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# Heat Transfer on MHD Flow of Second Grade Fluid Through a Porous Medium in Rotating Channel with Hall Effects

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**ABSTRACT**

In this paper, we consider hydromagnetic convective flow of an electrically conducting visco-elastic fluid through a rotating porous channel taking hall current into account. The governing equations are framed using Brinkman model. The exact solutions of the velocity and the temperature distributions are evaluated analytically using Laplace transform technique and which consist of the both steady and transient states. Thought is centered on the physical character of the solutions, and the construction of the various kinds of boundary layers outward appearance on the plates. The ultimate steady state velocity and temperature distributions are numerically discussed for various values of the flow parameters. The shear stresses and the Nusselt number are tabulated and discussed.

**Keywords:** Convective flows, heat source, heat transfer, porous medium, rotating channels and unsteady state flows.

## 1. INTRODUCTION

The hydromagnetic rotating flow of non-Newtonian fluids between parallel plates has important applications in magnetohydrodynamic (MHD) power generators and pumps, accelerators … etc. The flow through porous medium is very important particularly in the fields of agricultural engineering and technology for irrigation processes, especially in petroleum industry to study petroleum extraction process and transport, also in chemical engineering and technology for filtration and purification processes. A succession of explorations made by Raptis *et al*.[1, 2 & 3] into the study of two-dimensional flow through porous medium past an infinite vertical wall. Debnath *et al*.[4] have studied the effects of Hall current on unsteady hydro magnetic flow past a porous plate in a rotating fluid system. Rao and Krishna[5 and 6] studied combined effects on the non-torsionally generated unsteady hydro magnetic flow in semi-infinite expansion of an electrically conducting viscous rotating fluid. Krishna and Suneetha[7 and 8] discussed the Hall current effects on unsteady flow of Newtonian fluid between two rigid non-conducting rotating plates. Recently, Hall effects on an unsteady MHD flow of a viscous incompressible electrically conducting fluid in a horizontal porous channel with variable pressure gradient in a rotating system have discussed by Sanatan Das and Rabindranath Jana[9]. Significant concern has been originated in the study of magnetic field and the electrically conducting fluids flow, while medium is porous[10]. The unsteady free convection fluid flows past porous infinite plate are investigated by Toki and Tokis[11]. The chemical reaction on an electrically conducting fluid through a porous medium with slip effects have presented by Senapati *et al*.[12]. MHD free convection flow of an incompressible viscous fluid near an oscillating plate embedded in a porous medium has been presented by Khan *et al*.[13]. Therefore, many researchers have studied free convection flow past a vertical plate with thermal radiation[14-16]. Recently, Krishna and Prakash[17] discussed the hall current effects on Unsteady MHD flow in a Rotating parallel plate channel bounded by Porous bed on the Lower half. The effects of radiation and hall current on MHD free convection three dimensional flow in a vertical channel filled with a porous medium has been studied by Krishna and Basha[18]. Krishna[19] discussed the unsteady flow of an incompressible electrically conducting viscous fluid in a rotating porous media, with a variable pressure gradient and in the presence of hall current. Recently, Krishna and Swarnalathamma[20] discussed the peristaltic MHD flow of an incompressible and electrically conducting Williamson fluid in a symmetric planar channel with heat and mass transfer under the effect of inclined magnetic field. Krishna and M.G. Reddy[21] discussed MHD free convective rotating flow of visco-elastic fluid past an infinite vertical oscillating plate. Krishna and G.S. Reddy[22] discussed unsteady MHD convective flow of second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source. Swarnalathamma and Krishna[23] discussed the theoretical and computational study of peristaltic hemodynamic flow of couple stress fluids through a porous medium under the influence of magnetic field with wall slip condition.

In this paper, we have considered hall effects on the hydromagnetic convective flow of an electrically conducting second grade fluid through a rotating porous channel using Brinkman model.

## 2. FORMULATION AND SOLUTION OF THE PROBLEM

We have considered the unsteady hydromagnetic convective flow of an electrically conducting second grade fluid through porous medium two parallel non conducting plates under a uniform transverse magnetic field $H_o$ taking hall current into account. At initial stage, both the plates and the fluid rotate with the same angular velocity $\Omega$. At $t > 0$, the fluid obsessed by a invariable pressure gradient parallel to the plate and in addition the lower plate performs non-torsional oscillation in its individual plane. We promoted the plates are cooled or heated by a unvarying temperature gradient in same direction parallel to the plane at the plates. The physical configuration of the problem is as shown in Figure. 1.

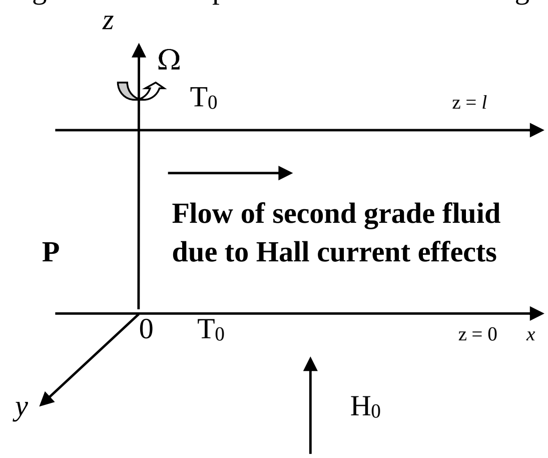


**Figure. 1 Physical configuration of the Problem**

We choose a Cartesian co-ordinate system O(*x, y, z*) such that the plates are at $z=0$ and $z=l$. The boundary layer equation of motion are given by

$$\frac{\partial u}{\partial t}-2\Omega v=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2\partial t}+\mu_e J_y H_0-\frac{\nu}{k}u \tag{1}$$

$$\frac{\partial v}{\partial t}+2\Omega u=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2\partial t}-\mu_e J_x H_0-\frac{\nu}{k}v \tag{2}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}-g(1-\beta(T-T_0))=0 \tag{3}$$

The energy equation is

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)(T-T_0)=\alpha_1\frac{\partial}{\partial z^2}(T-T_0)+\frac{Q}{\rho c_p}(T-T_0) \tag{4}$$

Since the plates extends to infinity along *x* and *y* directions, all the physical quantities except the pressure depend on z and t alone. When the potency of the magnetic field is very hefty, the generalized Ohm's law is tailored to include the Hall current, so that

$$J+\frac{\omega_e\tau_e}{B_0}J\times B=\sigma\,(E+V\times B+\frac{1}{en_e}\nabla p_e) \tag{5}$$

In equation (5) the electron pressure gradient, the ion-slip and thermo-electric effects are ignored and also the electric field $E=0$ under these assumptions trim downs to

$$J_x+m\ J_y=\sigma B_0 v \tag{6}$$

$$J_y + m\ J_x = -\sigma B_0 u \tag{7}$$

Where, $m = \omega_e \tau_e$ is the hall parameter.

On solving equations (6) and (2), we obtain

$$J_x = \frac{\sigma B_0}{1+m^2}(v + mu) \tag{8}$$

$$J_y = \frac{\sigma B_0}{1+m^2}(mv - u) \tag{9}$$

Substituting the equation (8) and (9) in the equations (1) and (2), we obtain**,**

$$\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} + \frac{\sigma\mu_e^2 H_0^2}{\rho\ 1+m^2}\ mv - u\ - \frac{\nu}{k}u \tag{10}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\sigma\mu_e^2 H_0^2}{\rho\ 1+m^2}\ v + mu\ - \frac{\nu}{k}v \tag{11}$$

Combining equations (10) and (11), let $q = u + iv$ and therefore we obtain

$$\frac{\partial q}{\partial t} + 2i\Omega q = -\frac{1}{\rho}\frac{\partial p}{\partial \xi} + \nu\frac{\partial^2 q}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 q}{\partial z^2 \partial t} + \frac{\sigma\mu_e^2 H_0^2}{\rho\ 1-im}q - \frac{\nu}{k}q \tag{12}$$

Integrating (3) we get,

$$\frac{p}{\rho} = -gz + \beta g\int (T - T_0)\,dz + \phi(\xi,\bar{\xi})H(t)$$

Where, $\xi = x - iy$, $\bar{\xi} = x + iy$

We use (3) in equation (12) and obtain,

$$\frac{\partial}{\partial z}\left(\frac{\partial q}{\partial t} + 2i\Omega q - \nu\frac{\partial^2 q}{\partial z^2} - \frac{\alpha_1}{\rho}\frac{\partial^3 q}{\partial z^2 \partial t} + \left(\frac{\sigma\mu_e^2 H_0^2}{\rho\ 1-im} + \frac{\nu}{k}\right)q\right) = -2\beta g\frac{\partial}{\partial \xi}\ T - T_0 \tag{13}$$

For the completeness of equation (2.13) we assume that

$$T - T_0 = (Ax + By)\,H(t) + \theta_1(z,t)$$

Where, $\theta_1(z,t)$ is an arbitrary function of $z$ and $t$, taking $T_0 + Ax + By + \theta_1\omega_1$ as the dimensional temperature of the lower and upper plates, respectively, for $t > 0$, we obtain the equation.

$$\left(\frac{\partial}{\partial t} + 2i\Omega - \nu\frac{\partial^2}{\partial z^2} - \frac{\alpha_1}{\rho}\frac{\partial^3}{\partial z^2 \partial t} + \frac{\sigma\mu_e^2 H_0^2}{\rho\ 1-im} + \frac{\nu}{k}\right)q = \beta g\ \ A + iB\ \ zH(t) + D \tag{14}$$

Where $D = \frac{\partial}{\partial \xi}\left[\phi(\xi,\bar{\xi})\,H(t)\right]$

The initial and boundary conditions are

$$q(z,t) = ae^{i\omega t} + be^{-i\omega t} \quad \text{at} \quad z = 0 \tag{15}$$

$$q(z,t) = 0 \quad \text{at} \quad z = l \quad \forall \; t \le 0 \text{ and } \forall \, z \tag{16}$$

$$q(z,t) = 0 \quad \text{at} \quad z = 0 \quad \forall \; t \le 0 \text{ and } \forall \, z \tag{17}$$

$$\theta(z,t) = \frac{\beta g l^3 (\theta_1\omega_2 - \theta_1\omega_1)}{\nu^2} = \theta_0 \quad \text{at} \quad z = l \tag{18}$$

Introducing the non-dimensional variables are,

$$z^* = \frac{z}{l}, q^* = \frac{ql}{\nu}, \; t^* = \frac{t\nu}{l^2}, \; \omega^* = \frac{\omega l^2}{\nu}, \; \theta^* = \frac{\beta g l^3 (\theta_1 - \theta_1\omega_1)}{\nu^2}$$

Using the non-dimensionalization process, the unsteady governing equations reduces to (dropping asterisks),

$$\frac{\partial^2 q}{\partial z^2} + S\frac{\partial^3 q}{\partial z^2 \partial t} - \left(\frac{M^2}{1-im} + 2iE^{-1} + D^{-2}\right)q - \frac{\partial q}{\partial t} = GrzH(t) + R \tag{19}$$

$$\frac{\partial^2 \theta}{\partial z^2} - \alpha\theta - Pr\left(\frac{\partial \theta}{\partial t} + (Gr_1 u + Gr_2 v)H(t)\right) = 0 \tag{20}$$

Where,

$M^2 = \frac{\sigma\mu_e^2 H_0^2 l^2}{\rho\nu}$ is the Hartmann number (Magnetic field parameter), $E = \frac{\nu}{\Omega l^2}$ is the Ekman number, $S = \frac{\alpha_1}{\rho l^2}$ is the second grade fluid parameter, $D^{-2} = \frac{l^2}{k}$ is the inverse Darcy Parameter, , $P_r = \frac{\mu C_p}{k_1}$ is the Prandtl number, $\alpha = \frac{Ql^2}{k_1}$ is the Heat source Parameter, $R = \left(-\frac{l^3}{\nu^3}\right)D$ is the Pressure gradient Parameter, $Gr = Gr_1 + iGr_2$ is the Grashof number,

The corresponding initial and boundary conditions are

$$q(z,t) = ae^{i\omega t} + be^{-i\omega t} \quad \text{at} \quad z = 0 \tag{21}$$

$$q(z,t) = 0 \quad \text{at} \quad z = 1 \quad \forall \, t \le 0, \forall \, z \tag{22}$$

$$\theta(z,t) = 0 \quad \text{at} \quad z = 0 \quad \forall \, t \le 0, \forall \, z \tag{23}$$

$$\theta(z,t) = \frac{\beta g l^3 (\theta_1\omega_2 - \theta_1\omega_1)}{\nu^2} = \theta_0 \quad \text{at} \quad z = 1 \tag{24}$$

Taking Laplace transforms in the equations (19) and (20), we obtain

$$1+sS\ \frac{d^2\bar{q}}{dz^2}-\left(s+\frac{M^2}{1-im}+2iE^{-1}+D^{-2}\right)\bar{q}=Gr\,z\,H(t)+R\ \frac{1}{s} \tag{25}$$

$$\frac{d^2\bar{\theta}}{dz^2}-\ s\,Pr+\alpha\ \ \bar{\theta}-Pr\ \ Gr_1u+Gr_2v\ \ H(t)=0 \tag{26}$$

Relavant transformed boundary conditions,

$$\bar{q}\ \ z,s\ =\frac{a}{s-i\omega}+\frac{b}{s+i\omega}\quad \text{at}\quad z=0 \tag{27}$$

$$\bar{q}\ \ z,s\ =0\quad \text{at}\quad z=1 \tag{28}$$

$$\bar{q}\ \ z,s\ =0\quad \text{at}\quad z=0 \tag{29}$$

$$\bar{\theta}\ \ z,s\ =\frac{\beta g l^3\ \ \theta_1\omega_2-\theta_1\omega_1}{\nu^2}=\theta_0\quad \text{at}\quad z=1 \tag{30}$$

We evaluated the constants involved in the transformed variables and the transformed velocity and temperature are given by

$$\bar{q}=\left(\frac{a}{s-i\omega}+\frac{b}{s+i\omega}+\frac{R}{s(1+sS)\lambda_1^2}+\frac{Grz}{(1+sS)\lambda_1^2}\right)\cosh\lambda_1 z+$$

$$\left\{-\left[\frac{a}{s-i\omega}+\frac{b}{s+i\omega}+\frac{R}{s(1+sS)\lambda_1^2}+\frac{Grz}{(1+sS)\lambda_1^2}\right]\frac{\cosh\lambda_1}{\sinh\lambda_1}+\frac{R}{s(1+sS)\lambda_1^2\sinh\lambda_1}\right.$$

$$\left.+\frac{Gr}{(1+sS)\lambda_1^2\sinh\lambda_1}\right\}\sinh\lambda_1 z-\frac{R}{s(1+sS)\lambda_1^2}-\frac{Grz}{(1+sS)\lambda_1^2} \tag{31}$$

$$\bar{\theta}=-\frac{\Pr\mathrm{Gr}}{\lambda_2^2\mathrm{s}}\cosh\lambda_2\mathrm{z}+\left\{\frac{\theta_0}{\sinh\lambda_2}+\frac{\Pr\mathrm{Gr}}{\lambda_2^2\mathrm{s}}\frac{\cosh\lambda_2}{\sinh\lambda_2}-\frac{\Pr\mathrm{Gr}}{\lambda_2^2\mathrm{s}}\frac{1}{\sinh\lambda_2}\right\}\sinh\lambda_2\mathrm{z}+\frac{\Pr\mathrm{Gr}}{\lambda_2^2\mathrm{s}} \tag{32}$$

Where, $\lambda_1^2=\dfrac{s+\dfrac{M^2}{1-im}+2iE^{-1}+D^{-2}}{1+sS}$ and $\lambda_2^2=Prs+\alpha$

We are taking inverse Laplace transforms (Bromwich contour integral formula) in the equations (31) and (32). Therefore we obtained the expressions for the velocity and temperature. The dimensional shear stresses $\tau_x$ and $\tau_y$ are obtained at the lower and upper plates and are given by

$$(\tau_x+\mathrm{i}\,\tau_y)_{\mathrm{z}=0}=\left(\frac{\mathrm{dq}}{\mathrm{qz}}\right)_{\mathrm{z}=0}\quad \text{and}\quad (\tau_x+\mathrm{i}\,\tau_y)_{\mathrm{z}=1}=\left(\frac{\mathrm{dq}}{\mathrm{qz}}\right)_{\mathrm{z}=1}$$

The rate of heat transfer coefficient (Nusselt number) on the plates and are given by

$$(\mathrm{Nu})_{z=0} = \left(\frac{\mathrm{d}\theta}{\mathrm{q}z}\right)_{z=0} \text{ and } (\mathrm{Nu})_{z=1} = \left(\frac{\mathrm{d}\theta}{\mathrm{q}z}\right)_{z=1}$$

## 3. RESULTS AND DISCUSSIONS

We have considered hydromagnetic convective flow of an electrically conducting second grade fluid through a porous medium in a rotating parallel plate channel taking hall current into account and in the presence of a temperature dependent heat source. The perturbations in the flow are produced by a constant pressure gradient along the plates in addition to non-torsional oscillations of the lower plate. The governing equations are framed using Brinkman model. The exact solutions of the velocity and the temperature distributions are evaluated analytically using Laplace transform technique. The analytical solution consists of the both steady and transient states. The quasi-steady parts of the velocity and temperature representing the ultimate flow have been computed numerically for different sets of governing parameters namely viz. The Hartmann parameter M, the inverse Darcy parameter $D^{-1}$, the Ekman number E, the hall parameter *m*, S second grade fluid parameter, Grashof number Gr, the frequency of oscillation $\omega$, Prandtl number Pr and Heat source parameter α, and their profiles are plotted in Figures (1-10) for the oscillating lower plate and for plates are in rest respectively. For computational purpose we have assumed Gr to be real so that the applied pressure gradient in the *y*-direction is zero and Gr is positive or negative according as the plates are heated or cooled along the direction of the *x*-axis (non-zero pressure gradient $R = 10$), also Prandtl number Pr is chosen to be $\mathrm{Pr} = 0.71$. Since the thermal buoyancy balances the vertical pressure gradient in the absence of any other applied force in the direction of rotation, the flow takes place in planes parallel to the boundary plates. However the flow is three dimensional and all the perturbed variables have been obtained using boundary layer type equations, which reduce to two coupled partial differential equations for a complex velocity and the real temperature. Figures (1-7) correspond to profiles for the velocity components *u* and *v*, Figures (8-10) correspond to profiles for temperature when one of the plates (lower) is oscillating with given amplitude and other is at rest. Tables (1-2) represent the shear stresses at the stationary and oscillatory plates while table (3) signify the rate of heat transfer at both the plates.

It is evident from the figures (1-7) that the velocity profiles are parabolic in nature. We noticed that, the magnitudes of the velocity components *u* enhance and *v* diminish throughout the fluid region with increasing Ekman number E or Second grade fluid parameter S or Hall parameter *m* being the parameters fixed (Fig 1, 4-5). The resultant velocity is also increases with increasing E, S and *m*. Both the velocity components *u* and *v* experiences retardation with increasing the intensity of the magnetic field (Hartmann number M). The application of the transverse magnetic field plays the important role of a resistive type force (Lorentz force) similar to drag force (that acts in the opposite direction of the fluid motion) which tends to resist the flow thereby decreasing its velocity (Fig 2). Similar behaviour is observed for the resultant velocity. It is also noted from the Fig 3 the magnitude of the velocity component *u* diminish throughout the fluid region and the behaviour of the velocity component *v* remains the same with increasing the inverse Darcy parameter $D^{-1}$. We observe that lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. The resultant velocity is also trim downs throughout the fluid region.

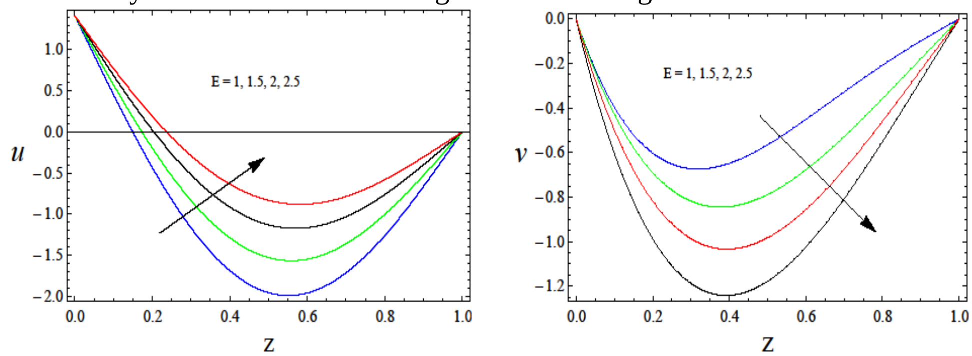


**Figure 1. The velocity profiles for *u* and *v* against E**

$$M = 1, m = 1, D^{-1} = 50, S = 1, \omega = \pi / 4, Gr = 2$$

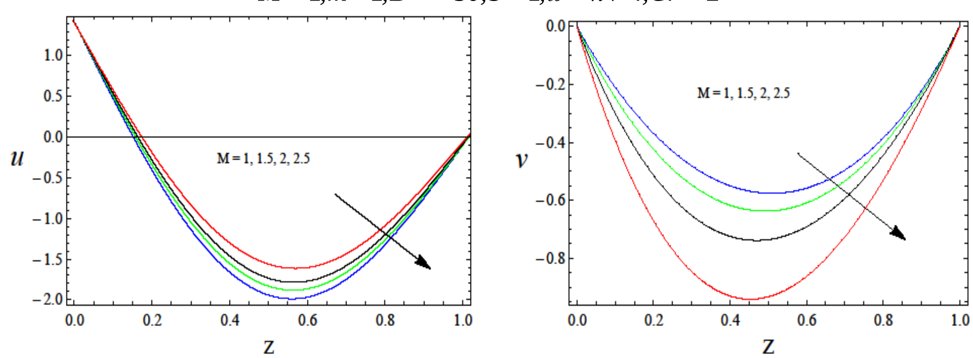


**Figure 2. The velocity profiles for *u* and *v* against M**

$$E = 1, m = 1, D^{-1} = 50, S = 1, \omega = \pi / 4, Gr = 2$$

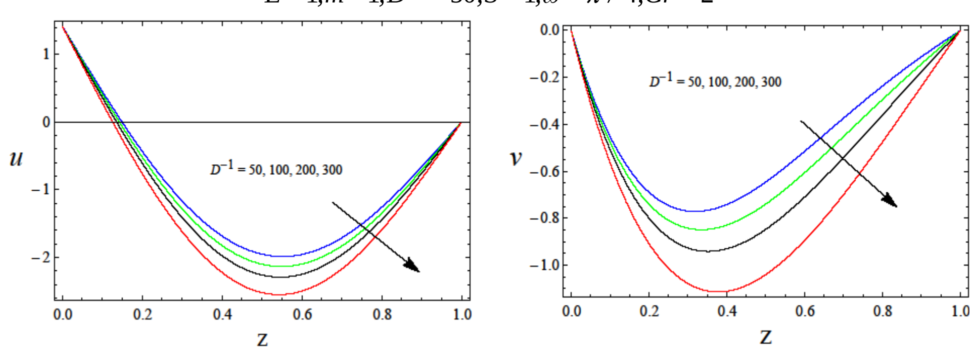


**Figure 3. The velocity profiles for *u* and *v* against $D^{-1}$**

$$E = 1, m = 1, M = 1, S = 1, \omega = \pi / 4, Gr = 2$$

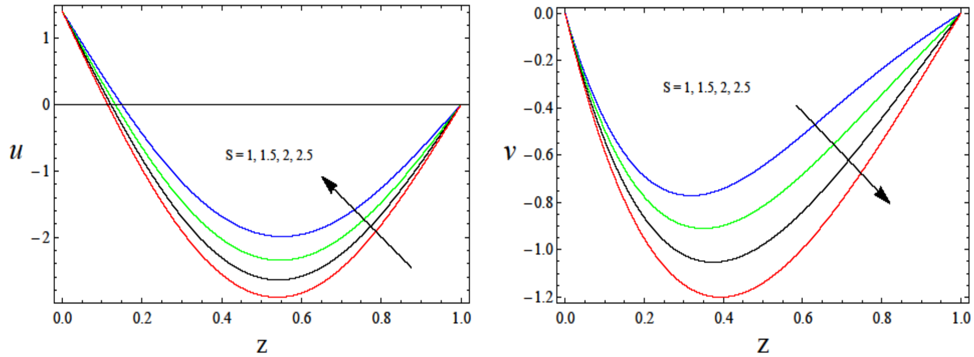


**Figure 4. The velocity profiles for *u* and *v* against S**

$E=1, m=1, D^{-1}=50, M=1, \omega=\pi/4, Gr=2$

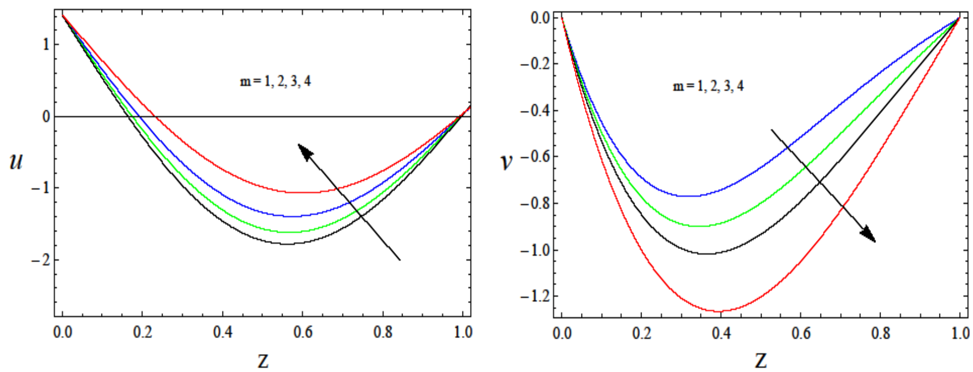


**Figure 5. The velocity profiles for *u* and *v* against *m***

$E=1, M=1, D^{-1}=50, S=1, \omega=\pi/4, Gr=2$

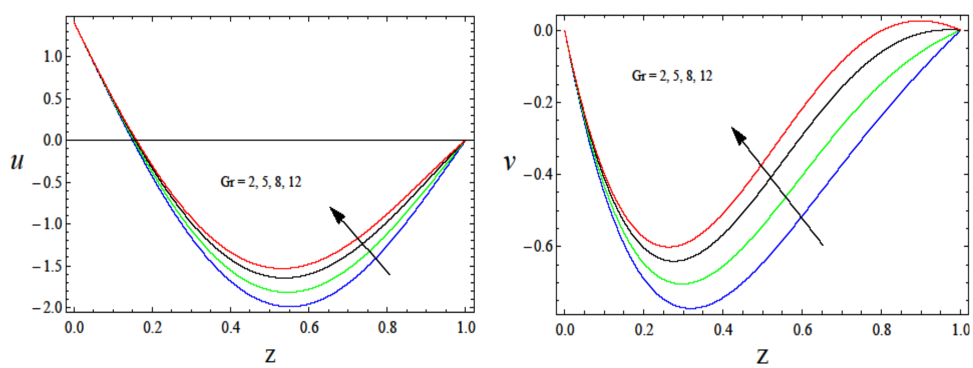


**Figure 6. The velocity profiles for *u* and *v* against Gr**

$E=1, m=1, D^{-1}=50, S=1, \omega=\pi/4, M=1$

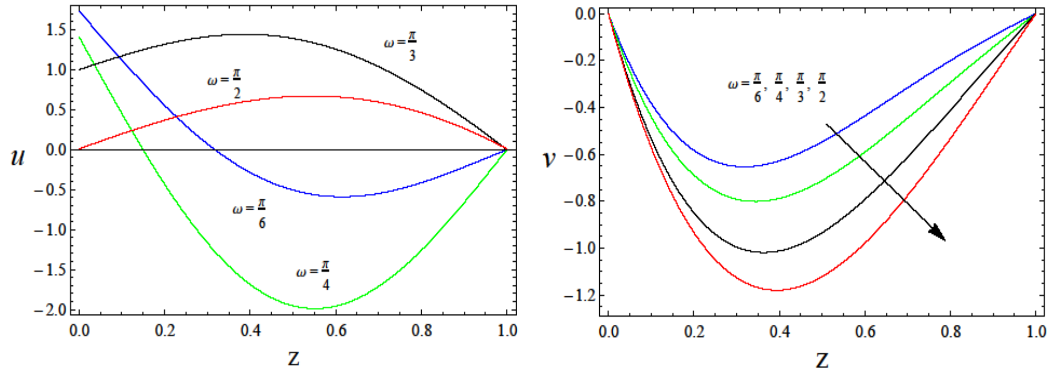


**Figure 7. The velocity profiles for *u* and *v* against $\omega$**

$$E = 1, m = 1, D^{-1} = 50, S = 1, M = 1, Gr = 2$$

An increase in Grashof number leads to raise both the primary velocity *u* and the secondary velocity *v* as shown in Fig (6). This is because; increase in Grashof number Gr means more heating and less density. The resultant velocity is also boost up throughout the fluid region. From the figure (7) depicts the magnitude of the velocity component *u* oscillates in the entire fluid region where as the velocity component *v* diminishes with increasing the frequency of oscillation *ω*. The resultant velocity is also reduces throughout the fluid region *ω*.

It is evident from the Fig (8-9) displays that the fluid temperature increases with an increase in Hartmann number M, Second grade fluid parameter S, the inverse Darcy parameter $D^{-1}$, Prandtl number Pr, Hall parameter *m* and the frequency of oscillation *ω*. The temperature reduces with increasing Ekman number E, Heat source parameter $\alpha$ and Grashof number Gr.

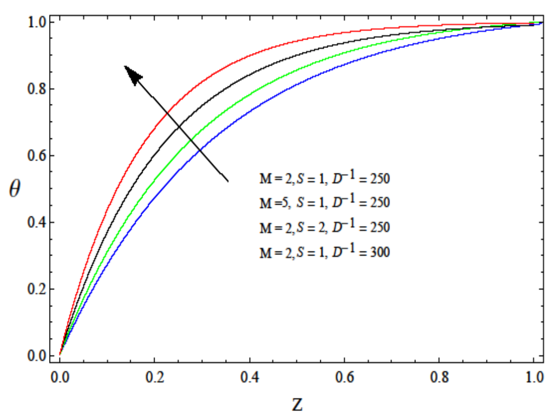


**Figure 8. The Temperature Profiles for $\theta$ with M, S, and D[-1]**

$$E = 1, m = 1, Gr = 2, \alpha = 5, Pr = 0.71, \omega = \pi / 4,$$

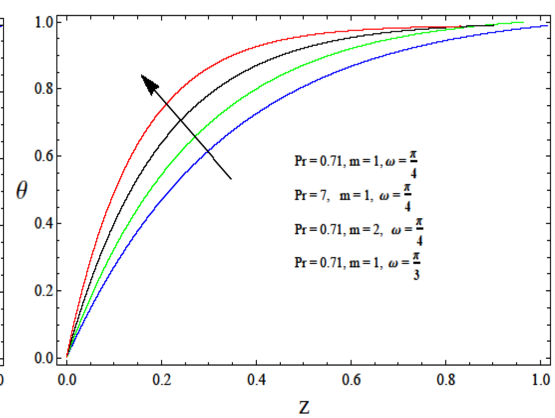


**Figure 9. The Temperature Profiles for $\theta$ with Pr, m, and $\omega$**

$$E = 1, D^{-1} = 250, S = 1, M = 1, Gr = 2, \alpha = 5$$

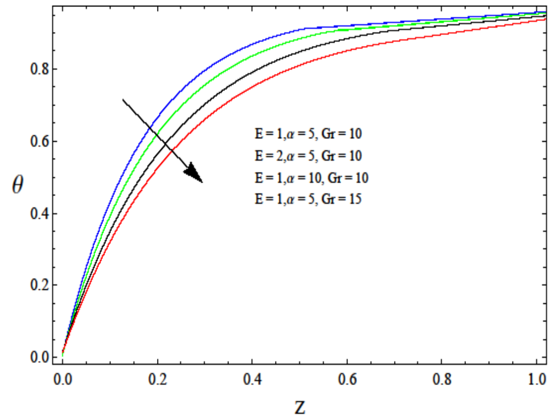


**Figure 10. The Temperature Profiles for $\theta$ with E, $\alpha$ and Gr**

$Pr = 0.71, m = 1, D^{-1} = 250, S = 1, M = 1, \omega = \pi/4$

**Table 1: The Shear stress ($\tau_x$ & $\tau_y$) on the stationary plate (upper plate)**

| E | M | $D^{-1}$ | $S$ | $m$ | Gr | $\omega$ | $\tau_x$ | $\tau_y$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi/4$ | 2.524785 | -4.36658 |
| 1.5 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi/4$ | 2.635525 | -4.98855 |
| 2 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi/4$ | 2.855474 | -5.45885 |
| 1 | 3 | 100 | 1 | 1 | 5 | $\omega = \pi/4$ | 2.422458 | -8.45784 |
| 1 | 5 | 100 | 1 | 1 | 5 | $\omega = \pi/4$ | 2.002544 | -10.5885 |
| 1 | 2 | 200 | 1 | 1 | 5 | $\omega = \pi/4$ | 2.144581 | -5.47854 |
| 1 | 2 | 300 | 1 | 1 | 5 | $\omega = \pi/4$ | 1.520248 | -7.45887 |
| 1 | 2 | 100 | 2 | 1 | 5 | $\omega = \pi/4$ | 2.855255 | -3.22145 |
| 1 | 2 | 100 | 3 | 1 | 5 | $\omega = \pi/4$ | 3.554855 | -2.74466 |
| 1 | 2 | 100 | 1 | 2 | 5 | $\omega = \pi/4$ | 4.255857 | -5.14114 |
| 1 | 2 | 100 | 1 | 3 | 5 | $\omega = \pi/4$ | 6.477845 | -6.57485 |
| 1 | 2 | 100 | 1 | 1 | 10 | $\omega = \pi/4$ | 1.545587 | -2.44812 |
| 1 | 2 | 100 | 1 | 1 | 15 | $\omega = \pi/4$ | 0.378547 | -1.85547 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi/3$ | 6.665485 | -7.14579 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi/2$ | 8.788514 | -9.55854 |

The shear stresses ($\tau_x$ & $\tau_y$) and Nusselt number (Nu) are calculated computationally at the stationary and oscillatory plates and tabulated on the tables (1-3). At the stationary plate, the magnitude of the stresses $\tau_x$ and $\tau_y$ enhances with increasing Ekman number E, hall parameter $m$ and the frequency of oscillation $\omega$. The reversal behavior is observed for increasing Grashof number Gr. The stresses $\tau_x$ reduce and $\tau_y$ increase in magnitude with increasing the intensity of the magnetic field M or inverse Darcy parameter $D^{-1}$, whereas stresses $\tau_x$ increases and $\tau_y$ decreases with increasing second grade fluid parameter S (Table 1). At the oscillatory plate, the magnitude of the stresses $\tau_x$ and $\tau_y$ enhances with increasing E, $D^{-1}$ and S. The stresses $\tau_x$ reduce and $\tau_y$ increase in magnitude with increasing M or Gr. The reversal behavior is observed for increasing *m* or $\omega$ (Table 2).

**Table 2: The Shear stress ( $\tau_x$ and $\tau_y$ ) on the oscillatory plate (Lower plate)**

| E | M | $D^{-1}$ | *S* | *m* | Gr | $\omega$ | $\tau_x$ | $\tau_y$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 0.684758 | -0.082547 |
| 1.5 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 0.817566 | -0.095478 |
| 2 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 0.985547 | -0.245781 |
| 1 | 3 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 0.547885 | -0.098854 |
| 1 | 5 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 0.366589 | -0.154421 |
| 1 | 2 | 200 | 1 | 1 | 5 | $\omega = \pi / 4$ | 0.945252 | -0.088774 |
| 1 | 2 | 300 | 1 | 1 | 5 | $\omega = \pi / 4$ | 1.255478 | -0.097485 |
| 1 | 2 | 100 | 2 | 1 | 5 | $\omega = \pi / 4$ | 1.547889 | -0.214588 |
| 1 | 2 | 100 | 3 | 1 | 5 | $\omega = \pi / 4$ | 2.544785 | -0.874855 |
| 1 | 2 | 100 | 1 | 2 | 5 | $\omega = \pi / 4$ | 1.244748 | -0.068547 |
| 1 | 2 | 100 | 1 | 3 | 5 | $\omega = \pi / 4$ | 1.869558 | -0.024577 |
| 1 | 2 | 100 | 1 | 1 | 10 | $\omega = \pi / 4$ | 0.455874 | -0.145585 |
| 1 | 2 | 100 | 1 | 1 | 15 | $\omega = \pi / 4$ | 0.211452 | -0.524884 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 3$ | 0.714996 | -0.065847 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 2$ | 0.9147485 | -0.032554 |

The magnitudes of the rate heat transfer Nu enhance with increasing Ekman number E, inverse Darcy parameter $D^{-1}$, hall parameter *m*, Grashof number Gr, the frequency of oscillation $\omega$ and Prandtl number Pr at either plates. The Nusselt number increases at stationary plate and reduces at oscillatory plate with increasing second grade fluid parameter S, where as reversal trend observed with increasing the intensity of the magnetic field M or Heat source parameter $\alpha$ (Table 3).

**Table 3: The Rate of Heat transfer (Nusselt number) at stationary plate and oscillatory plate**

| E | M | $D^{-1}$ | $S$ | $m$ | Gr | $\omega$ | $\alpha$ | Pr | Stationary plate | Oscillatory plate |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 1.588748 | 3.452287 |
| 1.5 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 2.554887 | 5.647855 |
| 2 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 3.221142 | 6.958447 |
| 1 | 3 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 1.255447 | 4.899657 |
| 1 | 5 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 0.855475 | 6.458874 |
| 1 | 2 | 200 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 2.855478 | 2.457848 |
| 1 | 2 | 300 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 3.988758 | 2.011475 |
| 1 | 2 | 100 | 2 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 1.885574 | 2.855148 |
| 1 | 2 | 100 | 3 | 1 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 2.155245 | 2.255485 |
| 1 | 2 | 100 | 1 | 2 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 5.788547 | 7.899658 |
| 1 | 2 | 100 | 1 | 3 | 5 | $\omega = \pi / 4$ | 5 | 0.71 | 8.966585 | 11.25485 |
| 1 | 2 | 100 | 1 | 1 | 10 | $\omega = \pi / 4$ | 5 | 0.71 | 5.785547 | 9.855478 |
| 1 | 2 | 100 | 1 | 1 | 15 | $\omega = \pi / 4$ | 5 | 0.71 | 8.477485 | 15.44748 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 3$ | 5 | 0.71 | 1.988548 | 5.668547 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 2$ | 5 | 0.71 | 2.688995 | 8.014225 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 8 | 0.71 | 1.255485 | 8.744856 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 10 | 0.71 | 0.007588 | 13.66254 |
| 1 | 2 | 100 | 1 | 1 | 5 | $\omega = \pi / 4$ | 5 | 7 | 2.855479 | 8.255478 |

## 4. CONCLUSIONS

1. The resultant velocity is increases with increasing E, S and *m*.
2. Lower the permeability of the porous medium lesser the fluid speed in the entire fluid region.
3. An increase in Grashof number leads to raise both the primary velocity *u* and the secondary velocity *v*.
4. The temperature reduces with increasing E, $\alpha$ and Gr.
5. The magnitude of the both stresses enhances with increasing E, *m* and *ω*, also reduces for increasing Gr at the stationary plate.
6. Nu increases at stationary plate and reduces at oscillatory plate with increasing S, where as reversal trend observed with increasing M or $\alpha$.

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