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The Strong Nonsplit Dom Strong Domination Number of A Graph

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ABSTRACT

A dom strong dominating set D of a graph is a strong nonsplit dom strong dominating set if the induced subgraph $< V - D >$ is complete. The strong nonsplit dom strong domination number $\gamma_{snsds}(G)$ of G is the minimum cardinality of a strong nonsplit dom strong dominating set. In this paper, the study of this parameter is initiated and the exact values of some standard graphs are also obtained.

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1. INTRODUCTION

Let $G = (V, E)$ be a finite, undirected graph without loops or multiple edges. Notations and terms used may found in Haynes[9] and Harary[1]. A set $D \subseteq V$ is said to be a dominating set of G, if every vertex in $V - D$ is adjacent to some vertex in D. The domination number is the minimum cardinality of a dominating set in G and it is denoted by $\gamma(G)$. Double domination introduced by Harary *et al.*[1] serves as a model for the type of fault tolerance where each computer has access to atleast two fileservers and each of the fileservers has direct access to atleast one backup fileserver. Sampathkumar and Pushpalatha[8] have introduced the concept of strong weak domination in graphs.

A combination of the concepts of double domination and strong weak domination is the concept of domination strong domination where in for every vertex outside the dominating set, there are two vertices inside the dominating set, one of which dominates the outside vertex and the other strongly dominates the outside vertex. The concept of dom strong domination was introduced by Namasivayam P[6].

A subset D of V is called a dom strong dominating set if for every $v \in V - D,$ there exists $u_1, u_2 \in D$ such that $u_1 v, u_2 v \in E(G)$ and $\deg(u_1) \leq \deg(v)$. The minimum cardinality of a dom strong dominating set is called dom strong domination number and is denoted by γ_{ds}.

The concept of nonsplit domination number of a graph was defined by Kulli and Janakiram[2]. A dominating set $D$of a graph G is an non split dominating set if the induced subgraph $< V - D >$ is connected. The nonsplit domination number $\gamma_{ns}(G)$ of G is the minimum cardinality of a nonsplit dominating set.

The concept of nonsplit dom strong domination number of a graph was defined by Mahadevan *et al*.[5]. A dom strong dominating set D of a graph G is a nonsplit dom strong dominating set if the induced subgraph $< V - D >$ is connected. The nonsplit dom strong domination number $\gamma_{nsds}(G)$ of G is the minimum cardinality of a nonsplit dom strong dominating set.

Dominating sets whose complements induce a complete subgraph have a great diversity of applications. One such applications mentioned in[3] is the following.

In setting up the communication links in a network one might want a strong core group that can communicate with each member of the core group and so that everyone in the group receives the message from someone outside the group and communicates it to every other in the group. This motivated Kulli and Janakiram[3] to define strong nonsplit dominating set.

In this paper, the strong nonsplit dom strong domination number of the graph is introduced and determined the bounds and exact values of this parameter.

2. STRONG NONSPLIT DOM STRONG DOMINATION NUMBER OF THE GRAPH

Definition 2.1

A dom strong dominating set D of a graph is a strong nonsplit dom strong dominating set if the induced subgraph $< V - D >$ is complete. The strong nonsplit dom strong domination number $\gamma_{snsds}(G)$ of G is the minimum cardinality of a strong nonsplit dom strong dominating set.

Proposition 2.2

For any graph G, $\gamma_{ds}(G) \leq \gamma_{nsds}(G) \leq \gamma_{snsds}(G)$

Proof. Since every strong nonsplit dom strong dominating set is a nonsplit dom strong dominating set and every nonsplit dom strong dominating set is a dom strong dominating set.

Proposition 2.3

For any graph, $\beta_0(G) < \gamma_{snsds}(G)$ where $\beta_0(G)$ is the independence number of G.

Proof. Let D be a strong nonsplit dom strong dominating set. Let S be an independent set of vertices in G. Since S contains atmost one vertex from $V - D$ and atmost $|D| - 1$ vertices from D or $S \subseteq D$.

Theorem 2.4

Let G be a graph with no isolates. Then $2 \leq \gamma_{snsds}(G) \leq n$ and the bounds are sharp.

Proof. Since any dom strong dominating set has atleast two elements and atmost n elements. Any strong nonsplit dom strong dominating set has atleast two elements and atmost n elements. For a star, $\gamma_{snsds}(K_{1,n-1}) = n$ and for $K_n, \gamma_{snsds}(K_n) = 2$. Therefore the bounds are sharp.

Theorem 2.5

In a graph G, every strong nonsplit dom strong dominating set contains all pendant vertices.

Proof. Let D be any strong nonsplit dom strong dominating set of G. Let v be a pendant vertex with support say u. If v does not belong to D, then there must be two points say x, y belong to D such that x dominates v and y dominates v. Therefore x and y are adjacent to v and hence $\deg v \geq 2$ which is a contradiction. So v belongs to D.

Observation 2.6

If $G \cong K_n o K_1$, then $\gamma_{ds}(G) = \gamma_{nsds}(G) = \gamma_{snsds}(G)$.

The strong nonsplit dom strong domination number of some of the standard classes of graphs are given below

1. $\gamma_{snsds}(P_n) = n - 1 \; for \; n \geq 4$, where P_n is a path on n vertices.
2. $\gamma_{snsds}(C_n) = n - 1 \; for \; n \geq 4$, where C_n is a cycle on n vertices
3. $\gamma_{snsds}(K_n) = 2 \; for \; n \geq 3$, where $K_n$is a complete graph on n vertices.
4. $\gamma_{snsds}(K_{1,n-1}) = n$.
5. $\gamma_{snsds}(K_{r,s}) = \begin{cases} r + (s-1) for \; r < s \\ r + s - 2 \; for \; r = s \end{cases}$ where $K_{r,s}$ is a bipartite graph on $r + s$ vertices
6. $\gamma_{snsds}(P) = 8$, where P is the Peterson graph.
7. $\gamma_{snsds}(W_n) = n - 2$.
8. If G is the corona $C_n \circ K_1$, then $\gamma_{snsds}(G) = 2n - 2 \; for \; n \geq 3$

RELATION BETWEEN THE STRONG NONSPLIT DOM STRONG DOMINATION NUMBER AND CHROMATIC NUMBER

In this section, the upper bound for the sum of the strong nonsplit dom strong domination number and chromatic number is obtained and characterized the corresponding extremal graphs.

Theorem 2.7[7]

For any connected graph G, $\chi(G) \leq \Delta(G) + 1$.

Theorem 2.8.
For any graph, $\gamma_{snsds}(G) + \chi(G) \leq 2n$ and equality holds if and only if $G \cong K_2$.
Proof. By theorem 2.4 and 2.7, it follows that $\gamma_{snsds}(G) + \chi(G) = n + \Delta + 1 \leq 2n$. Now we assume that $\gamma_{snsds}(G) + \chi(G) = 2n$. This is possible only if $\gamma_{snsds}(G) = n$ and $\chi(G) = n$. Since $\chi(G) = n$, G is complete. But for complete graph, $\gamma_{snsds}(G) = 2$. Hence $G \cong K_2$. Converse is obvious.

Theorem 2.9.
For any graph G, $\gamma_{snsds}(G) + \chi(G) = 2n - 1$ if and only if $G \cong P_3 \; or \; K_3$.
Proof. If G is either P_3 or K_3, then clearly $\gamma_{snsds}(G) + \chi(G) = 2n - 1$. Conversely, assume that $\gamma_{snsds}(G) + \chi(G) = 2n - 1$. This is possible only if $\gamma_{snsds}(G) = n$ and $\chi(G) = n - 1$ (or) $\gamma_{snsds}(G) = n - 1$ and $\chi(G) = n$.

Case (i). $\gamma_{snsds}(G) = n$ and $\chi(G) = n - 1$.
Since $\gamma_{snsds}(G) = n$, G is a star. Therefore $n = 3$. Hence $G \cong P_3$. On increasing the degree we get a contradiction.
Case (ii). $\gamma_{snsds}(G) = n - 1$ and $\chi(G) = n$.
Since $\chi(G) = n, G$ is complete. But for K_n, $\gamma_{snsds}(G) = 2$, so that $n = 3$. Hence $G \cong K_3$.

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