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A Survey on Role of Graph Theory in Various Approaches

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ABSTRACT

Graph Theory is one of the best known, popular and extensively researched subject, having many applications and conjectures, which are still open and studied by various mathematicians and computer scientists along the world.Now a days the role of graph theory in various filed is increasing, currently it provide greater functionality, combination, and low cost system into real world designed systems. Graph theory is in spot to play extensive roles in real application and hence, this paper illustrates could do with by bringing together all known current approaches of graph theory.This paper presents a survey on the graph theory challenges relevant to their approaches and techniques. We extend this review and explore the field of graph theory further, recitation various results obtained by other authors.

Keywords: Graph Theory, Graph, Edge, Spectral graph theory, Stochastic coupled systems.

I. INTRODUCTION

[1]Graphs consist of points called vertices, lines called edges, Edges connect two vertices, Edges only intersect at vertices and Edges joining a vertex to itself are called loops fig 1. A graph is formed by vertices and edges connecting the vertices. Formally, a graph is a pair of sets (V, E), where V is the set of vertices and E is the set of edges, formed by pairs of vertices.

E is a multiset, in other words, its elements can occur more than once so that every element has a multiplicity. Often, we label the vertices with letters (for example: a, b, c, .. or v1, v2, . . .) or numbers 1, 2, . . . Throughout this lecture material, we will label the elements of V in this way.

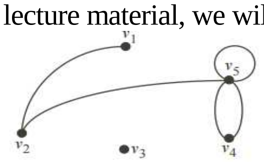

Fig 1 Example of Graph

We have V = {v1, . . . , v5} for the vertices and E = {(v1, v2),(v2, v5),(v5, v5),(v5, v4),(v5, v4)} for the edges. Similarly, we often label the edges with letters (for example: a, b, c, . . . or e1, e2, . . .) or numbers 1, 2, . . . for simplicity.

A walk in the graph G = (V, E) is a finite sequence of the form vi0 , ej1 , vi1 , ej2 , . . . , ejk , vik , which consists of alternating vertices and edges of G. The walk starts at a vertex. Vertices vit−1 and vit are end vertices of ejt (t = 1, . . . , k). vi0 is the initial vertex and vik is the terminal vertex. k is the length of the walk. A zero length walk is just a single vertex vi0 . It is allowed to visit a vertex or go through an edge more than once. A walk is open if vi0 6= vik . Otherwise it is closed refer in fig 2.

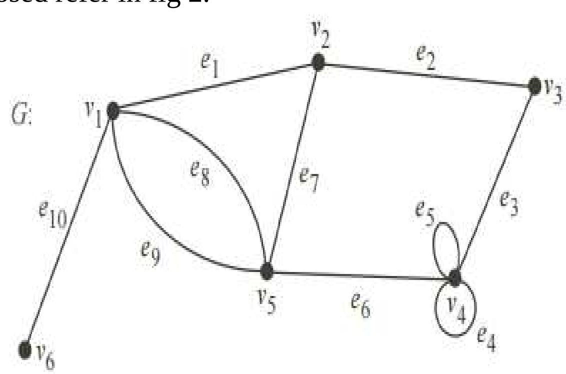

Fig 2 Example of Graph with edges and vertices

The walk v2, e7, v5, e8, v1, e8, v5, e6, v4, e5, v4, e5, v4 is open. On the other hand, the walk v4, e5, v4, e3, v3, e2, v2, e7, v5, e6, v4 is closed. A walk is a trail if any edge is traversed at most once. Then, the number of times that the vertex pair u, v can appear as consecutive vertices in a trail is at most the number of parallel edges connecting u and v.

II. SURVEY

[7]Using spectral graph theory and especially its graph comparison techniques, proposes new methodologies to allocate coupling strengths to guarantee global complete management in complex networks. The key step is that all the eigen values of the Laplacian matrix associated with a given network can be estimated by utilizing flexibly topological features of the network. The proposed methodologies enable the structure of different coupling-strength

combinations in response to different knowledge about sub networks. Adaptive allocation strategies can be carried out as well using only local network topological information. Besides recognized analysis, use simulation examples to demonstrate how to apply the methodologies to typical complex networks. In this paper presented new ways to allocate coupling strengths using spectral graph theory in order to achieve synchronization in complex networks. The main idea is to bind the second-smallest eigen values of the Laplacian matrices associated with the given networks by comparing the parallel network graphs to complete or other typical graphs with the same vertex sets. The obtained results can simplify the computation and be applied to growing networks. Currently, looking into applying the proposed methodologies to networks with directed topologies, some preliminary results have been presented in[9]. The main challenge is then how to deal with the fact that the Laplacian matrices associated with directed graphs are not guaranteed to be positive semi definite anymore. The constructed synchronization criteria to develop optimal or suboptimal solutions for adding or deleting edges in a network to achieve better synchronizability. It is of great interest to apply the results to practical engineered complex networks, such as the synchronization of generators in electric power grids and data union for signal processing in sensor networks.

[6]In this paper, a novel class of stochastic coupled systems with Lévy noise on networks (SCSLNNs) is presented. Both white noise and Lévy noise are considered in the networks. By exploiting graph theory and Lyapunov stability theory, criteria ensuring *p*th moment exponential stability and stability in probability of these SCSLNNs are established, respectively. These principles are closely related to the topology of the network and the perturbation intensity of white noise and Lévy noise. Moreover, to verify the theoretical results, stochastic coupled oscillators with Lévy noise on a network and stochastic Volterra predator–prey system with Lévy noise are performed. Finally, a numerical example about oscillators' network is provided to illustrate the feasibility of our analytical results. This paper investigates in detail the stability of SCSLNNs. It has applications in many branches of science and industry. Based on graph theory and Lyapunov stability theory, some criteria about the *p*th moment exponential stability and stability in probability of SCSLNNs are derived, respectively. These criteria can be successfully applied to any other stochastic coupled networks with Lévy noise. In addition, have given a network of exponentially stable stochastic coupled oscillators with Lévy noise, and obtained an allowable bound of perturbation intensity of white noise and Lévy noise for stability of such oscillators. Moreover, stochastic Volterra predator–prey system with Lévy noise has been studied. Have provided the existence and uniqueness result for global positive solution and given the stochastic stability of positive equilibrium. Above two well-known examples fully demonstrate the applicability and effectiveness of our results.

In addition, simulation results in this paper can be helpful in understanding the role of Lévy noise in network stability. This paper is not a simple modification of[10] and other\ relevant papers. On the one hand, Lévy noise has been firstly incorporated into the coupled networks in this paper, which is much more realistic in the concrete applications. It should be mentioned that the stability analysis for the model including Lévy noise is more complicated than one without Lévy noise, since we must deal with the jump part. On the other hand,[10] mainly studied the stochastic stability, while our paper aims to investigate both the moment exponential stability and stochastic stability. In a word, our results improveand extend the earlier work. In our future work, we will further look into a more complicated Lyapunov function for coupled systems, not just at the weighted sum of the given ones. theoretically address some sensitivity analysis and robustness issues for stochastic coupled systems of differential equations on networks under Lyapunov conditions in view of the work for stochastic difference equations under Lyapunov conditions in[11].

[5]With multimedia dominating the digital contents, device-to-device communication has been proposed as a promising data offloading solution in the big data area. As the quality of experience (QoE) is a major determining factor in the success of new multimedia applications, it proposes a QoE-driven cooperative content dissemination (QeCS) scheme in this paper. In particular, all users predict the QoE of the potential connections characterized by the mean opinion score (MOS), and send the results to the content provider (CP). Then, the CP formulates a weighted directed graph according to the network topology and MOS of each potential connection. In order to stimulate cooperation among the users, the content dissemination mechanism is designed through seeking one-factor of the weighted directed graph with the maximum weight thus achieving maximum total user MOS. In addition, a debt mechanism is adopted to combat the cheat attacks. Furthermore, the proposed QeCS scheme by considering a constrained condition to the optimization problem for fairness improvement. Extensive simulation results demonstrate that the proposed QeCS scheme achieves both efficiency and fairness especially in large scale and density networks.

In this paper proposed a QoE-driven cooperation stimulation multimedia content dissemination strategy for D2D communication in IMT-A networks. Considering the features of D2D communication that over two-thirds of the mobile data traffic will be multimedia contents, designed the content dissemination mechanism to maximize the total user MOS as it is an important user centric measurement in multimedia communication. In order to stimulate the users to be cooperative in sharing contents with others, employed resource-exchange based incentive strategy, resolved by introducing graph theory, i.e., the issue is formulated as graph factorization and optimization problems. Meanwhile, a debt mechanism was introduced to realize cheat proof. Moreover, it extended the QeCS scheme by adding a constrained condition to the optimization problem to achieve a certain amount of fairness in QeCSC. To our best knowledge, the proposed scheme is the rest study on multimedia content dissemination strategy in D2D communication thus the work sheds light on this research line. For future work, we will consider a completely distributed content dissemination scheme with a more general QoE estimation model. Meanwhile, we will extend this work by investigating coalitional game theory framework to enhance the robustness of the system.

[4]A method of applying network flow analyses during real time power system operation, to provide better network connectivity visualization, is developed and presented.

Graph theory network flow analysis is capable of determining the maximum flow that can be transported between two nodes within a directed graph. These network flow algorithms are applied to a graphical representation of a power system topology to determine the minimum number of system branches needed to be lost in order to guarantee disconnecting the two nodes in the system that are selected. The number of system branches that are found serves as an approximate indicator of system vulnerabilities. The method used in these connectivity analyses makes use of well known graph theory network flow maximum flow algorithms, but also introduces a new algorithm for updating an old network flow solution for the loss of only a single system branch. The proposed new algorithm allows for significantly decreased solution time that is desired in a real-time environment. The value of using the proposed method is illustrated by using a detailed example of the 2008 island formation that occurred in the Entergy power system. The method was applied to a recreation of the 2008 event using a 20,000-bus model of the Entergy system to show both the proposed method's benefits as well as practicality of implementation.

This work proposes a purely topology based analysis using graph theory network flows to assist with power system operations. The proposed algorithm utilizes well-documented maximum flow network flow algorithms, such as Edmonds and Karp, as well as a new method of updating a network flow solution using a single path search. The method was explained in detail and was applied to a recreation of the 2008 Entergy island formation. The results of the algorithm after each outage clearly indicate that a large area containing Baton Rogue and New Orleans is being slowly disconnected from the rest of the network. The method is able to supplement conventional monitoring tools or serve as an alternative visualization tool in the event state estimation is not available. The work uses a purely topology based representation of a power system to approximate connectivity vulnerabilities. A large reduction in computational complexity is the primary advantage to using this representation. However, without including the electrical properties of the network within the analysis, the applicability of the method has limitations. The method is not capable of determining at what point another outage is not survivable except for the special case where a cut set of size one is found. Instead, the method identifies vulnerabilities as a result of topological changes. Problems that are not preceded or caused by significant changes in topology cannot be identified using this method.

[3]This paper is concerned with the adaptive pinning synchronization problem of stochastic complex dynamical networks(CDNs). Based on algebraic graph theory and Lyapunov theory, pinning controller design conditions are derived, and the rigorous convergence analysis of synchronization errors in the probability sense is also conducted. Compared with the existing results, the topology structures of stochastic CDN are allowed to be unknown due to the use of graph theory. In particular, it is shown that the selection of nodes for pinning depends on the unknown lower bounds of coupling strengths. Finally, an example on a Chua's circuit network is given to validate the effectiveness of the theoretical results.

In this paper, the problem of adaptive pinning synchronization control of stochastic CDN with unknown topology structure has been considered. By combining graph theory and Lyapunov theory, pinning controller design conditions have been derived and the rigorous convergence analysis of synchronization errors in the probability sense has also been conducted. It is shown that the number of pinning nodes and the selection of nodes for pinning depend on the unknown lower bounds of coupling strengths. In particular, by further constructing an appropriate Lyapunov functional, the above controller design conditions have been proven to be still valid for the stochastic CDN with coupling delay. Finally, the simulation results on a Chua's circuit network have been givento illustrate the effectiveness of the obtained theoretical results.

[2]The world of system of systems engineering (SoSE), and consequently systems engineering, is currently being reformalized so as to provide greater functionality, integration, and extensibility into designed systems of systems (SoS). Graph theory and big data techniques are in position to play substantial roles in this reformalization, and hence, this paper provides a significant need by bringing together all known current applications of graph theory to SoSE in tutorial/summary form. In this paper, explore the applications of graph theory and known graph algorithms for system design (and SoS design), optimization techniques, complexity measures, and novel graph algorithms for real-time deployment of SoS's and managing SoS design. The work is straightforward with several examples, and is meant to be a standalone document that can be used to quickly come up to speed regarding the applications of graph theory in SoSE.

This paper has outlined several applications of graph theory to system of systems engineering. Some of these have already begun to be explored in the literature, while others are novel concepts. Certainly, graph algorithms and graph databases are tools that may be big players in the SoSE space due to their inherent extensibility and their ability to model relationships. After all, relationships between systems are where the crux of the difficultly lies in SoSE. We need better techniques for designing, simulating, and deploying systems and interfaces between systems, and graph theory may be one vehicle that can solve some of these problems. As mentioned throughout this paper, each of the concepts presented can be extended in future work so as to eventually provide a suite of mature algorithms and techniques for modeling, simulating, building, and deploying complex systems of systems.

[8]A unified propagation graph modeling approach is proposed, which is applicable to predicting multipath radio propagation by considering both specular components (SCs) and diffuse components (DCs). In this approach, the semi-deterministic graph modeling is used to generate multibounce DCs, and a single-lobe directive model is applied to reproducing individual single-bounce SCs together with DCs associated. The performance of the proposed approach is assessed by comparing measured channel characteristics with simulation results for 60 GHz millimeter-wave propagation in an office environment.

In this paper, a simulation-based channel modeling approach using a unified propagation graph theory was proposed which generates multiple-bounce diffuse scattering components by using semi-deterministic graph modeling, and meanwhile simulates single-bounce specular components and the accompanying diffuse components by using a single-lobe directive model. Channel measurement results obtained in an office scenario at 60 GHz frequency bands were adopted for evaluating the performance of proposed approach. The results showed that the PDP and PAS simulated by using the unified graph modeling approach are consistent with their counterparts obtained through measurements. This indicates the necessity of extending the conventional graph modeling technique to including the directive models that help emulate power dispersion in directions more realistically.

III. COMPARISON

Table I
Comparison Study of Graph Theory

SLNO	GRAPH THEORY		
	Approaches	**Focused on**	**Advantage**
1	spectral graph theory	enable the construction of different coupling-strength combinations in response to different knowledge about sub networks	simplify the computation
2	stochastic coupled systems with Lévy noise on networks	obtained an allowable bound of perturbation intensity of white noise and Lévy noise for stability of such oscillators	provides stability
3	multimedia dominating the digital contents	a QoE-driven cooperative content dissemination (QeCS) real time power system operation	Capable power system provides
4	adaptive pinning synchronization	controller design conditions have been derived and the rigorous convergence analysis of synchronization errors in the probability sense has also been conducted	To detect the probability of synchronization error in the controller design
5	A unified propagation graph modeling approach	predicting multipath radio propagation by considering both specular components (SCs) and diffuse components (DCs)	Provides consistent with their counterparts obtained through measurements

IV. CONCLUSION

This paper has showed the role of graph theory in various approaches. Some of these have previously begun to be explored the literature, while others are fresh concepts. Graph theory may be one of transportation to solve various problems such as calculating, simulating, and deploying systems and interfaces between systems.

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