6}\Gamma\left(\frac{1}{4}\right) \tag{4.6}$$

In equation (2.3) put $\alpha = -3, \beta = 3, b = 1$ and value of $\sin\left(\frac{\pi}{3}\right)$, we get

$$\int_0^\infty \frac{\sin(x^{-3})}{x^3}\,\mathrm{d}x = \frac{1}{2\sqrt{3}}\Gamma\left(\frac{2}{3}\right) \tag{4.7}$$

In equation (2.3) put $\alpha = -2, \beta = 3, b = 1$ and value of $\sin\left(\frac{\pi}{6}\right)$, we obtain

$$\int_0^\infty \frac{\sin(x^{-3})}{x^2}\,\mathrm{d}x = \frac{1}{6}\Gamma\left(\frac{1}{3}\right) \tag{4.8}$$

In equation (2.3) put $\alpha = 0, \beta = 3, b = 1$ and value of $\sin\left(\frac{\pi}{6}\right)$, we have

$$\int_0^\infty \sin(x^{-3})\,\mathrm{d}x = \frac{1}{2}\Gamma\left(\frac{2}{3}\right) \tag{4.9}$$

In equation (2.3) put $\alpha = 1, \beta = 3, b = 1$ and value of $\sin\left(\frac{\pi}{3}\right)$, we get

$$\int_0^\infty x \sin(x^{-3})\,\mathrm{d}x = \frac{\sqrt{3}}{4}\Gamma\left(\frac{1}{3}\right) \tag{4.10}$$

In equation (2.3) put $\alpha = -2, \beta = 2, b = 1$ and value of $\sin\left(\frac{\pi}{4}\right)$, we obtain

$$\int_0^\infty \frac{\sin(x^{-2})}{x^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{2}} \tag{4.11}$$

In equation (2.3) put $\alpha = 0, \beta = 2, b = 1$ and value of $\sin\left(\frac{\pi}{4}\right)$, we have

$$\int_0^\infty \sin(x^{-2})\,\mathrm{d}x = \sqrt{\frac{\pi}{2}} \tag{4.12}$$

In equation (2.1) put $\alpha = -2, \beta = 2, b = 1$ and value of $\sin\left(\frac{\pi}{4}\right)$, we get

$$\int_0^\infty \frac{\sin(x^2)}{x^2}\mathrm{d}x = \sqrt{\frac{®}{2}} \tag{4.13}$$

In equation (2.1) put $ª = 0, « = 2, b = 1$ and value of $\sin\left(\frac{®}{4}\right)$, we obtain

$$\int_0^\infty \sin(x^2)\mathrm{d}x = \frac{1}{2}\sqrt{\frac{®}{2}} \tag{4.14}$$

In equation (2.1) put $ª = -3, « = 3, b = 1$ and value of $\sin\left(\frac{®}{3}\right)$, we have

$$\int_0^\infty \frac{\sin(x^3)}{x^3}\mathrm{d}x = \frac{\sqrt{3}}{4}\Gamma\left(\frac{1}{3}\right) \tag{4.15}$$

In equation (2.1) put $ª = -2, « = 3, b = 1$ and value of $\sin\left(\frac{®}{6}\right)$, we get

$$\int_0^\infty \frac{\sin(x^3)}{x^2}\mathrm{d}x = \frac{1}{2}\Gamma\left(\frac{2}{3}\right) \tag{4.16}$$

In equation (2.1) put $ª = 1, « = 3, b = 1$ and value of $\sin\left(\frac{®}{3}\right)$, we obtain

$$\int_0^\infty x\sin(x^3)\mathrm{d}x = \frac{1}{2\sqrt{3}}\Gamma\left(\frac{2}{3}\right) \tag{4.17}$$

In equation (2.1) put $ª = -4, « = 4, b = 1$ and value of $\cos\left(\frac{®}{8}\right)$, we have

$$\int_0^\infty \frac{\sin(x^4)}{x^4}\mathrm{d}x = \frac{\sqrt{(2+\sqrt{2})}}{6}\Gamma\left(\frac{1}{4}\right) \tag{4.18}$$

In equation (2.1) put $ª = -3, « = 4, b = 1$ and value of $\sin\left(\frac{®}{4}\right)$, we get

$$\int_0^\infty \frac{\sin(x^4)}{x^3}\mathrm{d}x = \frac{1}{2}\sqrt{\frac{®}{2}} \tag{4.19}$$

In equation (2.1) put $ª = -2, « = 4, b = 1$ and value of $\sin\left(\frac{®}{8}\right)$, we obtain

$$\int_0^\infty \frac{\sin(x^4)}{x^2}\mathrm{d}x = \frac{\sqrt{(2-\sqrt{2})}}{2}\Gamma\left(\frac{3}{4}\right) \tag{4.20}$$

In equation (2.1) put ª $= 0,$ « $= 4, b = 1$ and value of $\sin\left(\frac{\text{®}}{8}\right)$, we have

$$\int_0^\infty \sin(x^4)\,dx = \frac{\sqrt{(2-\sqrt{2})}}{8}\Gamma\left(\frac{1}{4}\right) \tag{4.21}$$

In equation (2.1) put ª $= 1,$ « $= 4, b = 1$ and value of $\sin\left(\frac{\text{®}}{4}\right)$, we get

$$\int_0^\infty x\sin(x^4)\,dx = \frac{1}{4}\sqrt{\frac{\text{®}}{2}} \tag{4.22}$$

In equation (2.1) put ª $= 2,$ « $= 4, b = 1$ and value of $\cos\left(\frac{\text{®}}{8}\right)$, we obtain

$$\int_0^\infty x^2\sin(x^4)\,dx = \frac{\sqrt{(2+\sqrt{2})}}{8}\Gamma\left(\frac{3}{4}\right) \tag{4.23}$$

In equation (2.4) put ª $= -4,$ « $= 4, b = 1$ and value of $\sin\left(\frac{\text{®}}{8}\right)$, we have

$$\int_0^\infty \frac{\cos(x^{-4})}{x^4}\,dx = \frac{\sqrt{(2-\sqrt{2})}}{8}\Gamma\left(\frac{3}{4}\right) \tag{4.24}$$

In equation (2.4) put ª $= -3,$ « $= 4, b = 1$ and value of $\cos\left(\frac{\text{®}}{4}\right)$, we get

$$\int_0^\infty \frac{\cos(x^{-4})}{x^3}\,dx = \frac{1}{4}\sqrt{\frac{\text{®}}{2}} \tag{4.25}$$

In equation (2.4) put ª $= -2,$ « $= 4, b = 1$ and value of $\cos\left(\frac{\text{®}}{8}\right)$, we obtain

$$\int_0^\infty \frac{\cos(x^{-4})}{x^2}\,dx = \frac{\sqrt{(2+\sqrt{2})}}{8}\Gamma\left(\frac{1}{4}\right) \tag{4.26}$$

In equation (2.4) put ª $= -3,$ « $= 3, b = 1$ and value of $\cos\left(\frac{\text{®}}{3}\right)$, we have

$$\int_0^\infty \frac{\cos(x^{-3})}{x^3}\,dx = \frac{1}{6}\Gamma\left(\frac{2}{3}\right) \tag{4.27}$$

In equation (2.4) put ª $= -2,$ « $= 3, b = 1$ and value of $\cos\left(\frac{\text{®}}{6}\right)$, we get

$$\int_0^\infty \frac{\cos(x^{-3})}{x^2}\,\mathrm{d}x = \frac{1}{2\sqrt{3}}\Gamma\left(\frac{1}{3}\right) \tag{4.28}$$

In equation (2.4) put $\text{ª} = -2, \text{«} = 2, b = 1$ and value of $\cos\left(\frac{\text{®}}{4}\right)$, we obtain

$$\int_0^\infty \frac{\cos(x^{-2})}{x^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\text{®}}{2}} \tag{4.29}$$

In equation (2.2) put $\text{ª} = 0, \text{«} = 2, b = 1$ and value of $\cos\left(\frac{\text{®}}{4}\right)$, we have

$$\int_0^\infty \cos(x^2)\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\text{®}}{2}} \tag{4.30}$$

In equation (2.2) put $\text{ª} = 0, \text{«} = 3, b = 1$ and value of $\cos\left(\frac{\text{®}}{6}\right)$, we get

$$\int_0^\infty \cos(x^3)\,\mathrm{d}x = \frac{1}{2\sqrt{3}}\Gamma\left(\frac{1}{3}\right) \tag{4.31}$$

In equation (2.2) put $\text{ª} = 1, \text{«} = 3, b = 1$ and value of $\cos\left(\frac{\text{®}}{3}\right)$, we obtain

$$\int_0^\infty x\cos(x^3)\,\mathrm{d}x = \frac{1}{6}\Gamma\left(\frac{2}{3}\right) \tag{4.32}$$

In equation (2.2) put $\text{ª} = 0, \text{«} = 4, b = 1$ and value of $\cos\left(\frac{\text{®}}{8}\right)$, we have

$$\int_0^\infty \cos(x^4)\,\mathrm{d}x = \frac{\sqrt{(2+\sqrt{2})}}{8}\Gamma\left(\frac{1}{4}\right) \tag{4.33}$$

In equation (2.2) put $\text{ª} = 1, \text{«} = 4, b = 1$ and value of $\cos\left(\frac{\text{®}}{4}\right)$, we get

$$\int_0^\infty x\cos(x^4)\,\mathrm{d}x = \frac{1}{4}\sqrt{\frac{\text{®}}{2}} \tag{4.34}$$

In equation (2.2) put $\text{ª} = 2, \text{«} = 4, b = 1$ and value of $\sin\left(\frac{\text{®}}{8}\right)$, we obtain

$$\int_0^\infty x^2\cos(x^4)\,\mathrm{d}x = \frac{\sqrt{(2-\sqrt{2})}}{8}\Gamma\left(\frac{3}{4}\right) \tag{4.35}$$

In equation (2.1) put $\text{ª} = 3, \text{«} = 96, b = 1$ and value of $\sin\left(\frac{\text{®}}{48}\right)$, we have

$$\int_0^\infty x^3 \sin(x^{96})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{24}\right)}{192}\sqrt{\frac{2\sqrt{2} - \sqrt{(4+\sqrt{2}+\sqrt{6})}}{\sqrt{2}}} \tag{4.36}$$

In equation (2.2) put $\text{ª} = 1, \text{«} = 48, b = 1$ and value of $\cos\left(\frac{\text{®}}{48}\right)$, we get

$$\int_0^\infty x \cos(x^{48})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{24}\right)}{96}\sqrt{\frac{2\sqrt{2} + \sqrt{(4+\sqrt{2}+\sqrt{6})}}{\sqrt{2}}} \tag{4.37}$$

In equation (2.1) put $\text{ª} = 2, \text{«} = 60, b = 1$ and value of $\sin\left(\frac{\text{®}}{40}\right)$, we obtain

$$\int_0^\infty x^2 \sin(x^{60})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{20}\right)}{120}\sqrt{\frac{2\sqrt{2} - \sqrt{\{4+\sqrt{(10+2\sqrt{5})}\}}}{\sqrt{2}}} \tag{4.38}$$

In equation (2.2) put $\text{ª} = 2, \text{«} = 60, b = 1$ and value of $\cos\left(\frac{\text{®}}{40}\right)$, we have

$$\int_0^\infty x^2 \cos(x^{60})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{20}\right)}{120}\sqrt{\frac{2\sqrt{2} + \sqrt{\{4+\sqrt{(10+2\sqrt{5})}\}}}{\sqrt{2}}} \tag{4.39}$$

In equation (2.1) put $\text{ª} = 1, \text{«} = 32, b = 1$ and value of $\sin\left(\frac{\text{®}}{32}\right)$, we get

$$\int_0^\infty x \sin(x^{32})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{16}\right)}{64}\sqrt{2 - \sqrt{\{2+\sqrt{(2+\sqrt{2})}\}}} \tag{4.40}$$

In equation (2.2) put $\text{ª} = 3, \text{«} = 64, b = 1$ and value of $\cos\left(\frac{\text{®}}{32}\right)$, we obtain

$$\int_0^\infty x^3 \cos(x^{64})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{16}\right)}{128}\sqrt{2 + \sqrt{\{2+\sqrt{(2+\sqrt{2})}\}}} \tag{4.41}$$

In equation (2.1) put $\text{ª} = 4, \text{«} = 60, b = 1$ and value of $\sin\left(\frac{\text{®}}{24}\right)$, we have

$$\int_0^\infty x^4 \sin(x^{60})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{12}\right)}{120}\sqrt{\frac{(4-\sqrt{6}-\sqrt{2})}{2}} \tag{4.42}$$

In equation (2.2) put ª $= 4,$ « $= 60, b = 1$ and value of $\cos\left(\frac{\text{®}}{24}\right)$, we get

$$\int_0^\infty x^4 \cos(x^{60})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{12}\right)}{120}\sqrt{\frac{(4+\sqrt{6}+\sqrt{2})}{2}} \tag{4.43}$$

In equation (2.1) put ª $= 5,$ « $= 60, b = 1$ and value of $\sin\left(\frac{\text{®}}{20}\right)$, we obtain

$$\int_0^\infty x^5 \sin(x^{60})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{10}\right)}{120}\sqrt{\frac{\{4-\sqrt{(10+2\sqrt{5})}\}}{2}} \tag{4.44}$$

In equation (2.2) put ª $= 6,$ « $= 70, b = 1$ and value of $\cos\left(\frac{\text{®}}{20}\right)$, we have

$$\int_0^\infty x^6 \cos(x^{70})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{10}\right)}{140}\sqrt{\frac{\{4+\sqrt{(10+2\sqrt{5})}\}}{2}} \tag{4.45}$$

In equation (2.1) put ª $= 6,$ « $= 56, b = 1$ and value of $\sin\left(\frac{\text{®}}{16}\right)$, we get

$$\int_0^\infty x^6 \sin(x^{56})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{8}\right)}{112}\sqrt{2-\sqrt{(2+\sqrt{2})}} \tag{4.46}$$

In equation (2.2) put ª $= 7,$ « $= 64, b = 1$ and value of $\cos\left(\frac{\text{®}}{16}\right)$, we obtain

$$\int_0^\infty x^7 \cos(x^{64})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{8}\right)}{128}\sqrt{2+\sqrt{(2+\sqrt{2})}} \tag{4.47}$$

In equation (2.1) put ª $= 7,$ « $= 48, b = 1$ and value of $\sin\left(\frac{\text{®}}{12}\right)$, we have

$$\int_0^\infty x^7 \sin(x^{48})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{6}\right)}{192}(\sqrt{6}-\sqrt{2}) \tag{4.48}$$

In equation (2.2) put ª $=8,$ « $=54, b=1$ and value of $\cos\left(\frac{®}{12}\right)$, we get

$$\int_0^\infty x^8 \cos(x^{54})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{6}\right)}{216}(\sqrt{6}+\sqrt{2}) \tag{4.49}$$

In equation (2.1) put ª $=9,$ « $=48, b=1$ and value of $\sin\left(\frac{5®}{48}\right)$, we obtain

$$\int_0^\infty x^9 \sin(x^{48})\,\mathrm{d}x = \frac{\Gamma\left(\frac{5}{24}\right)}{96}\sqrt{\frac{2\sqrt{2}-\sqrt{(4+\sqrt{6}-\sqrt{2})}}{\sqrt{2}}} \tag{4.50}$$

In equation (2.2) put ª $=14,$ « $=72, b=1$ and value of $\cos\left(\frac{5®}{48}\right)$, we have

$$\int_0^\infty x^{14} \cos(x^{72})\,\mathrm{d}x = \frac{\Gamma\left(\frac{5}{24}\right)}{144}\sqrt{\frac{2\sqrt{2}+\sqrt{(4+\sqrt{6}-\sqrt{2})}}{\sqrt{2}}} \tag{4.51}$$

In equation (2.1) put ª $=8,$ « $=45, b=1$ and value of $\sin\left(\frac{®}{10}\right)$, we get

$$\int_0^\infty x^8 \sin(x^{45})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{5}\right)}{180}(\sqrt{5}-1) \tag{4.52}$$

In equation (2.2) put ª $=9,$ « $=50, b=1$ and value of $\cos\left(\frac{®}{10}\right)$, we obtain

$$\int_0^\infty x^9 \cos(x^{50})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{5}\right)}{200}\sqrt{(10+2\sqrt{5})} \tag{4.53}$$

In equation (2.1) put ª $=10,$ « $=44, b=1$ and value of $\sin\left(\frac{®}{8}\right)$, we have

$$\int_0^\infty x^{10} \sin(x^{44})\,\mathrm{d}x = \frac{\Gamma\left(\frac{1}{4}\right)}{88}(2-\sqrt{2}) \tag{4.54}$$

In equation (2.2) put ª $=11,$ « $=48, b=1$ and value of $\cos\left(\frac{®}{8}\right)$, we get

$$\int_0^\infty x^{11}\cos(x^{48})\,\mathrm{d}x=\frac{\Gamma\left(\frac{1}{4}\right)}{96}(2+\sqrt{2}) \tag{4.55}$$

In equation (2.1) put $\alpha=13, \beta=48, b=1$ and value of $\sin\left(\frac{7\pi}{48}\right)$, we obtain

$$\int_0^\infty x^{13}\sin(x^{48})\,\mathrm{d}x=\frac{\Gamma\left(\frac{7}{24}\right)}{96}\sqrt{\frac{2\sqrt{2}-\sqrt{(4-\sqrt{6}+\sqrt{2})}}{\sqrt{2}}} \tag{4.56}$$

In equation (2.2) put $\alpha=20, \beta=72, b=1$ and value of $\cos\left(\frac{7\pi}{48}\right)$, we have

$$\int_0^\infty x^{20}\cos(x^{72})\,\mathrm{d}x=\frac{\Gamma\left(\frac{7}{24}\right)}{144}\sqrt{\frac{2\sqrt{2}+\sqrt{(4-\sqrt{6}+\sqrt{2})}}{\sqrt{2}}} \tag{4.57}$$

In equation (2.1) put $\alpha=11, \beta=32, b=1$ and value of $\sin\left(\frac{3\pi}{16}\right)$, we get

$$\int_0^\infty x^{11}\sin(x^{32})\,\mathrm{d}x=\frac{\Gamma\left(\frac{3}{8}\right)}{64}\sqrt{\{2-\sqrt{(2-\sqrt{2})}\}} \tag{4.58}$$

In equation (2.2) put $\alpha=17, \beta=48, b=1$ and value of $\cos\left(\frac{3\pi}{16}\right)$, we obtain

$$\int_0^\infty x^{17}\cos(x^{48})\,\mathrm{d}x=\frac{\Gamma\left(\frac{3}{8}\right)}{96}\sqrt{\{2+\sqrt{(2-\sqrt{2})}\}} \tag{4.59}$$

In equation (2.1) put $\alpha=15, \beta=40, b=1$ and value of $\sin\left(\frac{\pi}{5}\right)$, we have

$$\int_0^\infty x^{15}\sin(x^{40})\,\mathrm{d}x=\frac{\Gamma\left(\frac{2}{5}\right)}{80}\sqrt{\frac{(5-\sqrt{5})}{2}} \tag{4.60}$$

In equation (2.2) put $\alpha=17, \beta=45, b=1$ and value of $\cos\left(\frac{\pi}{5}\right)$, we get

$$\int_0^\infty x^{17}\cos(x^{45})\,\mathrm{d}x=\frac{\Gamma\left(\frac{2}{5}\right)}{180}(\sqrt{5}+1) \tag{4.61}$$

In equation (2.1) put ª $=14,$ « $=36, b=1$ and value of $\sin\left(\frac{5®}{24}\right)$, we obtain

$$\int_0^\infty x^{14}\sin(x^{36})\,\mathrm{d}x=\frac{\Gamma\left(\frac{5}{12}\right)}{72}\sqrt{\frac{(4+\sqrt{2}-\sqrt{6})}{2}} \tag{4.62}$$

In equation (2.2) put ª $=19,$ « $=48, b=1$ and value of $\cos\left(\frac{5®}{24}\right)$, we have

$$\int_0^\infty x^{19}\cos(x^{48})\,\mathrm{d}x=\frac{\Gamma\left(\frac{5}{12}\right)}{96}\sqrt{\frac{(4-\sqrt{2}+\sqrt{6})}{2}} \tag{4.63}$$

In equation (2.1) put ª $=21,$ « $=48, b=1$ and value of $\sin\left(\frac{11®}{48}\right)$, we get

$$\int_0^\infty x^{21}\sin(x^{48})\,\mathrm{d}x=\frac{\Gamma\left(\frac{11}{24}\right)}{96}\sqrt{\frac{2\sqrt{2}-\sqrt{(4-\sqrt{2}-\sqrt{6})}}{\sqrt{2}}} \tag{4.64}$$

In equation (2.2) put ª $=32,$ « $=72, b=1$ and value of $\cos\left(\frac{11®}{48}\right)$, we obtain

$$\int_0^\infty x^{32}\cos(x^{72})\,\mathrm{d}x=\frac{\Gamma\left(\frac{11}{24}\right)}{144}\sqrt{\frac{2\sqrt{2}+\sqrt{(4-\sqrt{2}-\sqrt{6})}}{\sqrt{2}}} \tag{4.65}$$

In equation (2.1) put ª $=-\frac{1}{2},$ « $=1, b=1$ and value of $\sin\left(\frac{®}{4}\right)$, we have

$$\int_0^\infty \frac{\sin(x)}{\sqrt{x}}\,\mathrm{d}x=\sqrt{\frac{®}{2}} \tag{4.66}$$

In equation (2.2) put ª $=-\frac{1}{2},$ « $=1, b=1$ and value of $\cos\left(\frac{®}{4}\right)$, we get

$$\int_0^\infty \frac{\cos(x)}{\sqrt{x}}\,\mathrm{d}x=\sqrt{\frac{®}{2}} \tag{4.67}$$

Similarly we can evaluate some interesting integrals obtained from our results (2.5) to (2.8) by setting the suitable numerical values of parameters and numerical values of Sine and Cosine functions.

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