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Some New Operators in Neutrosophic Topological Spaces

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ABSTRACT

Focus of this paper is to introduce some new operators namely neutrosophic feebly border and neutrosophic feebly exterior using neutrosophic feebly open sets. And we discuss their properties in neutrosophic topological spaces.

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Keywords: Neutrosophic feebly closed set, Neutrosophic feebly open set, Neutrosophic feebly border and Neutrosophic feebly exterior.

1. INTRODUCTION

Neutrosophy, as a new branch of Philosophy has been introduced by Smrandache[5-8] and explained, neutrosophic set is a generalization of Intuitionistic fuzzy set[2-4]. Smrandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy and non membership values respectively, where $\rfloor^{-}0, 1^{+}\lfloor$ is non standard unit interval. In 2012, Salama, Alblowi[12] introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological spaces and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non- membership of each element.
Let T, I, F be real standard or not standard subset of $\rfloor^{-}0,1^{+}\lfloor$, with
$sup\_T = t\_sup, inf\_ = t\_inf$
$sup\_I = i\_sup, inf\_ = i\_inf$
$sup\_F = f\_sup, inf\_ = f\_inf$

$n - sup = t\_sup + i\_sup + f\_sup$
$n - inf = t\_inf + i\_inf + f_{inf}, T, I, F$ are called neutrosophic components.

In 2014, Salama, Smarandache and Valeri[11] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions. P. Jeya Puvaneswari *et al*.[9] defined the notion of neutrosophic feebly open sets and neutrosophic feebly closed sets in neutrosophic topological spaces. Since then, it has been widely investigated in the literature. For these sets, we introduce the notions of neutrosophic feebly border and neutrosophic feebly exterior of a neutrosophic set and show that some of their properties are analogous to those for open sets. Throughout this paper, (X, τ) (simply X) always mean neutrosophic topological spaces.

This paper consists of four sections. Section I gives the introduction. The Section II consists of the basic definitions of neutrosophic feebly open sets, neutrosophic feebly closed sets and their properties which are used in the later sections. The Section III deals with the concept of neutrosophic feebly border and their properties. Section IV explains neutrosophic feebly exterior of a neutrosophic set in neutrosophic topological space.

2. PRELIMINARIES

For basic notations and definitions of Neutrosophic Theory not given here, the reader can refer[1-13].

Definition 2.1.[9] A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly open if there is a neutrosophic open set U in X such that $U \leq A \leq NScl(U)$.

Lemma 2.2. [9] A neutrosophic subset A is neutrosophic feebly open if and only if
(i) $A \leq Nint\left(Ncl\big(Nint(A)\big)\right)$.
(ii) $A \leq NScl\big(Nint(A)\big)$.

Lemma 2.3. [9] (i) Every neutrosophic open set is a neutrosophic feebly open set.

Lemma 2.4. [9] (i) If A and B are two neutrosophic feebly open set then $A \vee B$ is neutrosophic feebly open set.
(ii)Arbitrary union of neutrosophic feebly open sets is a neutrosophic feebly open set.

Definition 2.5. [9] A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly closed if there is a neutrosophic closed set U in X such that $NSint(U) \leq A \leq U$.

Lemma 2.6. [9] A neutrosophic subset A of a neutrosophic topological space (X, τ) is
(i) neutrosophic feebly closed if and only if $Ncl\left(Nint\big(Ncl(A)\big)\right) \leq A$.
(ii) A neutrosophic subset A is neutrosophic feebly closed iff if $NSint\big(Ncl(A)\big) \leq A$.
(iii) A neutrosophic subset A is a neutrosophic feebly closed set if and only if A^c is neutrosophic feebly open.

(iv) Every neutrosophic closed set is a neutrosophic feebly closed set.
(v) If A and B are two neutrosophic feebly closed sets, then $A \wedge B$ is a neutrosophic feebly closed set.
(vi) Finite intersection of a neutrosophic feebly closed sets is a neutrosophic feebly closed set.

Definition 2.7 [9] Let (X, τ) be neutrosophic topological space and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ be a neutrosophic set in X. Then neutrosophic feebly interior of A is defined by $NFint(A)$ $= \vee \{G: G$ is a neutrosophic feebly open set in X and $G \leq A\}$.

Lemma 2.8. [9] Let (X, τ) be neutrosophic topological space. Then for any neutrosophic feebly subsets A and B of a neutrosophic topological space X, we have
(i) $NFint(A) \leq A$
(ii) A is neutrosophic feebly open set in $X \Leftrightarrow NFint(A) = A$
(iii) $NFint(NFint(A)) = NFint(A)$
(iv) If $A \leq B$, $NFint(A) \leq NFint(B)$

Definition 2.9. [9] Let (X, τ) be neutrosophic topological space and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ be a neutrosophic set in X. Then the neutrosophic feebly closure is defined by $NFcl(A) = \wedge\{K: K$ is a neutrosophic feebly closed set in X and $A \leq K\}$.

Lemma 2.10. [9] Let (X, τ) be a neutrosophic topological space. Then for any neutrosophic subsets A and B of a neutrosophic topological space X,
(i) $A \leq NFcl(A)$
(ii) A is a neutrosophic feebly closed set in $X \Leftrightarrow NFcl(A) = A$
(iii) $NFcl(NFcl(A)) = NFcl(A)$
(iv) If $A \leq B$ then $NFcl(A) \leq NFcl(B)$.

Definition 2.11.[9] Let A be a neutrosophic subset in the neutrosophic topological space X. Then the neutrosophic feebly frontier of a neutrosophic subset A is defined as neutrosophic subset $NFFr(A) = NFcl(A) \wedge NFcl(A^C)$.

3. NEUTROSOPHIC FEEBLY BORDER

In this section, we introduce the neutrosophic feebly border and neutrosophic feebly exterior operators using neutrosophic feebly open sets and their properties are discussed in neutrosophic topological spaces.

Definition 3.1. Let A be a neutrosophic subset of neutrosophic topological space X. Then the set $NFBr(A) = A - NFint(A)$ is called the neutrosophic feebly border of A.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{0_N, A, B, C, D, 1_N\}$. Then (X, τ) is a neutrosophic topological space. The neutrosophic closed sets are $\tau^C = \{1_N, F, G, H, I, 0_N\}$ where,
$A = \langle x, (0, 0.2, 0.9), (0.1, 0.8, 0.4), (0.5, 0.4, 0.3) \rangle$,

$B = <x, (0.4, 0.3, 0.3), (0.3, 0.7, 0.2), (0.3, 0.1, 0.7)>$,
$C = <x, (0.4, 0.3, 0.3), (0.3, 0.8, 0.2), (0.5, 0.4, 0.3)>$,
$D = <x, (0, 0.2, 0.9), (0.1, 0.7, 0.4), (0.3, 0.1, 0.7)>$,
$E = <x, (0.9, 0.8, 0), (0.4, 0.2, 0.1), (0.3, 0.6, 0.5)>$,
$F = <x, (0.3, 0.7, 0.4), (0.2, 0.3, 0.3), (0.7, 0.9, 0.3)>$,
$G = <x, (0.3, 0.7, 0.4), (0.2, 0.2, 0.3), (0.3, 0.6, 0.5)>$,
$H = <x, (0.9, 0.8, 0), (0.4, 0.3, 0.1), (0.7, 0.9, 0.3)>$,
$I = <x, (0.7, 0.4, 0.5), (0.4, 0.6, 0.1), (0.7, 0.8, 0.7)>$ and
$J = <x, (0.4, 0.2, 0.8), (0.1, 0.2, 0.5), (0.2, 0.3, 0.9)>$. Define
$A = <x, (0.5, 0.6, 0.5), (0.5, 0.8, 0.4), (0.7, 0.6, 0.3)>$. Then
$Nint(A) = <x, (0.5, 0.6, 0.7), (0.1, 0.8, 0.4), (0.7, 0.2, 0.3)>$ also
$NFint(A) = <x, (0.5, 0.4, 0.3), (0.1, 0.4, 0.4), (0.7, 0.2, 0)>$. $NFBr(A) = A - NFint(A) =$
$<x, (0, 0.2, 0.4), (0.4, 0.4, 0), (0, 0.4, 0.3)>$.

Remark 3.3. Since every neutrosophic open set is a neutrosophic feebly open set, for a neutrosophic subset A of X, $NFBr(A) \subset NBr(A)$.

Theorem 3.4. If a subset A of X is neutrosophic feebly closed, then $NFBr(A) = NFFr(A)$.

Proof. Let A be a neutrosophic feebly closed subset of X. Then by Lemma 2.10 (ii), $NFcl(A) = A$. Now, $NFFr(A) = NFcl(A) - NFint(A) = A - NFint(A) = NFBr(A)$.

Theorem 3.5. For a neutrosophic subset A of X, $A = NFint(A) \vee NFBr(A)$.

Proof. Let $x_{(\alpha,\beta,\gamma)} \in A$. If $x_{(\alpha,\beta,\gamma)} \in NFint(A)$, then the result is obvious. If $x_{(\alpha,\beta,\gamma)} \notin NFint(A)$, then by the definition of $NFBr(A)$, $x_{(\alpha,\beta,\gamma)} \in NFBr(A)$. Hence $x_{(\alpha,\beta,\gamma)} \in NFint(A) \vee NFBr(A)$ and so $A \leq NFint(A) \vee NFBr(A)$. On the other hand, since $NFint(A) \leq A$ and $NFBr(A) \leq A$, we have $NFint(A) \vee NFBr(A) \leq A$.

Theorem 3.6. For a neutrosophic subset A of X, $NFint(A) \wedge NFBr(A) = 0_N$

Proof. Suppose $NFint(A) \wedge NFBr(A) \neq 0_N$. Let $x_{(\alpha,\beta,\gamma)} \in NFint(A) \wedge NFBr(A)$. Then $x_{(\alpha,\beta,\gamma)} \in NFint(A)$ and $x_{(\alpha,\beta,\gamma)} \in NFBr(A)$. Since $NFBr(A) = A - NFint(A)$, then $x_{(\alpha,\beta,\gamma)} \in A$. But $x_{(\alpha,\beta,\gamma)} \in NFint(A)$, $x_{(\alpha,\beta,\gamma)} \in A$. There is a contradiction. Hence $NFint(A) \wedge NFBr(A) = 0_N$.

Theorem 3.7. For a neutrosophic subset A of X, A is a neutrosophic feebly open set if and only if $NFBr(A) = 0_N$.

Proof. Necessity: Suppose A is neutrosophic feebly open. Then by Lemma 2.10 (ii), $NFint(A) = A$. Now, $NFBr(A) = A - NFint(A) = A - A = 0_N$.
Sufficiency: Suppose $NFBr(A) = 0_N$. This implies, $A - NFint(A) = 0_N$. Therefore $A = NFint(A)$ and hence A is neutrosophic feebly open.

Corollary 3.8. For a neutrosophic topological space, $NFBr(0_N) = 0_N$ and $NFBr(1_N) = 0_N$.

Proof. Since 0_N and 1_N are neutrosophic feebly open, by Theorem 3.6, $NFBr(0_N) = 0_N$ and $NFBr(1_N) = 0_N$.

Theorem 3.9. For a neutrosophic subset A of X, $NFBr(NFint(A)) = 0_N$.

Proof. By the definition of neutrosophic feebly border, $NFBr\big(NFint(A)\big) = NFint(A) - NFint(NFint(A))$. By Lemma 2.8 (iii), $NFint(NFint(A)) = NFint(A)$ and hence $NFBr(NFint(A)) = 0_N$.

Theorem 3.10. For a neutrosophic subset A of X, $NFint\big(NFBr(A)\big) = 0_N$.

Proof. Let $x_{(\alpha,\beta,\gamma)} \in NFint(NFBr(A))$. Since $NFBr(A) \leq A$, by Lemma 2.8 (i), $NFint(NFBr(A)) \leq NFint(A)$. Hence $x_{(\alpha,\beta,\gamma)} \in NFint(A)$. Since $NFint(NFBr(A)) \leq NFBr(A)$, $x_{(\alpha,\beta,\gamma)} \in NFBr(A)$. Therefore $x_{(\alpha,\beta,\gamma)} \in NFint(A) \wedge NFBr(A)$, $x_{(\alpha,\beta,\gamma)} = 0_N$.

Theorem 3.11. For a neutrosophic subset A of X, $NFBr\big(NFBr(A)\big) = NFBr(A)$.

Proof. By the definition of neutrosophic feebly border, $NFBr\big(NFBr(A)\big) = NFBr(A) - NFint(NFBr(A))$. By Theorem 3.9 $NFint(NFBr(A)) = 0_N$ and hence $NFBr(NFBr(A)) = NFBr(A)$.

Theorem 3.12. For a subset A of a space X, the following statements are equivalent.
(i) A is neutrosophic feebly open
(ii) $A = NFint(A)$
(iii) $NFBr(A) = 0_N$.
Proof: (i) → (ii) Obvious from Lemma 2.8.
(ii) → (iii). Suppose that $A = NFint(A)$. Then by Definition 3.1, $NFBr(A) = NFint(A) - NFint(A) = 0_N$.
(iii) → (i). Let $NFBr(A) = 0_N$. Then by Definition 3.1, $A - NFint(A) = 0_N$ and hence $A = NFint(A)$.

Theorem 3.13. Let A be a neutrosophic subset of X. Then, $NFBr(A) = A \wedge NFcl(A^c)$
Proof. Since $NFBr(A) = A - NFint(A)$ and by Lemma 2.10, $NFBr(A) = A - \big(NFcl(A^c)\big)^c = A \wedge (NFcl(A^c)^c) = A \wedge NFcl(A^c)$.

Theorem 3.14. For a neutrosophic subset A of X, $NFBr(A) \leq NFFr(A)$.
Proof. Since $A \leq NFcl(A)$, $A - NFint(A) \leq NFcl(A) - NFint(A)$. That implies, $NFBr(A) \leq NFFr(A)$.

4. NEUTROSOPHIC FEEBLY EXTERIOR

Definition 4.1. Let A be a neutrosophic subset of a neutrosophic topological space (X, τ). The neutrosophic feebly interior of A^c is called the neutrosophic feebly exterior of A and it is denoted $NFExt(A)$. That is, $NFExt(A) = NFint(A^c)$.

Example 4.2. Let $X = \{a, b, c\}$ and $\tau = \{0_N, A, B, C, D, 1_N\}$. Then (X, τ) is a neutrosophic topological space. The neutrosophic closed sets are $\tau^C = \{1_N, F, G, H, I, 0_N\}$ where,
$A = < x, (0.5, 0.6, 0.7), (0.1, 0.8, 0.4), (0.7, 0.2, 0.3) >$,
$B = < x, (0.8, 0.8, 0.5), (0.5, 0.4, 0.2), (0.9, 0.6, 0.7) >$,
$C = < x, (0.8, 0.8, 0.5), (0.5, 0.8, 0.2) , (0.9, 0.6, 0.3) >$,
$D = < x, (0.5, 0.6, 0.7), (0.1, 0.4, 0.4), (0.7, 0.2, 0.7) >$,
$E = < x, (0.8, 0.8, 0.4), (0.5, 0.8, 0.1), (0.9, 0.7, 0.2) >$,
$F = < x, (0.7, 0.4, 0.5), (0.4, 0.2, 0.1), (0.3, 0.8, 0.7) >$,
$G = < x, (0.5, 0.2, 0.8), (0.2, 0.6, 0.5), (0.7, 0.4, 0.9) >$,
$H = < x, (0.5, 0.2, 0.8), (0.2, 0.2, 0.5), (0.3, 0.4, 0.9) >$,
$I = < x, (0.7, 0.4, 0.5), (0.4, 0.6, 0.1), (0.7, 0.8, 0.7) >$ and
$J = < x, (0.4, 0.2, 0.8), (0.1, 0.2, 0.5) , (0.2, 0.3, 0.9) >$. Here E and J are neutrosophic feebly open and neutrosophic feebly closed set respectively. Some of the neutrosophicfeeblyopen are 0_N, A, B, C, D, E, 1_N and neutrosophic feebly closed set are 1_N, F, G, H, I, J, 0_N. Therefore $NFFr(C)$=H≤C. But C is not a neutrosophic feebly closed set.$NFFr$(J)≤ J^c= E. But J is not a neutrosophic feebly open set. We define
A=<x, (0.5, 0.6, 0.5), (0.5, 0.8, 0.4), (0.7, 0.6,0.3)>.Then
A^c=<x, (0.7, 0.4, 0.5), (0.4, 0.2, 0.5), (0.3, 0.4, 0.7)>. Now
$NFExt(A) = NFint(A^c)$= <x, (0.5, 0.4, 0.5), (0, 0, 0.7), (0.2, 0, 1)>.

Remark 4.3. Since every neutrosophic open set is a neutrosophic feebly open set, for a neutrosophic subset A of X, $NFExt(A) \subset NExt(A)$.

Theorem 4.4. For a neutrosophic subset A of X, $NFExt(A) = (NFcl(A))^c$.
Proof. We know that, $X - NFcl(A) = NFint(A^c)$, then $NFExt(A) = NFint(A^c) = (NFcl(A))^c$.

Theorem 4.5. For a neutrosophic subset A of X, $NFExt(NFExt(A)) = NFint(NFcl(A)) \geq NFint(A)$.

Proof. Now, $NFExt(NFExt(A)) = NFExt(NFint(A^c)) = NFint(\left(NFint(A^c)\right)^c) = NFint(NFcl(A)) \geq NFint(A)$.

Theorem 4.6. For a neutrosophic subset A of X, If $A \leq B$, then $NFExt\ (B) \leq NFExt(A)$.
Proof. Suppose $A \leq B$. Now, $NFExt(B) = NFint(B^c) \leq NFint(A^c) = NFExt(A)$.

Theorem 4.7. For a neutrosophic subset A of X, $NFExt(1_N) = 0_N$ and $NFExt(0_N) = 1_N$.
Proof. Now, $NFExt(1_N) = NFint((1_N)^c) = NFint(0_N)$ and $NFExt(\ 0_N) = NFint((0_N)^c) = NFint(1_N)$. Since 0_N and 1_N are neutrosophic feebly open sets, then $NFint(0_N) = 0_N$ and $NFint(1_N) = 1_N$. Hence $NFExt(\ 0_N) = 1_N$ and $NFExt(1_N) = 0_N$.

Theorem 4.8. For a neutrosophic subset A of X, $NFExt(A) = NFExt((NFExt(A))^c)$.
Proof. Now,$NFExt\left(\left(NFExt(A)\right)^c\right) = NFExt\left(\left(NFint(A^c)\right)^c\right) = NFint((((NFint(A^c))^c))^c) = NFint(NFint(A^c)) = NFint(A^c) = NFExt(A)$.

Theorem 4.9. For a sub sets A and B of X, the followings are valid.

(i) $NFExt(A \vee B) \leq NFExt(A) \wedge NFExt(B)$.

(ii) $NFExt(A \wedge B) \geq NFExt(A) \vee NFExt(B)$.

Proof.

(i) $NFExt(A \vee B) = NFint((A \vee B)^c) = NFint((A^c) \wedge (B^c)) \leq NFcl(A^c) \wedge NFcl(B^c) = NFExt(A) \wedge NFExt(B)$.

(ii) $NFExt(A \wedge B) = NFint((A \wedge B)^c) = NFint((A^c) \vee (B^c)) \geq NFcl(A^c) \vee NFcl(B^c) = NFExt(A) \vee NFExt(B)$.

CONCLUSION

So far, we have studied some new operators called neutrosophic feebly border and neutrosophic feebly exterior with the help of neutrosophic feebly open sets in neutrosophic space. And we discussed the important properties of them and the relations between them.

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