Journal of Computer and Mathematical Sciences, *Vol.8(5), 157-170 May 2017* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Stress Work Effects on MHD Natural Convection Flow Along a Vertical Flat Plate with Heat Conduction and Nonuniform Surface Temperature

M. H. Rashid[1], Mohammad Mahfuzul Islam[2], MD. M. ALAM[3] and M. M. Parvez[4]

Department of Mathematics,
Dhaka University of Engineering and Technology,
Gazipur-1700, BANGLADESH.

(Received on: April 30, Accepted: May 2, 2017)

ABSTRACT

In this paper is presented to study conjugate effects of viscous dissipation and pressure work on MHD natural convection flow along a vertical flat plate with heat conduction and power law variation of surface temperature. Viscous dissipation and pressure work effects on magneto-hydrodynamics natural convection flows with heat conduction are considered in this investigation. With a goal to attain similarity solutions of the problem, the developed equations are made dimensionless by using suitable transformations. The non-dimensional equations are then transformed into non-similar forms by introducing non- similarity transformations. The resulting non-similar equations together with their corresponding boundary conditions based on conduction and convection are solved numerically by using the shooting method of Nachtsheim-swigert iteration technique and finite difference method together with Keller box Scheme. Numerically calculated velocity profiles and temperature profiles, skin friction and the rate of heat transfer coefficient are shown on graphs for different values of the parameters entering into the problem.

Keywords: Free convection, Prandtl's number, magneto-hydrodynamics, heat generation, viscous dissipation and pressure work.

1. INTRODUCTION

The Problem of natural convection flow along a vertical isothermal plate is a classical problem of fluid mechanics that has been solved with the similarity method near about 70 years ago. Ackroyd[1] was the first who introduced the viscous dissipation and the pressure work in the energy equation. He proved that, for this problem, the pressure work effect is more important than that of viscous dissipation. Gebhart[2], was the first who studied the problem of laminar natural convection flow along a vertical heated plate taking into account the viscous dissipation. Also, he found that the non-dimensional parameter $g\,\beta\,l\,/\,C_p$ (g= acceleration due to gravity, β is the coefficient of thermal expansion, l = length scale of the problem, C_p = specific heat under constant pressure) determined the influence of viscous dissipation and it has been called the dissipation number. Gebhart and Mollendorf[3] have shown that the problem of natural convection flow over a vertical plate with viscous dissipation admits similarity solution only when the plate temperature varies in exponential law. Effect of pressure stress work and viscous dissipation on MHD and Joule heating natural convection flow along a vertical flat plate and over a sphere have been investigated by Alam *et al*.,[4,5]. Alim *et al*.,[6] studied combined effect of viscous dissipation and Joule heating on the coupling of conduction and free convection along a vertical flat plate. Joshi and Gebhart[7] treated with the perturbation method, the problem of natural convection over a vertical isothermal plate taking into account both the viscous dissipation and the pressure work in the energy equation using two non-dimensional numbers. The effect of axial heat conduction in a vertical flat plate on free convection heat transfer are studied by Miyamoto *et al*.,[9]. The coupling of conduction with laminar natural convection along a flat plate is studied by Pozzi, and Lupo[9]. Emad *et al*.,[10] investigated viscous dissipation and Joule heating effects on MHD-free convection flow from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. But our investigation the effects of stress work and heat generation on MHD natural convection flow along a vertical plate with power law variation with uniform surface temperature.

Nomenclature

B_0	Magnetic field strength
C_P	Specific heat at constant pressure.
g	Acceleration due to gravity.
Gr	The local Grashof number.
k	Thermal conductivity of the fluid.
M	The magnetic parameter.
P	Pressure of the fluid.
P_x	Fluid pressure along x-direction.
P_r	Prandtl number.
Q	Heat generation parameter.
Q_o	Heat generation coefficient
q_x	Heat flux
T	Temperature of the fluid.
T_w	Wall temperature
T_∞	Ambient temperature.
u	Velocity component in the x- direction.

v	Velocity component in the y-direction.
x	Measured from the leading edge.
y	Distance normal to the surface.

Greek symbols

α	The thermal diffusivity.
β	Co-efficient of volume expansion
η	The pseudo-similarity variable.
ξ	The similarity variable.
ν	Kinematic viscosity
μ	Viscosity of the fluid
φ	Dimensionless temperature
ρ	Density of the fluid inside the boundary layer.
ψ	Stream function
σ	The electric conductivity.

2. FORMULATION OF THE PROBLEM

Consider the laminar free convection flow along a vertical p placed in a calm environment with u and v denoting respectively the velocity components in the x and y direction, where x is vertically upwards and y is the coordinate perpendicular to x.

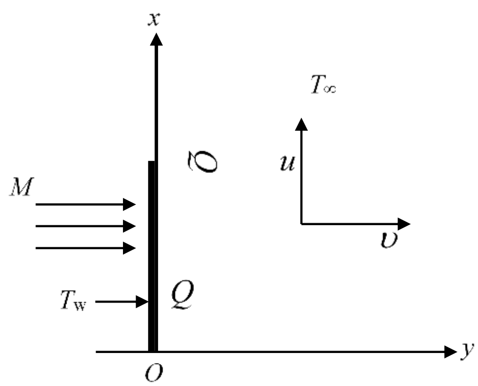


Fig :1 Physical Co-ordinate system

For steady, two-dimensional flow of the boundary layer continuity equation, momentum equation and energy equation, including viscous dissipation, pressure work and heat generation term are given below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial v}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$+\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\beta T}{\rho c_p}u\frac{\partial Px}{\partial x}+\frac{Q_0}{\rho C_p}\left(T-T_\infty\right)$$

Where $\alpha=\dfrac{k}{\rho C_p}$ is the thermal diffusivity.

The temperature of quiescent ambient fluid T_∞ at large values of *y* is taken to be constant. Where T_w is the temperature on the wall, T is the fluid temperature, ν is the kinematics velocity, β is the fluid thermal expansion coefficient, B_0 is the magnetic field strength, Q_0 is the heat generated coefficient, C_p is the specific heat at constant pressure, ρ is the fluid density and P is the pressure. The last two terms in the energy equation are the viscous dissipation and the pressure work respectively.

Equations (1)-(3) are to solved subject to the boundary conditions

$$\begin{aligned} &u=v=0, T=T_w \text{ on } y=0\\ &u\to U, T\to T_\infty \text{ as } y\to\infty\\ &u=U, T=T_w \text{ at } x=0 \end{aligned} \tag{4}$$

where U is the free stream velocity.

The following generalizations are introduced to obtain the equations in terms of generalized stream $f(x,y)$ and temperature functions and $\phi(x,y)$. Now letting

$$u=\nu\frac{\partial\psi}{\partial y},\ d(x)=\Delta T=T_w-T_\infty$$

$$v=-\nu\frac{\partial\psi}{\partial x},\ \phi(x,y)=\frac{T-T_\infty}{T_w-T_\infty}=\frac{T-T_\infty}{\Delta T},$$

$$\therefore \phi(x,y)\Delta T=T-T_\infty, i.e. T=T_\infty+\phi\Delta T$$

$$\frac{\partial T}{\partial x} = \Delta T\phi_X \;\therefore \frac{\partial u}{\partial x} = \nu \frac{\partial^2 \psi}{\partial x \partial y}$$

$$, \frac{\partial u}{\partial y} = \nu \frac{\partial^2 \psi}{\partial y^2}, \frac{\partial^2 u}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3}$$

Equation (2) becomes

$$\psi_y \psi_{yx} - \psi_x \psi_{yy} \tag{5}$$

$$= \psi_{yyy} + \frac{g\beta\phi\Delta T}{\nu^2} - \frac{\sigma B_0^2}{\rho\nu}\psi_y$$

Again, we know

$$\frac{\partial T}{\partial x} = \Delta T\phi_x + \phi(\Delta T)_x \;\; \frac{\partial T}{\partial y} = \Delta T\phi_y$$

$$\frac{\partial^2 T}{\partial y^2} = \Delta T\phi_{yy}, \frac{\partial u}{\partial y} = \nu \frac{\partial^2 \psi}{\partial y^2}$$

$$P_r = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}, R_e = \frac{\nu l_c}{\nu} = \frac{\rho \nu l_c}{\mu} \;\; \therefore \frac{1}{\rho} = \frac{\alpha}{k} C_p$$

Now the equation (3) becomes

$$\therefore \psi_y \phi_x \Delta T + \phi \psi_y (\Delta T)_x - \psi_x \phi_y \Delta T \tag{6}$$

$$= \frac{k}{\nu\rho c_p}\Delta T\phi_{yy} - \frac{\beta T g}{c_p}\psi_y + \frac{\nu^2}{c_p}\psi_{yy}{}^2 + \frac{Q_0}{\nu\rho C_p}\Delta T$$

Where $(T_w - T_\infty)$ is the downstream temperature difference (along the *x*-axis) which would result without the inclusion of the viscous dissipation and hydrostatic pressure effects that are both $\varepsilon(x)$ and $\lambda(x)$ are zero. Gr_x is related to the actual physical Grashof number $Gr_x = Gr_x \phi(0)$.

3. TRANSFORMATION OF THE GOVERNING EQUATIONS

Now using the similarity variable and the stream function of the following form $x = \xi, \eta = yb(x), \psi(x, y) = c(\xi) f(\eta), \phi(x, y) = \phi(\eta)$ In equation (5) and (6) and also their boundary conditions (4),

So, viscous dissipation terms can be written as

$$\frac{\nu^2}{c_p}\psi_{yy}^2 = \frac{\nu^2}{c_p} c^2(\xi) b^4(\xi) f''^2(\eta)$$

and also the pressure effect term

$$\frac{g\beta T}{c_p}\psi_y = \frac{g\beta T}{c_p}c(\xi)b(\xi)f'(\eta)$$

Temperature profiles

$$\phi_y = \frac{\partial}{\partial y}[\phi(\eta)] = \phi'(\eta)b(\xi), \phi_{yy} = \phi''(\eta)b^2(\xi),$$

$$\phi_x = \frac{\partial}{\partial x}[\phi(\eta)] = \frac{\eta}{b(\xi)}b_\xi(\xi).\phi'(\eta)$$

Therefore, the momentum equation

$$c(\xi)b(\xi)f'(\eta)[c_\xi(\xi)b(\xi)f'(\eta)$$
$$+c(\xi)b_y(\xi)f'(\eta)+\eta c(\xi)b_\xi(\xi)f''(\eta)]$$
$$-\left[c_\xi(\xi)f(\eta)+\eta.\frac{c(\xi)f'(\eta)b_\xi(\xi)}{b(\xi)}\right]c(\xi)b^2(\xi)f''(\eta)$$
$$=c(\xi)b^3(\xi)f'''(\eta)+\frac{g\beta\phi\Delta T}{\nu^2}-\frac{\sigma B_0^2}{\rho\nu}cb\,f'$$

$$\therefore f'''(\eta)+\frac{g\beta\phi\Delta T}{c(\xi)b^3(\xi)\nu^2}+\frac{c_\xi(\xi)}{b(\xi)}f(\eta)f''(\eta)$$
$$-\left[\frac{c_\xi(\xi)}{b(\xi)}+\frac{c(\xi)b_\xi(\xi)}{b^2(\xi)}\right]f'^2(\eta)+-\frac{\sigma B_0^2}{\mu b^2}f'=0 \quad (7)$$

Also the energy equation

$$\frac{1}{p_r}\phi''(\eta)+\frac{c_\xi(\xi)}{b(\xi)}f(n)\phi'(\eta)$$
$$-\frac{c(\xi)(\Delta T)_\xi}{b(\xi)\Delta T}f'(\eta)\phi(n)-\frac{g\beta T}{c_p}\frac{c(\xi)}{b(\xi)\Delta T}f'(\eta) \quad (8)$$
$$+\frac{\nu^2}{c_p\Delta T}c^2(\xi)b^2(\xi)f''^2(\eta)-\frac{Q_o}{b^2(\xi)\nu\rho C_p}\phi=0$$

The last two terms in the energy equation (8) are the pressure work and viscous dissipation effect respectively. Here we consider the power law $\Delta T = Nx^n = N\xi^n$ surface (at η =0) temperature distributions. Considering for similarity solutions, we use the following transformations:

$$c(\xi)=4\left(\frac{1}{4}Gr_\xi\right)^{\frac{1}{4}},\ b(\xi)=\frac{1}{\xi}\left(\frac{1}{4}Gr_\xi\right)^{1/4},\ Gr_\xi=\frac{g\beta\xi^3 n\xi^n}{\nu^2}$$

$$\therefore c_\xi = \frac{d}{d\xi}[4(\frac{1}{4}Gr_\xi)^{1/4}] = \frac{n+3}{\xi}\left(\frac{1}{4}Gr_\xi\right)^{1/4},\ b_\xi = \frac{n-1}{4\xi^2}(\frac{1}{4}Gr_\xi)^{1/4}$$

Now $(\Delta T)_\xi = \frac{d}{d\xi}(N\xi^{\eta)} = nN\xi^{\eta-1} = \frac{n}{\xi}\Delta T$

The for the momentum equation

$$\begin{aligned} &\therefore\ f'''(\eta) + (n+3) f(\eta) f''(\eta) \\ &-2(n+1) f'^2(\eta) + \phi(\eta) + M\ f' = 0 \end{aligned} \tag{9}$$

Where, $M = \frac{2\sigma B_0^2 \xi^2}{\mu\left(Gr_\xi\right)^{\frac{1}{2}}}$, Hartman number related to MHD.

And also energy equation

$$\begin{aligned} &\phi''(\eta) + p_r[\left(n+3\right) f(\eta)\phi'(\eta) \\ &-4n\ f'(\eta)\phi/\eta) - \frac{4g\,\beta\,\xi}{C_p}\frac{T}{\Delta T}\ f'(\eta) \\ &+\frac{4g\,\beta\,\xi}{C_p}\ f''^2(\eta) - \frac{Q_o}{b^2(\xi)\nu\,\rho C_p}\phi] = 0 \end{aligned} \tag{10}$$

Therefore, $f(\eta)$ and $\phi(\eta)$ are functions of η, p_r and ξ^n for the power law case. To retain both the viscous dissipation and hydrostatic pressure effects to the first order, $\varepsilon(\xi)$ and $\lambda(\xi)$ are chosen as

$\varepsilon(\xi) = \frac{4g\beta\,\xi}{C_p}$ and $\lambda(\xi) = \frac{4g\,\beta\,\xi}{C_p}\frac{T}{\Delta T}$, So the equation

$$\begin{aligned} &\therefore \phi''(\eta) + p_r[(n+3) f(\eta)\phi'(\eta) \\ &-4n\ f'(\eta)\phi(\eta) - \lambda(\xi) f'(\eta) \\ &+\varepsilon(\xi) f''^2(\eta) - Q\phi\left(\eta\right)] = 0 \end{aligned} \tag{11}$$

Where, $Q = \frac{Q_0\,\xi^2}{\nu\,\rho C_p\,(\frac{1}{4}Gr_\xi)^{\frac{1}{2}}}$, Q is the dimensionless heat generation parameter.

Equations (10) and (12) are numerically integrated in the vertical surface case, with the boundary conditions

$$\eta = 0: \quad f(0) = 0, \quad f^{'}(0) = 0, \quad \phi(0) = 1; \qquad (12)$$
$$\eta \to \infty: \quad f^{'}(\infty) \to 0, \quad \phi(\infty) \to 0,$$

Local Nussult Number $Nu_{\xi} = -(\frac{1}{4}Gr_{\xi})^{\frac{1}{4}} \phi^{'}(\xi, 0)$ which implies that

$-\phi^{'}(\xi, 0) = Nu_{\xi} (\frac{1}{4}Gr_{\xi})^{-\frac{1}{4}}$.

The viscous stress $\tau(\xi)$ and skin friction coefficient $C_{f_{\xi}}$ defined on a convection velocity $U = \nu c(\xi) b(\xi) f'_{\max}(0) \propto \nu c(\xi) b(\xi)$ are known for $f(\eta)$

Therefore Skin friction coefficient

$$C_{f\,\xi} = \frac{\tau(\xi)}{\frac{1}{2}\rho U^2} = \frac{f^{''}(\xi, 0)}{2(\frac{1}{4}Gr_{\xi})^{\frac{1}{4}}}$$

which implies that, $f^{''}(\xi, 0) = 2(\frac{1}{4}Gr_{\xi})^{\frac{1}{4}} Cf_{\xi}$

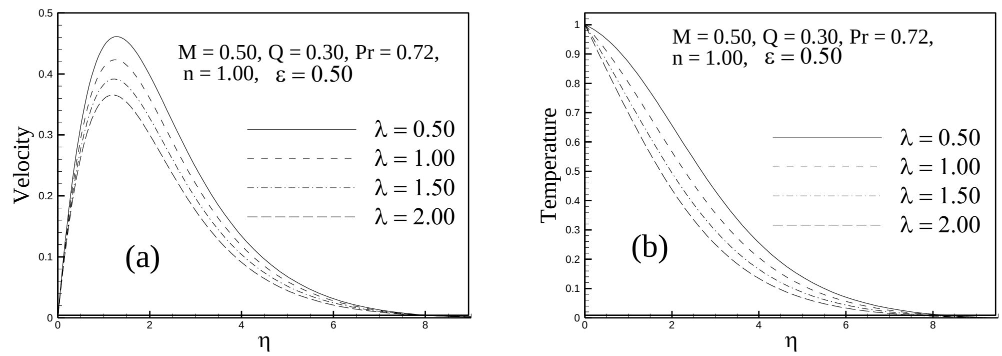
4. NUMERICAL METHODS

The theoretical treatment of Magneto-hydrodynamics flows or any other flows both in horizontal or incline planes have so far been made mostly analytically and applying perturbation methods. In some cases the asymptotic method has been applied. However, our solutions would be based mainly on numerical methods. For this purpose the shooting method of Nachtsheim-Swigert iteration technique and finite difference method together with Keller box Scheme will be used for problems for which similarity solutions of ordinary differential equations are sought. In order to obtain non-similar solutions to partial differential the shooting method of Nachtsheim-swigert iteration technique together with Keller box Scheme will be used for these problems for which similarity solutions of ordinary differential equations are sought

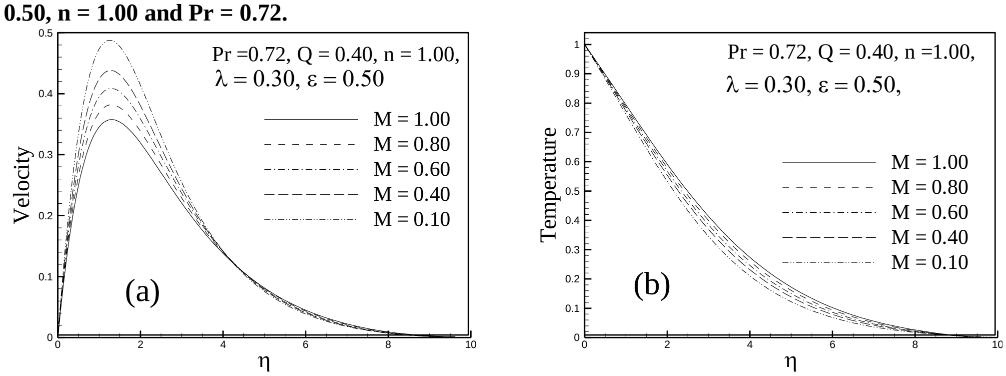
5. RESULTS AND DISCUSSION

The system of non-linear ordinary differential equations (9) and (11) together with the boundary condition (12) have been solved numerically by employing shooting method of Nachtsheim-swigert iteration technique and finite difference method together with Keller-box elimination technique. With the flow parameters M, Q, λ, ε and Pr, the results are displayed in figures (2) to figures (9) for predicting velocity profiles, temperature profiles, skin friction and rate of heat transfer coefficient.

From figure 2(a) it depicts that the velocity distribution increases slightly as the pressure work parameter λ increases in the region $\eta \in [0, 9]$ but near the surface of the plate, velocity increases and becomes maximum and then decreases and at a certain point velocity profiles coincide and finally approaches to zero. The maximum values of the velocity are recorded to be 0.43056, 0.44332, 0.47589 and 0.53463 for λ = 0.50, 1.00, 1.50 and 2.00 respectively which occur at η = 1.17520 for first, second and third maximum values and at η = 1.23788 for last maximum values. Here we see that the velocity increases by 24.17 % as λ increases from 0.50 to 2.00. From figure 2(b), it is seen that when the values of pressure work parameter λ increases in the region $\eta \in$ [0, 9], the temperature distribution also increases. But near the surface of the plate temperature profiles are maximum and then decreases away from the surface and finally take asymptotic value.



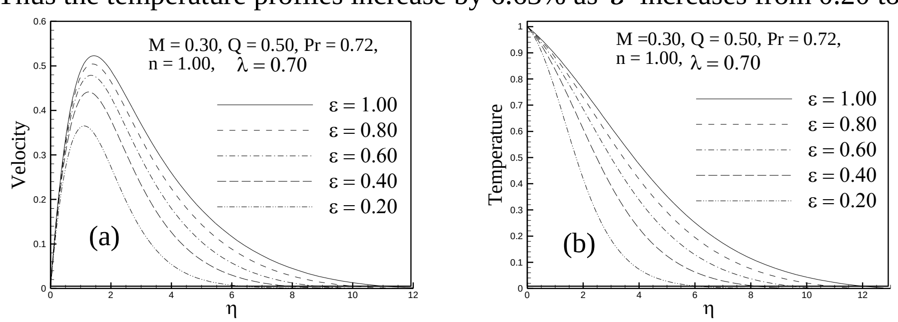
Variation of 2(a) velocity and 2(b) temperature profiles against η for λ with M = 0.50, Q = 0.30, ε = 0.50, n = 1.00 and Pr = 0.72.



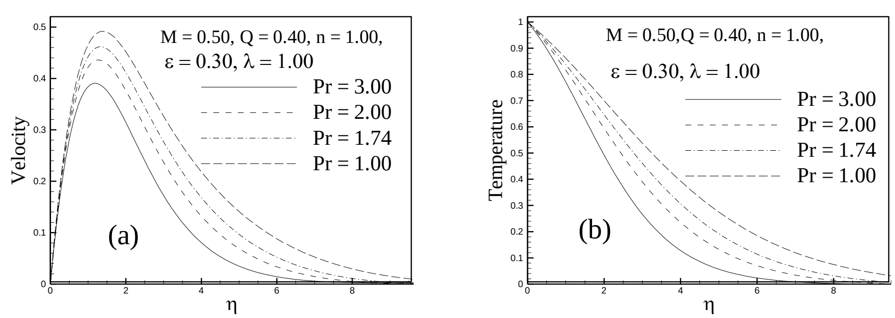
Variation of 3(a) velocity and 3(b) temperature profiles against η for *M* with *Pr* = 0.72, *Q* = 0.40, ε = 0.50, *n* = 1.00 and λ = 0.30.

Figure 3(a) and figure 3(b) display results for the velocity and temperature profiles, for different small values of magnetic parameter or Hartmann Number *M* = 0.10, 0.40, 0.60, 0.80,1.00 plotted against η at *Pr* = 0.72, ε = 0.50, *Q* = 0.40, λ = 0.30 and *n* = 1.00. It is seen from figure 3(a) that the velocity profile is influenced considerably and decreases when the value of magnetic parameter M increases. But near the surface of the plate velocity increases significantly and then decreases slowly and finally approaches to zero. Also it has been observed that the temperature field increases for increasing values of magnetic parameter or Hartmann Number M in figure 3(b).

Figure 4(a) and figure 4(b) display results for the velocity and temperature profiles, for different small values of viscous dissipation parameter ε (= 0.20, 0.40, 0.60, 0.80, 1.00) plotted against η at Pr = 0.72, M = 0.30, Q = 0.50, λ = 0.70 and n = 1.00. From figure 4(a), it is seen that an increase in the viscous dissipation parameter ε is associated with a considerable increase in velocity profiles but near the surface of the plate, the velocity increases and become maximum and then decreases and finally approaches to zero asymptotically. The maximum values of the velocity are 0.3735, 0.3671, 0.3609 and 0.3568 for ε = 0.20, 0.40, 0.60, 0.80, 1.00 respectively which occur at η = 1.3693 for all maximum values. Here we see that the velocity increases by 14.68% as ε increases from 0.20 to 1.00. A similar situation is also observed from figure 4(b) in the case of temperature field. Here it is seen that the local maximum values of the temperature profiles are 0.9630, 0.9416, 0.9216, 0.9031 for ε = 0.20, 0.40, 0.60, 0.80, 1.00 respectively and each of which occurs at the surface. Thus the temperature profiles increase by 6.63% as ε increases from 0.20 to 1.00.

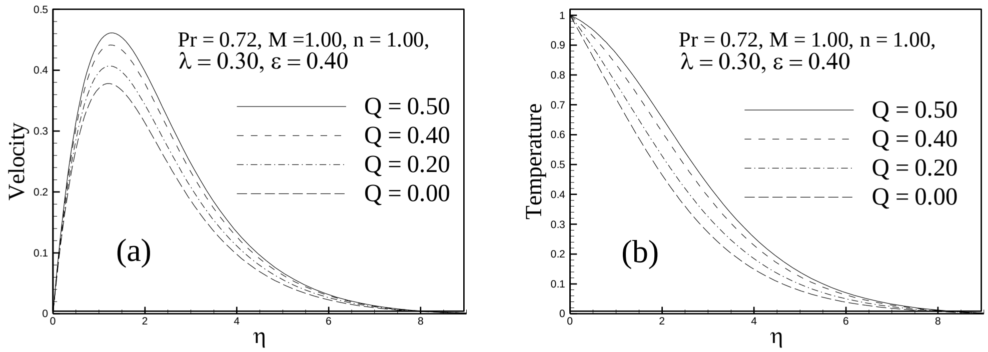


Variation of 4(a) velocity and 4(b) temperature profiles against η for ε with M = 0.30, Q = 0.50, Pr = 0.72, n = 1.00 and λ = 0.70.



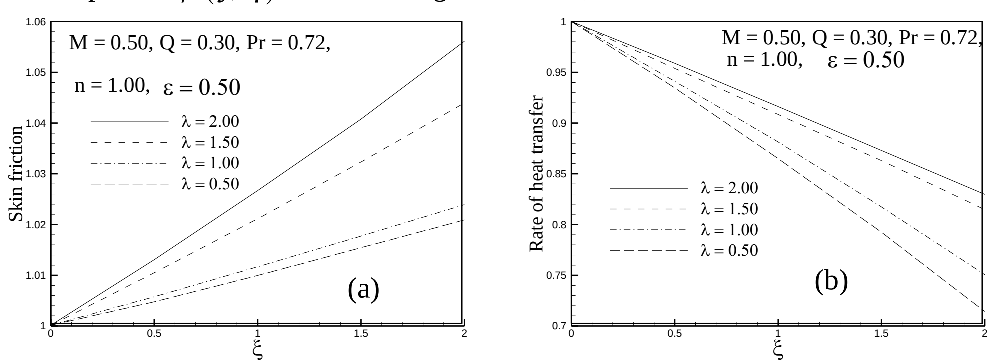
Variation of 5(a) velocity and 5(b) temperature profiles against η for Pr with M = 0.50, Q = 0.40, ε = 0.30, n = 1.00 and λ = 1.00

Figure 5 (a) depicts the velocity profiles for different values of the Prandtl's number *Pr* (= 1.00, 1.74, 2.00, 3.00) with the others controlling parameters M = 0.50, ε = 0.30, Q = 0.40, n = 1.00 and λ = 1.00. Corresponding distribution of the temperature profile $\phi(\xi,\eta)$ in the fluids is shown in figure 5(b). From figure 5(a), it can be seen that if the Prandtl's number increases the velocity of the fluid decreases. On the other hand, from figure 5(b) it is observed that the temperature profiles decrease within the boundary layer due to increase of the Prandtl's number *Pr*.



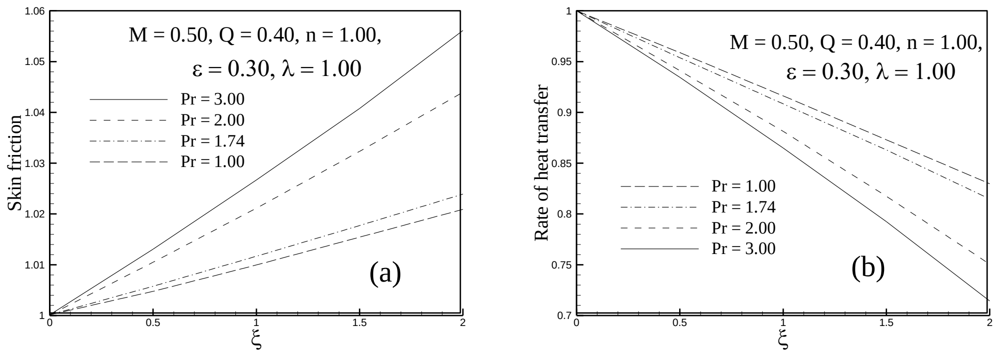
Variation of 6(a) velocity and 6(b) temperature profiles against η for Q with M = 1.00, Pr = 0.72, ε = 0.40, n = 1.00 and λ = 0.30.

Figure 6(a) and figure 6(b) deal with the effect of the heat generation parameter Q = 0.00, 0.20, 0.40, 0.50 for different values of the controlling parameters Pr = 0.72, M = 1.00, ε = 0.40, n = 1.00 and λ = 0.30 on the velocity profile $f'(\xi,\eta)$ and the temperature profiles $\phi(\xi,\eta)$. From figure 6(a), it is revealed that the velocity profile $f'(\xi,\eta)$ increases slightly with the increase of the heating generation parameter Q which indicates that heating generation parameter accelerates the fluid motion. Small increment is shown from figure 6(b) on the temperature profile $\phi(\xi,\eta)$ for increasing values of Q.



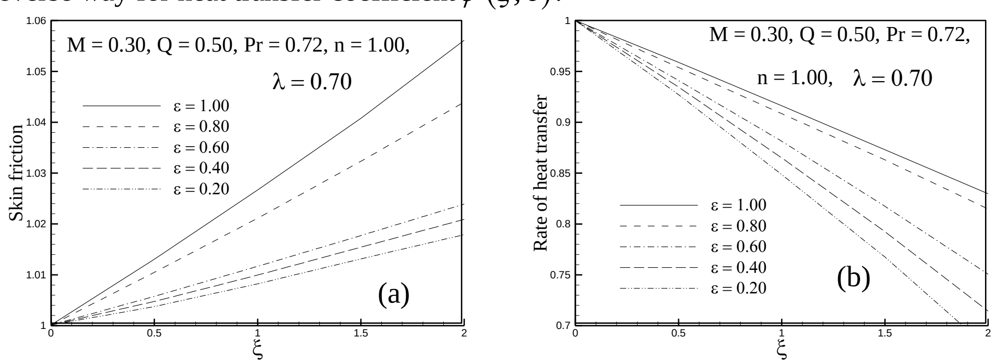
Variation of 7(a) skin friction and 7(b) heat transfer coefficient against ξ for λ with M = 0.50, Q = 0.30, ε = 0.50, n = 1.00 and Pr = 0.72.

Figure 7(a) and figure 7(b), represent the effects for different values of pressure work parameter λ for the magnetic parameter *M* =0.50, viscous dissipation parameter ε = 0.50, *n* = 1.00, heat generation parameter *Q* = 0.30 and Prandtl number *Pr* = 0.72 on the reduced skin friction coefficient $f''(\xi, 0)$ and rate of heat transfer $\phi'(\xi,0)$. The skin friction coefficient $f''(\xi, 0)$ and heat transfer coefficient $\phi'(\xi,0)$ increase with the increasing values of pressure work parameter λ.



Variation of 8(a) skin friction and 8(b) heat transfer coefficient against ξ for *Pr* with *M* = 0.50, *Q* = 0.40, ε = 0.30, *n* = 1.00 and λ = 1.00.

In figure 8(a), the skin friction coefficient $f''(\xi, 0)$ and figure 8(b), the rate of heat transfer $\phi'(\xi,0)$ are shown graphically for different values of the Prandtl number *Pr* (=1.00, 1.74, 2.00, 3.00) when other values of the controlling parameters are *M* = 0.50, ε = 0.30, *Q* = 0.40, *n* = 1.00 and λ = 1.00. Here skin friction is increase with the increasing values of *Pr*, but reverse way for heat transfer coefficient $\phi'(\xi,0)$.



Variation of 9(a) skin friction and 9(b) heat transfer coefficient against ξ for ε with *M* = 0.30, *Q* = 0.50, λ = 0.70, *n* = 1.00 and *Pr* = 0.72.

In figure 9(a) and it is seen that the skin friction coefficient $f''(\xi, 0)$ and the rate of heat transfer $\phi'(\xi,0)$ both are increases with the the increasing values of the pressure work parameter when other values of the controlling parameters are M = 0.30, λ = 0.70, Q = 0.50, n = 1.00 and Pr = 0.72.

6. CONCLUSIONS

From the present investigation, the following conclusions may be drawn:

- Both the skin friction coefficient and the velocity profile increase for increasing values of the viscous dissipation parameter ε and the pressure works parameter λ.
- Increased values of the viscous dissipation parameter ε leads to increase the rate of heat transfer coefficient as well as the temperature distribution.
- Increased values of the pressure work parameter λ leads to increase the rate of heat transfer coefficient as well as the temperature distribution.
- It has been observed that the skin friction coefficient, the rate of heat transfer coefficient, the temperature profile and the velocity profile decrease over the whole boundary layer with the increase of the Prandtl number *Pr*.
- The effect of magnetic parameter or Hartmann Number *M* is to decrease the velocity distribution over the whole boundary layer decrease, but reverse case happens for temperature distributions.

REFERENCES

1. J.A.D Ackroyd., Stress work effects in laminar flat plate natural convection. *J. Fluid Mech*. 62 pp. 677-695 (1962).
2. Gebhart, B., "Effects of viscous dissipation in natural convection". *J. Fluid Mech*. Vol.14, pp. 225-232 (1962).
3. B. Gebhart, J. 11Moendorf., viscous dissipation in external natural convection flows. *J. Fluid Mech*. 38 pp. 97-107 (1965).
4. Alam, Md. M., Alim, M. A. and Chowdhury, Md. M. K., (2006): Viscous Dissipation Effects on MHD Natural Convection Flow Along a Sphere. Journal of Mechanical Engineering, Vol.ME 36, No.2, pp. 44-48.
5. Alam, Md. M., Alim, M. A., and Chowdhury, Md. M. K., Effect of pressure stress work and viscous dissipation in natural convection flow along a vertical flat plate with heat conduction. *Jounal of Naval Architecture and Marine Engineering*, Vol.3, No.2, pp. 69-76 (2006).
6. Alim, M. A., Alam, Md. M., Abdullah-Al-Mamun and Belal Hossain, Md., Combined effect of viscous dissipation and Joule heating on the coupling of conduction and free convection along a vertical flat plate. *International Communication of Heat and Mass Transfer*, Vol.35, pp. 338-346 (2008).
7. Joshi and Gebhart., The effect of pressure stress work and viscous dissipation in some natural convection flows. *Int. J. Heat Mass Transfer*, Vol.24, No.10, pp. 1377-1388 (1981).

8. Miyamoto. M., Sumikawa. J., Akiyoshi.T. & Nakamura. T., Effect of axial heat conduction in a vertical flat plate on free convection heat transfer. *Int. J. Heat Mass Transfer*, Vol.23, pp.1545-1533 (1980).
9. Pozzi, A. and Lupo, M., The coupling of conduction with laminar natural convection along a flat plate. *Int. J. Heat Mass Transfer*, Vol.31, No.9, pp. 1807-1814 (1988).
10. Emad M. Abo-Eldahab*, Mohamed A. El Aziz., Viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. *J. Applied Mathematical Modelling* Vol. 29. pp. 579–595 (2005).