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Rigidity Holder for Cubic Matrices

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ABSTRACT

Let P be a field, and $M^{(3)}(P)$ is ring of cubic matrices on a field P. In this paper is defined (C_1, C_2)-Holder valuation on a ring $M^{(3)}(P)$ and then is proved, that (C_1, C_2)-Holder valuation on a ring $M^{(3)}(P)$ is $(2, \alpha)$-Holder equivalent to some classical valuation, where $\alpha = (\log(2C_1))^{-1}$. It gives expansion of the theorem of Garcia ([4]) on some class of noncommutative rings (class of cubic matrices on a field P).

Keywords: cubic matrices; (C_1, C_2)-Holder.

1. INTRODUCTION

Definition 1.1. Let Γ be a totally ordered abelian group. A matrix valuation on skew field D (simple valuation on $A = M_n(D)$) is a function $\mu: A \to \Gamma \cup \{\infty\}$ satisfying:

1. $\mu(XY) = \mu(X) + \mu(Y)$, for all $X, Y \in A$;
2. $\mu(X\nabla Y) \geq \min\{\mu(X), \mu(Y)\}$, for all $X, Y \in A$ such that $X\nabla Y$ is defined;
3. $\mu(X)$ is unchanged if any row or column of X is multiplied by -1;
4. $\mu(X) = \infty$ for any singular matrix $X \in A$;
5. $\mu(I) = 0$, where I is an identity matrix.

Remark 1.2. We observe that when $n = 1$, (i.e., A is a division ring), then (1)-(5) simply say that μ is a valuation on A.

Proposition 1.3. Let μ be a simple valuation on $A = M_n(D)$. Then we have:

- If $\mu(X) \neq \mu(Y)$. Then $\mu(X\nabla Y) = \min\{\mu(X), \mu(Y)\}$, whenever $X\nabla Y$ is defined in A.
- $\mu(E) = 0$for any elementary matrix E in A.
- $\mu(X)$is unchanged if X multiplied on the left (or right) by an elementary matrix.
- $\mu(X)$remains unchanged under any permutation of rows (or columns).

2. CUBIC MATRICES AND RIGIDITY HOLDER

Now let given the numerical field P. Any system from n^3 elements $A_{i,j,k}(i, j, k =$

$1,2,\cdots,n)$ of field P that defined as coordinates i,j,k, is called a 3-dimensional (cubic) matrix of order n on P and it is denoted in abbreviated from by a symbol $\|A_{ijk}\|$ $(i,j,k = 1,2,\cdots,n)$.

Definition 2.1. The valuation on set of cubic matrices $M^{(3)}(P)$ on a field P is called map $|\ |: M^{(3)}(P) \to \mathbb{R} \cup \{\infty\}$, satisfying the following conditions:

1. if $A = \|A_{ijk}\| \in M_n^{(3)}(P)$ and A is a singular matrix, then $|A| = \infty$;
2. if $A = \|A_{ijk}\|, B = \|B_{ijk}\| \in M_n^{(3)}(P)$, $1 \le \lambda \le n, B_{\lambda jk} = -A_{\lambda jk}$ and $B_{ijk} = A_{ijk}$for $i \neq \lambda, 1 \le j,k \le n$, then $|A| = |B|$;
3. if $A = \|A_{ijk}\|, B = \|B_{ijk}\| \in M_n^{(3)}(P)$ and the determinant sum $A\nabla B$ is determined, then $|A\nabla B| \ge \min\{|A|,|B|\}$;
4. for any matrix $A = \|A_{ijk}\| \in M_n^{(3)}(P)$, $B = \|B_{ijk}\| \in M_m^{(3)}(P)$: $|A \oplus B| = |A| + |B|$;
5. $|I| = 0$, where $I = I_n \in M_n^{(3)}(P)$ is an identity cubic matrix.

Definition 2.2. Let $C_1, C_2 \ge 1$. Then a (C_1, C_2)-Holder valuation on set of cubic matrices $M^{(3)}(P)$ on a field P is called map $\|\ \|: M^{(3)}(P) \to \mathbb{R} \cup \{\infty\}$, satisfying the following conditions:

1. If $A = \|A_{ijk}\| \in M_n^{(3)}(P)$ and A is a singular matrix, then $|A| = \infty$;
2. if $A = \|A_{ijk}\|, B = \|B_{ijk}\| \in M_n^{(3)}(P)$, $1 \le \lambda \le n, B_{\lambda jk} = -A_{\lambda jk}andB_{ijk} = A_{ijk}$ for $i \neq \lambda, 1 \le j,k \le n$, then $|A| = |B|$;
3. if $A = \|A_{ijk}\|, B = \|B_{ijk}\| \in M_n^{(3)}(P)$ and the determinant sum $A\nabla B$ is determined, then $|A\nabla B| \ge C_2 \min\{|A|,|B|\}$;
4. for any matrix $A = \|A_{ijk}\| \in M_n^{(3)}(P)$, $B = \|B_{ijk}\| \in M_m^{(3)}(P)$: $C_1^{-1}(\|A\| + \|B\|) \le \|A \oplus B\| \le C_1(\|A\| + \|B\|)$;
5. $|I| = 0$, where $I = I_n \in M_n^{(3)}(P)$ is an identity cubic matrix.

Remark 2.3. We shall notice, that a (1,1)-Holder valuation on set of cubic matrices $M^{(3)}(P)$ on a field P is a classical valuation. Let $|\ |, \|\ \|$ are two valuations on set of cubic matrices $M^{(3)}(P)$ on a field P. Then we say, that are (C_0, α) equivalent, if:
$C_0^{-1}|A|^\alpha \le \|A\| \le C_0|A|^\alpha$
for all $A \in M^{(3)}(P)$.

Theorem 2.4.*(Rigidity Holder for set of cubic matrices).*

Let $\|\ \|: M^{(3)}(P) \longrightarrow \mathbb{R} \cup \{\infty\}$ is a (C_1, C_2)-Holder valuations from set of cubic matrices $M^{(3)}(P)$, where $C_1 \ge 1, C_2 \ge 1$. Then there exist a classical valuations $|A|$ on set of cubic matrices $M^{(3)}(P)$, which is $(2, \alpha)$-equivalent to valuation $\|\ \|$, where $\alpha = (\log_2(2C_1))^{-1}$.

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