Journal of Computer and Mathematical Sciences, *Vol.10(1),132-138 January 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

g_β^-Continuous Maps in Topological Spaces

K. Alli [1] and M. Arline Jeyamary[2*]

[1]Assistant Professor, Department of Mathematics,
The M.D.T. Hindu College, Tirunelveli-627 010, Tamil Nadu, INDIA.
[2*]Assistant Professor, Department of Mathematics,
Manonmaniam Sundaranar University College, Govindaperi,
Tirunelveli-627 414, Tamil Nadu, INDIA.
email: [1]allimdt@gmail.com, [2*]arlinebeno@gmail.com(mob:9751819631)
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti,
Tirunelveli, Tamil Nadu, INDIA).

(Received on: December 14, 2018)

ABSTRACT

The aim of this paper is to introduce g_β^-continuous map, g_β^-irresolute map in topological spaces. The inter relationships of these newly introduced functions with various functions are analysed and the dependence links with the other existing functions are discussed. Composition of two g_β^-continuous functions and composition of two g_β^-irresolute functions are also studied in this paper.

Keywords: g_β^-continuous map, g_β^-irresolute map.

INTRODUCTION

Norman Levine[9] introduced generalized closed sets (briefly g-closed sets) in topological spaces in 1970. M.K.R.S. Veerakumar has introduced g^-closed sets[22] and discussed their properties. Ramya.N and Parvathi.A introduced Ψg^-closed sets[20] in topological spaces in 2011. Using Ψg^-open, M.Arline Jeyamary introduced g_β^-closed set[2] and the relation between g_β^-closed set and the other closed sets are analysed. In this paper, g_β^-continuous map and g_β^-irresolute map are defined and their properties are investigated.

PRELIMINARIES

Definition 2.1 : A subset A of a topological space (X , τ) is called
1. a pre-open set[13] if $A \subseteq int (cl(A))$ and pre-closed if $cl(int A) \subseteq A$.

2. a semi-open set [8] if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A.
3. An α-open set[15] if A ⊆ int(cl(int(A))) and α-closed set if cl(int(cl(A))) ⊆ A.

The complement of pre-open(resp.semi-open, α-open) sets are called pre-closed(resp.semi-closed, α-closed) sets.

Definition 2.2: A subset A of a topological space (X , τ) is called

1. a generalized closed set (briefly g-closed)[9] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X , τ).
2. a generalized pre-closed set (gp-closed)[12] if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X , τ).
3. a α-generalized closed set (αg-closed)[11] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X , τ).
4. a generalized semi-preclosed set (gsp-closed)[5] if spcl(A)⊆U whenever A ⊆ U and U is open in (X , τ).
5. a g_β^-closed set [2] if spcl(A) ⊆ U whenever A ⊆ U and U is Ψg^-open in (X , τ).
6. a g*s-closed set[10] if scl(A) ⊆ U whenever A ⊆ U and U is gs-open in (X , τ).
7. a g*-closed set[21] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X , τ).
8. a (gs)*-closed set[6] if cl(A) ⊆ U whenever A ⊆ U and U is gs-open in (X , τ).
9. a $g^\#$-closed set[23] if cl(A) ⊆ U whenever A ⊆ U and U is αg-open in (X , τ).
10. a $g^\#$-preclosed set($g^\#$p-closed)[18] if pcl(A) ⊆ U whenever A⊆U and U is $g^\#$-open in (X,τ).
11. a $g^\#$-semiclosed set ($g^\#$s-closed)[24] if scl(A) ⊆ U whenever A ⊆ U and U is αg-open in (X,τ).
12. a $g^\#p^\#$-closed set [17] if cl(A) ⊆ U whenever A ⊆ U and U is $g^\#$-open in (X , τ).

Definition 2.3: A function f:(X, τ)→(Y, σ) is called a

1. g-continuous[4] if f^{-1}(V) is a g-closed set of (X, τ) for every closed set V of (Y, σ).
2. αg-continuous[7] if f^{-1}(V) is a αg-closed set of (X, τ) for every closed set V of (Y,σ).
3. gsp-continuous[5] if f^{-1}(V) is a gsp-closed set of (X, τ) for every closed set V of (Y,σ).
4. gp-continuous[3] if f^{-1}(V) is a gp-closed set of (X, τ) for every closed set V of (Y, σ).
5. g*-continuous[21] if f^{-1}(V) is a g*-closed set of (X, τ) for every closed set V of (Y, σ).
6. g*s-continuous[19] if f^{-1}(V) is a g*s-closed set of (X, τ) for every closed set V of (Y,σ).
7. $g^\#$s-continuous[24] if f^{-1}(V) is a $g^\#$s-closed set of (X, τ) for every closed set V of (Y, σ).
8. semi-continuous[8] if f^{-1}(V) is a semi-closed set of (X, τ) for every closed set V of (Y,σ).
9. pre-continuous[13] if f^{-1}(V) is a pre-closed set of (X,τ) for every closed set V of (Y,σ).
10. α-continuous[14] if f^{-1}(V) is a α-closed set of (X, τ) for every closed set V of (Y, σ).
11. $g^\#p^\#$-continuous[16] if f^{-1}(V) is a $g^\#p^\#$-closed set of (X, τ) for every closed set V of (Y,σ).
12. $g^\#$p-continuous[1] if f^{-1}(V) is a $g^\#$p-closed set of (X, τ) for every closed set V of (Y,σ).

13. $g^{\#}$-continuous[23] if $f^{-1}(V)$ is a $g^{\#}$-closed set of (X, τ) for every closed set V of (Y, σ).
14. (gs)*-continuous[6] if $f^{-1}(V)$ is a (gs)*-closed set of (X, τ) for every closed set V of (Y, σ).

Definition 2.4: A topological space (X , τ) is called a

1. Tg_{β}^-space[2] if every g_{β}^-closed set in it is closed.
2. αTg_{β}^-space[2] if every g_{β}^-closed set in it is α-closed.
3. sTg_{β}^-space[2] if every g_{β}^-closed set in it is semi-closed.
4. pTg_{β}^-space[2] if every g_{β}^-closed set in it is pre-closed.
5. gTg_{β}^-space[2] if every g_{β}^-closed set in it is g-closed.
6. αgTg_{β}^-space[2] if every g_{β}^-closed set in it is αg-closed.

3. g_{β}^-CONTINUOUS MAPS AND g_{β}^-IRRESOLUTE MAPS

Definition 3.1: A function f : (X, τ)→(Y, σ) is called **g_{β}^-continuous** if $f^{-1}(V)$ is a g_{β}^-closed set of (X, τ) for every closed set V of (Y, σ).

Definition 3.2: A function f : (X, τ)→(Y, σ) is called **g_{β}^-irresolute** if $f^{-1}(V)$ is a g_{β}^-closed set of (X, τ) for every g_{β}^-closed set V of (Y, σ).

Theorem 3.3: Every continuous map is g_{β}^-continuous but not conversely.

Proof: It follows from the fact that every closed set is a g_{β}^-closed set.

Example 3.4: Let X=Y={a,b,c}, τ ={ {b,c}, X, φ} and σ ={ {b},{b,c}, Y, φ }. Define f:(X,τ)→ (Y,σ) by f(a)=a, f(b)=c, f(c)=b. Then f^{-1}{a,c}={a,b} and f^{-1}{a}={a} are g_{β}^-closed in (X, τ) but {a,b} is not closed in (X, τ). Hence f is g_{β}^-continuous but not continuous.

Theorem 3.5: Every (gs)*-continuous[resp.g*s-continuous, g#s-continuous] is g_{β}^-continuous but not conversely.

Proof: It follows from the fact that every (gs)*-closed set[resp.g*s-closed, g#s-closed] is a g_{β}^-closed set.

Example 3.6: Let X=Y={a,b,c}, τ ={{b},{a,c}, X, φ } and σ ={ {c},Y, φ }. Define f : (X, τ)→(Y, σ) by f(a)=c, f(b)=a, f(c)=b. Then f^{-1}{a,b}={b,c} is g_{β}^-closed in (X, τ) but not (gs)*-closed[resp.g*s-closed, g#s-closed] in (X, τ). Hence f is g_{β}^-continuous but not (gs)*-continuous[resp.g*s-continuous, g#s-continuous]

Theorem 3.7: Every g_{β}^-continuous is gsp-continuous but not conversely.

Proof: It follows from the fact that every g_{β}^-closed set is gsp-closed.

Example 3.8: Let X=Y={a,b,c}, τ ={ {a}, X, φ } and σ ={ {a,b}, Y, φ }. Define f:(X, τ)→ (Y, σ) by f(a)=c, f(b)=c, f(c)=a. Then f^{-1}{c}={a,b} is gsp-closed in (X, τ) but not g_{β}^-closed in (X, τ). Hence f is gsp-continuous but not g_{β}^-continuous.

Theorem 3.9: Every semi-continuous map is g_β^-continuous but not conversely.

Proof: Since every semi-closed set is g_β^-closed set, the result follows.

Example 3.10: Let X=Y={a,b,c}, τ ={{a}, {b,c}, X, φ} and σ ={{a,c},Y, φ}. Define f:(X,τ)→(Y,σ) by f(a)=b, f(b)=b,f(c)=a. Then f^{-1}{b}={a,b} is g_β^-closed set in (X, τ) but not semi-closed in (X, τ). Hence f is g_β^-continuous but not semi-continuous.

Remark 3.11: g_β^-continuous is independent from g-continuous[resp.g*-continuous, $g^\# p^\#$-continuous] which can be seen from the following examples.

Example 3.12: Let X=Y={a,b,c}, τ ={{a,b}, X, φ } and σ ={ {b,c},Y, φ }. Define f : (X, τ)→(Y, σ) be the identity mapping. {a} is closed in (Y, σ). Then f^{-1}{a}={a} is g_β^-closed set in (X, τ) but not g-closed[resp.g*-closed and $g^\# p^\#$-closed] in (X, τ). Hence f is g_β^-continuous but not g-continuous[resp.g*-continuous and $g^\# p^\#$-continuous].

Example 3.13: Let X=Y={a,b,c}, τ ={{b},{a,b}, X, φ} and σ={{a,c},Y, φ}. Define f:(X, τ)→(Y, σ) by f(a)=a, f(b)=b, f(c)=b.{b} is closed in (Y, σ). Then f^{-1}{b}={b,c} is g-closed[resp.g*-closed and $g^\# p^\#$-closed] but not g_β^-closed in (X, τ). Hence f is g-continuous[resp.g*-continuous and $g^\# p^\#$-continuous] but not g_β^-continuous .

Remark 3.14: g_β^-continuous is independent from gp-continuous[resp.$g^\#$p-continuous] which can be seen from the following example.

Example 3.15: Let X=Y={a,b,c}, τ ={{b},{c},{b,c},X, φ} and σ ={{a,c},Y, φ}. Define f:(X,τ)→(Y,σ) be the identity mapping.{b} is closed in (Y, σ). Then f^{-1}{b}={b} is g_β^-closed set in (X, τ) but not gp-closed[resp. $g^\#$p-closed]. Hence f is g_β^-continuous but not gp-continuous[resp. $g^\#$p-continuous].

Example 3.16: Let X=Y={a,b,c}, τ ={{c},X, φ } and σ ={{b,c},Y, φ }. Define f : (X, τ)→(Y, σ) by f(a)=b, f(b)=a, f(c)=a. {a} is closed in (Y, σ). Then f^{-1}{a}={b,c} is gp-closed[resp. $g^\#$p-closed] but not g_β^-closed in (X, τ). Hence f is gp-continuous[resp. $g^\#$p-continuous] but not g_β^-continuous .

Theorem 3.17: Every $g^\#$-continuous map is g_β^-continuous but not conversely.

Proof: It follows from the fact that every $g^\#$-closed set is a g_β^-closed set.

Example 3.18: Let X=Y={a,b,c}, τ ={ {a,b}, X, φ } and σ ={{b,c},Y, φ }. Define f : (X, τ)→(Y, σ) be the identity mapping. Then f^{-1}{a}={a} is g_β^-closed in (X, τ) but not αg-closed in (X, τ). Hence f is g_β^-continuous but not $g^\#$-continuous.

Theorem 3.19: Composition of two g_β^-continuous functions need not be a g_β^-continuous function. This can be shown in the following example.

Example 3.20: Let X=Y=Z={a,b,c},τ ={{a,b}, X, φ}, σ ={{b,c},Y, φ} and η ={{a,c}, Z, φ}. Define f : (X, τ)→(Y, σ) be the identity map. Define g : (Y, σ)→(Z, η) by g(a)=b, g(b)=b, g(c)=a. Then f and g both are g_β^-continuous. {b} is closed in (X, τ). But $(fog)^{-1}${b}= g^{-1}[f^{-1}{b}]={a,b} is not a g_β^-closed in (X, τ). Hence fog is not g_β^-continuous.

Theorem 3.21: Every g_β^-irresolute function is g_β^-continuous but not conversely.

Proof: Let f : (X, τ)→(Y, σ) be g_β^-irresolute. Let V be a closed set in (Y, σ). Since every closed set is g_β^-closed, V is g_β^-closed in (Y, σ). Since f is g_β^-irresolute, f^{-1}(V) is g_β^-closed in (X, τ). Hence f is g_β^-continuous.

Example 3.22: Let X=Y={a,b,c}, τ ={{c}, X, φ} and σ ={{b,c},Y, φ }. Define f: (X, τ) → (Y, σ) by f(a)=a, f(b)=c, f(c)=b. Then f^{-1}{a}={a} is g_β^-closed in (X, τ). Therefore f is g_β^-continuous. {b} is g_β^-closed in (Y, σ). But f^{-1}{b}={c} is not g_β^-closed in (X, τ). Therefore f is not g_β^-irresolute. Hence f is g_β^-continuous but not g_β^-irresolute.

Theorem 3.23: Every g_β^-irresolute function is gsp-continuous but not conversely.

Proof: Let f:(X, τ)→(Y, σ) be g_β^-irresolute. Let V be a closed set in (Y, σ). Since every closed set is g_β^-closed, V is g_β^-closed in (Y, σ). Since f is g_β^-irresolute, f^{-1}(V) is g_β^-closed in (X, τ). Since every g_β^-closed set is gsp-closed, f^{-1}(V) is gsp-closed in (X, τ). Hence f is gsp-continuous.

Example 3.24: Let X=Y={a,b,c}, τ={{a},{a,b}, X, φ} and σ ={{a,c},Y, φ }. Define f:(X, τ)→(Y, σ) by f(a)=a, f(b)=c, f(c)=b. Then f^{-1}{b}={c} is gsp-closed in (X, τ). Therefore f is gsp-continuous. {a} is g_β^-closed in (Y, σ). But f^{-1}{a}={a} is not g_β^-closed in (X, τ). Therefore f is not g_β^-irresolute. Hence f is gsp-continuous but not g_β^-irresolute.

Theorem 3.25: Let (X, τ), (Y, σ) and (z, η) be any three topological spaces. Let f : (X, τ)→(Y, σ) and g : (Y, σ)→ (Z, η) be any two functions. Then
(i)gof is g_β^-continuous if g is continuous and f is g_β^-continuous.
(ii)gof is g_β^-continuous if g is g_β^-continuous and f is g_β^-irresolute.
(ii)gof is g_β^-irresolute if g is g_β^-irresolute and f is g_β^-irresolute.

Proof: The proof is obvious from the definitions [3.1] and [3.2].

Theorem 3.26: Let f:(X, τ)→(Y, σ) be a g_β^-continuous map. If (X, τ), the domain of f is an Tg_β^-space then f is continuous.

Proof: Let V be a closed set of (Y, σ). Since f is g_β^-continuous, f^{-1}(V) is a g_β^-closed set of (X, τ). Since (X, τ) is an Tg_β^-space f^{-1}(V) is a closed set of (X, τ). Therefore f is continuous.

Theorem 3.27: Let f:(X, τ)→(Y, σ) be a g_β^-continuous map. If (X, τ), the domain of f is an αTg_β^-space then f is α-continuous.

Proof: Similar to that of the above theorem [3.26].

Theorem 3.28: Let f:(X, τ)→(Y, σ) be a g_β^-continuous map. If (X, τ), the domain of f is an sTg_β^-space then f is semi-continuous.

Proof: Similar to that of the above theorem [3.26].

Theorem 3.29: Let f:(X, τ)→(Y, σ) be a g_β^-continuous map. If (X, τ), the domain of f is an pTg_β^-space then f is pre-continuous.

Proof: Similar to that of the above theorem [3.26].

Theorem 3.30: Let f : (X, τ)→(Y, σ) be a g_β^-continuous map. If (X, τ), the domain of f is an gTg_β^-space then f is g-continuous.

Proof: Similar to that of the above theorem [3.26].

Theorem 3.31: Let f : (X, τ)→(Y, σ) be a g_β^-continuous map. If (X, τ), the domain of f is an αgTg_β^-space then f is αg-continuous.

Proof: Similar to that of the above theorem [3.26]

REFERENCES

1. Alli.K, Pious Missier.S, and Subramanian.A, On $g^{\#}$p-ccontinuous maps in topological spaces, *International Journal of Modern Engineering Research*, Vol 3, Issue 1, 360-364 Jan-Feb(2013).
2. Alli.K, Arline Jeyamary.M, On g_β^-closed sets in Topological spaces, Proceedings, Sadakathullah Appa College (2017).
3. Arokiarani.I, Balachandran.K and Dontchev.J, Some Characterisations of gp-continuous maps between Topological Spaces, *Mem. Fac. Sci. Kochi. Univ. Ser. A. Math.,* 20, 93-104 (1999).
4. Balachandran.K, Sundaram.P and Maki.H, On generalized continuous maps in topological spaces, *Mem. Fac. Kochi Univ. Ser. A. Math*., 12, 5-13 (1991).
5. Dontchev.J, On generalizing semi-preopen sets, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math*., 16,35-48 (1995).
6. Elvina Mary.L, (gs)*-closed sets in topological spaces, *International Journal of Mathematics Trends and Technology*, (7) 83-93 (2014).
7. Gnanambal.Y, On generalized preregular closed sets in topological spaces, *Indian J. Pure. Appl. Math*.,28(3), 35-48 (1997).
8. Levine.N, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70, 36-41 (1963).
9. Levine.N, Generalized closed sets in topological spaces, *Rend. Circ. Mat. Palermo*, 19 (2), 89-96 (1970).
10. Maghrabi.A.I.EI and Nasaf.A.A, Between semi closed and gs closed sets, *Journal of Taibath University for Science* 2:78-87 (2009).
11. Maki.H, Devi.R and Balachandran.K, Associated topologies of generalized α-closed sets and α-generalized closed sets, *Mem.Fac.Sci.Kochi Univ. Ser.A.Math*., 15, 51-63 (1994).

12. Maki.H, Umehara.J and Noiri.T, "Every topological space is pre-$T_{1/2}$", *Mem. Fac. Sci. Kochi Univ. Ser.A Math*, 17, 33-42 (1996).
13. Mashhour.A.S, Abd El-Monsef.M.E and El-Deep.S.N, On pre-continuous and weak pre-continuous mappings, *Proc.Math. and Phys. Soc. Egypt*, 53, 47-53 (1982).
14. Mashhour.A.S, Hasanein.I.A and El-Deeb.S.N, On Pre-continuous and weak pre-continuous mappings, *Proc. Math. and Phys.Soc.Egypt*. 53,47-53 (1982).
15. Njastad.O, On some classes of nearly open sets, *Pacific J. Math*., 15, 961-970 (1965).
16. Pious Missier.S, Subramanian.A, Alli.K On $g^{\#}p^{\#}$-ccontinuous maps in topological spaces, *Journal of Global Research in Mathematical Archives*, volume 1 , No.5, ISSN 2320-5822 (May 2013).
17. S.Pious Missier, K.Alli and A.Subramanian, $g^{\#}p^{\#}$-closed sets in topological spaces *International Journal of Mathematics and Soft Computing*, Vol.3, No.3, 55-60 (2013).
18. Pious Missier.S, Alli.K and Subramanian.A, $g^{\#}$p-closed sets in topological spaces, *International Journal of Mathematical Archive*, 4(1), 176-181 (2013).
19. Pushpalatha.A, Anitha.K, g*s-closed sets in Topological Spaces, *Int. J. Contemp. Math. Sciences*, Vol 6, no.19,917-929 (2011).
20. Ramya. N,and Parvathi.A, Ψg^-closed sets in topological spaces, *IJMA*, Vol.2(10),PP.1992-1996, (2011).
21. Veerakumar.M.K.R.S, Between closed sets and g-closed sets, *Mem. Fac. Sci. Kochi. Univ. Ser. A. Math*., 17, 33-42 (1996).
22. Veerakumar.M.K.R.S, On g^-closed sets in topological spaces, Bulletion Allahabad. *Math Soc*.,18, 99-112 (2003).
23. Veerakumar. M.K.R.S, $g^{\#}$-closed sets in topological spaces, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math* 24, 1-13 (2003).
24. Veerakumar.M.K.R.S, $g^{\#}$ semi-closed sets in topological spaces, *International Journal of Scientific and Research Publications*, Vol 2, Issue 6, ISSN 2250-3153 (June 2012).