Journal of Computer and Mathematical Sciences, *Vol.11(6),27-33 June 2020* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

L(2,1)-Labeling of Oxide and Silicate Networks

Kins Yenoke*, D. Francis Xavier and Temi Maryson

Department of Mathematics,
Loyola College, Chennai-600 034, INDIA.
Email:jecinthokins@rediffmail.com.

(Received on: May 31, 2020)

ABSTRACT

An **L(2,1)-*labeling*** of a graph G is a function from the vertex set $V(G)$ to the set of all non-negative integers such that $|f(u) - f(v)| \geq 2$ if $d(u,v) = 1$ and $|f(u) - f(v)| \geq 1$ if $d(u,v) = 2$, where $d(u,v)$ denotes the distance between u and v in G. The ***L(2,1)-labeling number*** of G, denoted by $\lambda_{2,1}(G)$, is the smallest number k such that there is an L(2,1)-labeling with maximum label k. In this paper, we have determined the bounds for L(2,1)- labeling number of Oxide and Silicate networks.

Keywords: Labeling, L(2,1)-labeling, L(2,1)-labeling number, Oxide network, Silicate network.

1. INTRODUCTION

The Frequency Assignment Problem (FAP) is to assign frequencies to transmitters in a wireless network. Each transmitter is assigned a frequency channel for its transmissions in a broadcasting network. In case if both their channels are too close transmissions can interfere. This problem was first formulated as a graph coloring problem by Hale[12] who introduced the notion of T-coloring of a graph. Griggs *et al*.[4] introduced L(2,1)-labeling in the year 1992. In 1991, Roberts[3] proposed a variation of the frequency assignment problem where he mentions that the close transmitters must receive different channels and also adds that very close transmitters must receive channels at least two apart. Usually the transmitters are represented by the vertices of a graph. Two transmitters are 'very close' if they are adjacent in the graph and 'close' if they are of distance two in the graph. The bounds $\lambda_{2,1}(G)$ where computed systematically for some graphs like cycle[4], path[2], tree[6]. Bounds have also been obtained for various graphs like unit interval graphs[4], chordal graphs[10], planar graphs[7] and hypercubes[8]. Simple graphs like path, cycle, complete bipartite graph, tree can give exact L(2,1)-labeling, but for complex graph structure like cartesian product between different graphs is not as smooth as simple graphs. In this paper we have investigated the bounds for L(2,1)-labeling number of Silicate and Oxide networks.

2. SILICATE AND OXIDE NETWORK

Paul Manuel *et al.*[9] introduced Silicate and Oxide Networks in 2009. Silicates are obtained by fusing metal oxides or metal carbonates with sand. Essentially all the silicates contain SiO_4 tetrahedra. In chemistry the corner vertices of SiO_4 tetrahedran represent oxygen ions and the center vertex represents the silicon ion. In graph theory, we call the corner vertices as oxygen nodes and the center vertex as silicon node. The minerals are obtained by successively fusing oxygen nodes of two tetrahedra of different silicates. The different types of silicate structure arise from the ways in which these tetrahedra are arranged: they may exist as separate unlinked entities, as linked finite arrays, as 1-dimension chains, as 2-dimensional sheets or as 3-dimensional frameworks. They are termed as orthosilicates, pyrosilicates, chain silicates, cyclic silicates and sheet silicates. The silicate network SL(1) is a cyclic silicate with six SiO_4 tetrahedra units. The silicate network SL(2) is obtained by adding six units of SL(1) such that each outer SL(1) shares two consecutive tetrahedra of inner SL(1). Inductively, silicate network SL(n) is obtained from SL(n-1) by adding a layer of SL(1) around the boundary of SL(n-1). There are 6n number of SL(1) in the outer layer of SL(n). The parameter n of SL(n) is called the dimension of SL(n). The number of nodes in SL(n) is $15n^2 + 3n$ and the number of edges of SL(n) is $36n^2$. When all the silicon nodes are deleted from a silicate network, we obtain a new network which we call as an Oxide network. An n-dimensional oxide network is denoted by OX(n) having number of vertices $9n^2 + 3n$ and number of edges is $18n^2$.

In this paper we named the vertices of Silicate network as follows: As the silicate network is divided symmetrically into two parts, we first name the vertices of symmetric line as $u_{0,1}, \ldots, u_{0,2n}$, then we name the oxygen nodes parallel to symmetric lines above and below as $u_{\pm 1,1}, \ldots, u_{\pm n,n}$ respectively. Next, we name the remaining oxygen nodes in upper and lower parts as $v_{1,1}, \ldots, v_{n,2(n+n)}$ and $v'_{1,1}, \ldots, v'_{n,2(n+n)}$ respectively. Finally, we name the silicon nodes in the upper half and lower half as $w_{1,1}, \ldots, w_{n,2n}$ and $w'_{1,1}, \ldots, w'_{n,n}$ respectively. See Figure 1.

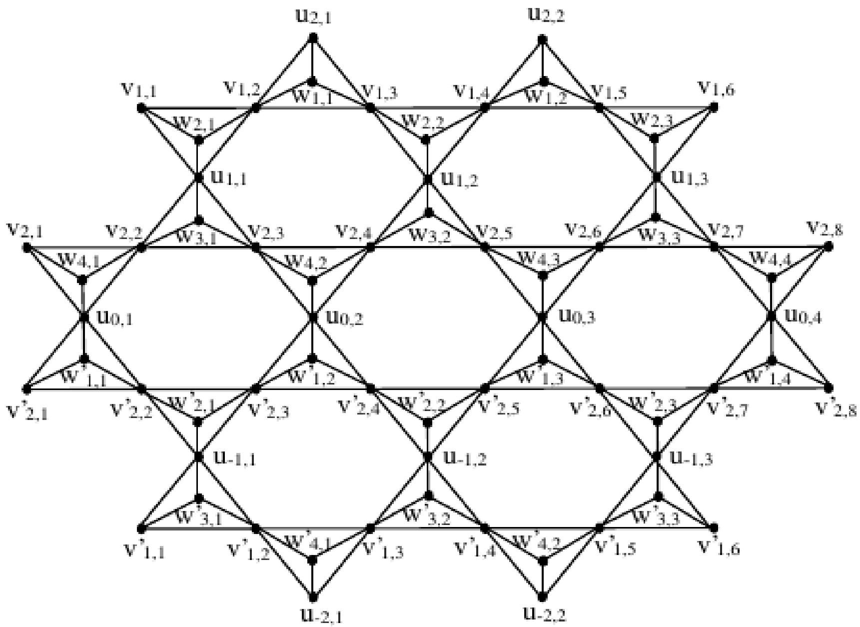

***Figure* 1*: Naming of nodes in SL*(2)**

3. L(2,1)-LABELING OF OXIDE AND SILICATE NETWORKS

Definition 3.1: An **L(2,1)-labeling** of a graph *G* is a function from the vertex set $V(G)$ to the set of all non-negative integers such that $|f(u)-f(v)| \geq 2$ if $d(u,v)=1$ and $|f(u)-f(v)| \geq 1$ if $d(u,v) = 2$, where $d(u,v)$ denotes the distance between *u* and *v* in *G*. The ***L*(2,1)-labeling number** of *G*, denoted by $\lambda_{2,1}(G)$, is the smallest number *k* such that there is an $L(2,1)$-labeling with maximum label *k*.

Theorem 3.1: Let G be an oxide network $OX(n)$. Then the $L(2,1)$-labeling number of G satisfies $\lambda_{2,1}(G) \leq 8$.

Proof: First we partition the vertex set into four disjoint sets as

$$V_1 = \{v_{1,1}, v_{1,2}, \ldots, v_{1,2(n+1)}, v_{2,1}, v_{2,2}, \ldots, v_{2,2(n+2)}, \ldots, v_{n,1}, v_{n,2}, \ldots, v_{n,2(n+n)}\}$$

$$V_2 = \{v'_{1,1}, v'_{1,2}, \ldots, v'_{1,2(n+1)}, v'_{2,1}, v'_{2,2}, \ldots, v'_{2,2(n+2)}, \ldots, v'_{n,1}, v'_{n,2}, \ldots, v'_{n,2(n+n)}\}$$

$$U_0 = \{u_{0,1}, u_{0,2}, \ldots, u_{0,2n}\} \text{ and}$$

$$U_1 = \{u_{\pm 1,1}, u_{\pm 1,2}, \ldots, u_{\pm 1,2n-1}, u_{\pm 2,1}, u_{\pm 2,2}, \ldots, u_{\pm 2,2n-2}, \ldots, u_{\pm n,1}, \ldots, u_{\pm n,n}\}$$

Define a mapping $f : V(G) \rightarrow \mathrm{N}$ as follows:

$$f(u_{\pm i,j}) = \begin{cases} 1 \equiv i \bmod 2 \\ 0 \equiv i \bmod 2 \end{cases}, \forall j \text{ and } 1 \leq i \leq n$$

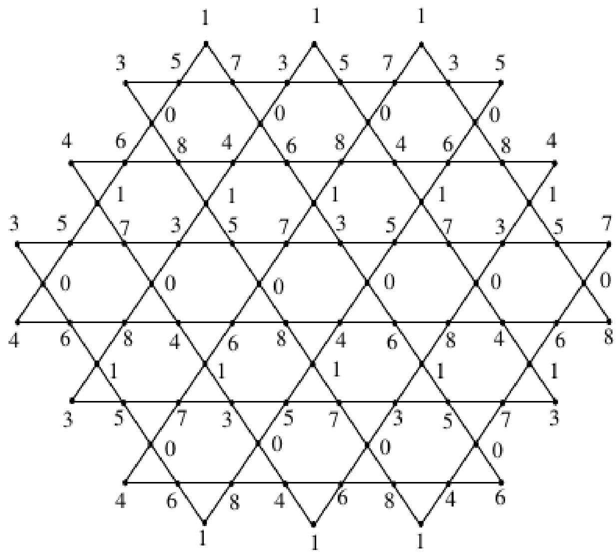

Figure 2: $L(2,1)$ −Labeling of $OX(3)$

$$f(v_{i,j}) = \begin{cases} 3, & 1 \equiv j \bmod 3 \\ 5, & 2 \equiv j \bmod 3 \\ 7, & 0 \equiv j \bmod 3 \end{cases}, \; i \text{ is odd and } 1 \le j \le 2(n+n)$$

$$f(v'_{i,j}) = \begin{cases} 4, & 1 \equiv j \bmod 3 \\ 6, & 2 \equiv j \bmod 3 \\ 8, & 0 \equiv j \bmod 3 \end{cases}, \; i \text{ is even and } 1 \le j \le 2(n+n)$$

Next we verify the L(2,1)-labeling condition $d(u,v) + |f(u) - f(v)| \geq 3 \; \forall \, u, v \in V(G)$. Let $u, v \in V(G)$.

***Case* 1:** Suppose $u, v \in U_1$ then $u = u_{l,m}, \; v = v_{s,t}$ where $1 \le l, s \le n$ and $1 \le m, t \le n$.

***Case* 1.1:** If $l = s$ and $m \neq t$ then $d(u,v) \geq 3$ and either $f(u) = 0, f(v) = 0$ or $f(u) = 1, f(v) = 1$. Therefore $d(u,v) + |f(u) - f(v)| \geq 3$.

***Case* 1.2:** If $l \neq s$ and $l = s + 1$ then either $f(u) = 0$ and $f(v) = 1$ or $f(u) = 1$ and $f(v) = 0$. Also the distance between *u* and *v* is exactly 2. Hence $d(u,v) + |f(u) - f(v)| = 2 + |0 - 1| = 3$ or $d(u,v) + |f(u) - f(v)| = 2 + |1 - 0| = 3$.

***Case* 1.3:** If $l \neq s$ and $l \neq s + 1$ then $f(u) = f(v) = 0$ or $f(u) = f(v) = 1$. Therefore $d(u,v) + |f(u) - f(v)| = 4 + |f(u) - f(v)| = 4$.

***Case* 2:** Suppose $u, v \in V_1$ then *u* and *v* are of the form $u = v_{k,j}$ and $v = v_{l,m}$ where $1 \le k, l \le n$ and $1 \le j, m \le 2(n+n)$.

***Case* 2.1:** Suppose *u* and *v* lie on the same line then the following possibilities occurs. If $1 \equiv k \bmod 3, 1 \equiv l \bmod 3$ and $l = k$ then $f(u) = f(v) = 3, d(u,v) \geq 3$. If $2 \equiv k \bmod 3, 2 \equiv l \bmod 3$ and $l = k$ then $f(u) = f(v) = 5, d(u,v) \geq 3$. If $0 \equiv k \bmod 3, 0 \equiv l \bmod 3$ and $l = k$ then $f(u) = f(v) = 7, d(u,v) \geq 3$. Thus in all possibilities $d(u,v)$

$+ |f(u) - f(v)| \geq 3$.

***Case* 2.2:** Suppose u and v lie in different lines then $u = v_{i,k}$ and $v = v_{l,k}$, $i \neq l$ then either $d(u,v) = 2$ *and* $|f(u) - f(v)| \geq 1$ or $d(u,v) = 3$ and $|f(u) - f(v)| \geq 0$. Hence in both the possibilities $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 3:** Suppose $u, v \in V_2$ then u and v are of the form $u = v'_{k,j}$ and $v = v'_{l,m}$ where $1 \leq k, l \leq n$ and $1 \leq j, m \leq 2(n+n)$.
***Case* 3.1:** Suppose u and v lie on the same line then the following possibilities occurs. If $1 \equiv k \bmod 3, 1 \equiv l \bmod 3$ and $l = k$ then $f(u) = f(v) = 4$ and $d(u,v) \geq 3$. If $2 \equiv k \bmod 3, 2 \equiv l \bmod 3$ and $l = k$ then $f(u) = f(v) = 6$ and $d(u,v) \geq 3$. If $0 \equiv k \bmod 3, 0 \equiv l \bmod 3$ and $l = k$ then $f(u) = f(v) = 8$ and $d(u,v) \geq 3$. Thus in all possibilities $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 3.2:** Suppose u and v lie in different lines then u and v are of the form $u = v'_{i,k}$ and $v = v'_{l,k}$, $i \neq l$ then either $d(u,v) = 2$ and $|f(u) - f(v)| \geq 1$ or $d(u,v) = 3$ and $|f(u) - f(v)| \geq 0$. Hence in both the possibilities $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 4:** Suppose $u \in V_1$ and $v \in V_2$ then $u = v_{i,k}$ and $v = v'_{l,k}$ where $1 \leq i, l \leq n$ and $1 \leq k \leq 2(n+n)$, then $f(u) = 3,5$ or 7 and $f(v) = 4,6$ or 8. Also $d(u,v) = 2, |f(u) - f(v)| \geq 1$. Hence $d(u,v) + |f(u) - f(v)| \geq 2 + 1 = 3$.
***Case* 5:** Suppose $u \in V_1$ and $v \in U_1$ then u and v are of the form $u = v_{i,j}$ where $1 \leq i \leq n, 1 \leq j \leq 2(n+n)$ and $v = v_{k,j}$ where $1 \leq k, j \leq n$. Also $f(u) = 3,5$ or 7 and $f(v) = 0$ or $f(u) = 4, 6$ or 8 and $f(v) = 1$. Hence in both the possibilities $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 6:** Suppose $u \in V_2$ and $v \in U_1$ then u and v are of the form $u = v'_{i,j}$ where $1 \leq i \leq n, 1 \leq j \leq 2(n+n)$ and $v = v_{k,j}$ where $1 \leq k, j \leq n$. Also $f(u) = 3,5$ or 7 and $f(v) = 1$ or $f(u) = 4, 6$ or 8 and $f(v) = 0$. Thus in both possibilities $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 7:** Suppose $u, v \in U_0$ then $d(u,v) = 3$ and $f(u) = f(v) = 0$ or $f(u) = f(v) = 1$. Hence in both possibilities $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 7.1:** Suppose $u \in U_0$ and $v \in V_1$, then $f(u) = 0$ or 1 and $f(v) = 3, 5$ or 7, $f(u) = 0$ or 1 and $f(v) = 4, 6$ or 8. Also the distance between u and v is at least 1. Hence in both cases $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 7.2:** Suppose $u \in U_0$ and $v \in V_2$. $f(u) = 0$ or 1, $f(v) = 3,5$ or 7 or $f(u) = 0$ or 1, $f(v) = 4, 6$ or 8 and $d(u,v) \geq 1$. Hence in both cases $d(u,v) + |f(u) - f(v)| \geq 3$.
***Case* 7.3:** Suppose $u \in U_0$ and $v \in U_1$, then either $f(u) = 0$ and $f(v) = 1$ or $f(u) = 1$and $f(v) = 0$. Hence in both cases we get $d(u,v) + |f(u) - f(v)| \geq 3$. Therefore f is a valid $L(2,1)$ –labeling and hence $\lambda_{2,1}(G) \leq 8$.

Theorem 3.2: Let G be an Silicate network SL(n). Then the L(2,1)-labeling number of G satisfies $\lambda_{2,1}(G) \leq 10$.

Proof: First we partition the vertex set into six disjoint sets say

$$V_1 = \{v_{1,1}, v_{1,2}, \dots, v_{1,2(n+1)}, v_{2,1}, v_{2,2}, \dots, v_{2,2(n+2)}, \dots, v_{n,1}, v_{n,2}, \dots, v_{n,2(n+n)}\}$$
$$V_2 = \{v'_{1,1}, v'_{1,2}, \dots, v'_{1,2(n+1)}, v'_{2,1}, v'_{2,2}, \dots, v'_{2,2(n+2)}, \dots, v'_{n,1}, v'_{n,2}, \dots, v'_{n,2(n+n)}\}$$
$$U_0 = \{u_{0,1}, u_{0,2}, \dots, u_{0,2n}\}$$
$$U_1 = \{u_{\pm 1,1}, u_{\pm 1,2}, \dots, u_{\pm 1,2n-1}, u_{\pm 2,1}, u_{\pm 2,2}, \dots, u_{\pm 2,2n-2}, \dots, u_{\pm n,1}, u_{\pm n,2}, \dots, u_{\pm n,n}\}$$
$$W_1 = \{w_{1,1}, w_{1,2}, \dots, w_{1,n}, w_{2,1}, w_{2,2}, \dots, w_{2,n+1}, \dots, w_{n,1}, w_{n,2}, \dots, w_{n,2n}\}$$
$$W_2 = \{w'_{1,1}, w'_{1,2}, \dots, w'_{1,2n}, w'_{2,1}, w'_{2,2}, \dots, w'_{2,n+1}, \dots, w'_{n,1}, w'_{n,2}, \dots, w'_{n,n}\}.$$

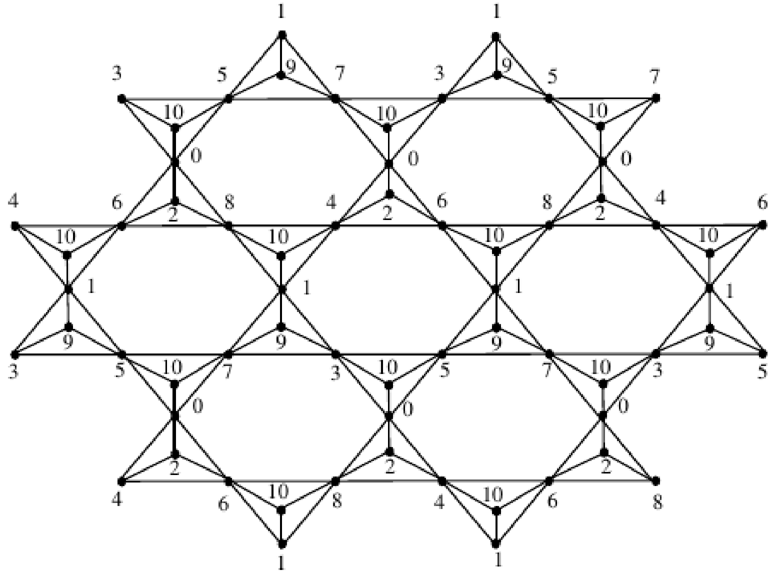

Figure 3: L(2,1)- Labeling of SL(2)

Next we define the map f : $V(\mathrm{G}) \rightarrow N$ for odd and even cases separately as follows:

when n is odd:

$f(w_{2i-1}) = 10$, $f(w_{4i-2}) = 9$ and $f(w_{4i}) = 2$

$f(w'_{2i}) = 10$, $f(w'_{4i-1}) = 9$ and $f(w'_{4i-3}) = 2$

when n is even:

$f(w_{2i-1}) = 10$, $f(w_{4i}) = 9$ and $f(w_{4i-2}) = 2$

$f(w'_{2i}) = 10$, $f(w'_{4i-3}) = 9$ and $f(w'_{4i-1}) = 2$

The remaining vertices in the silicate network are mapped as same as the oxide network. The rest of the proof is similar to Theorem 3.1.

REFERENCES

1. Andrew Lum, Upper Bounds on the L(2,1)-Labeling number of Graphs, *Journal of Applied and Computational Mathematics* (2018).

2. Chang, G.J., Kuo, D, The L(2,1)-Labeling on graphs, *SIAM J. Discrete Math*. 9, 309-316 (1996).
3. F.S. Roberts, T-Colorings of graphs: Recent results and open problems, *Discr. Math*, Vol 93, pp.229-245, (1991).
4. J.R. Griggs and R.K. Yeh, Labeling graphs with a condition at distance two, *SIAM J. Discrete Math*. 5, 586-595 (1992).
5. Gerard J. Chang and David Kuo, The L(2,1)-Labeling problem on Graphs, *SIAM J. Discrete Mathematics* Vol.9, No.2, pp.309-316.
6. Hasunuma.T, Ishii.T, Ono.H and Uno, A linear time algorithm for L(2,1)-Labeling of trees, *In: Lecture notes in Computer Science* 5757, 35-46 (2009).
7. K. Jonas, Graph coloring analogues with a condition at distance two, L(2,1)-Labelings, Ph.D. thesis, University of South Carolina, (1993).
8. M.A. Whittlesy, J.P. Georges, D.W. Mauro, On the λ number of Q_n and related graphs, *SIAM J. Discrete Math*., 8, 499-506 (1995).
9. Paul Manuel and Indra Rajasingh, Minimum Metric Dimension of Silicate networks, Proceedings of the International conference on *Mathematics and computer science* – (2009).
10. Sakai. D, Labeling chordal graphs with a condition at distance two, *SIAM J. Discrete Math*.7, 133-140 (1994).
11. S.K. Vaidya, Vihol, N. A. Nani and D. Bantva, L(2,1)- Labeling in the context of Some Graph Operations, *Journal of Mathematics Research* Vol.2, (2010).
12. W.K. Hale, Frequency Assignment: Theory and applications, proc. *IEEE* 68(12), 1497-1514 (1980).
13. Zehui Shao and Aleksander Vesel, L(2,1)-Labeling of the strong product of paths and cycles, *The Scientific World Journal* (2014).