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The Numerical Analysis of 2D Laminar Forced Convective Flow of Viscous Fluid over a Flat Plate

Jaionto Karmokar[1]*, Md. Sayeed Iftekhar Yousuf[2], M. M. Billah[3] and M. A. Haque[4]

[1,2,3]Department of Computer Science and Mathematics,
Bangladesh Agricultural University, Mymensingh-2202, BANGLADESH.
[4]Department of Applied Mathematics,
University of Rajshahi, Rajshahi-6205, BANGLADESH.
*Corresponding Author: jaiontoj@gmail.com

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ABSTRACT

This article provided to study the trend of heat transfer of two dimensional laminar force convective flow of viscous fluid over a flat plate. The governing nonlinear partial differential equations have been reduced to systems of nonlinear ordinary differential equations by employing the Adomian Decomposition Method (ADM). The transformed equations are solved numerically by using the software MAPLE. A comparative study of the numerical results with the results from an exact solution for the dimensionless similarity variables at the flat plate is also performed. In Pr=1, the ADM has a high accuracy than NM. The results are comparing between the numerical method and ADM, both of solution shows excellent agreement.

Keywords: Forced convection, Boundary layer, ADM, Heat transfer.

1. INTRODUCTION

Heat transfer measurement from the steady laminar forced convective flow in a heating element inside the sphere and temperatures of the flowing air of the surface employing corrections for heat transfer due to thermal radiation and natural convection. The most methodical problems such as heat transfer are integrally nonlinear. We know that excluding a limited number of these problems, most of them do not have numerical solutions. Using perturbation method and numerical techniques, some of the problems are solved. The boundary-layer models of nonlinear partial differential equations reduce to ordinary differential equations which are usually solved by numerical methods.

The influence of gravitational force occurs frequently in force convection flows as well as in science and engineering applications. Since there are some limitations of analytical solving problems upon the existence of a small parameter for different applications is very difficult. The newly developed method is Adomian decomposition method (ADM) based on series approximation for strongly nonlinear problem solving. Besides, the Homotopy Perturbation Method (HPM) used to finding the series solutions to boundary-layer equations, whereas the series in ADM (1994) are derived from functions in terms of consisting initial conditions.

When a heated surface is in contact with the fluid, the result of temperature difference causes the influence of the thermal and concentration buoyancy, which induces the natural convection. The flow is supposed to be in the *x*-direction and *y*-axis is normal to it. Many researchers are studies the problem of boundary layer force convection flow past a flat plate under different plate conditions. Hashim *et al*. (2006) applied ADM to the conventional Blasius equation and Wazwaz (2006) used ADM to solve the boundary layer equation of viscous flow due to a moving sheet. Awang Kechil *et al*. (2009) estimated the analytical solution of an unsteady boundary layer problem over a stretching sheet to applicability of ADM. Awang Kechil *et al*. (2007) applied ADM to a 2 by 2 system of nonlinear ordinary differential equations of free convective boundary layer equation. The MHD flow over a nonlinear stretching sheet by using the modified ADM are studied Hayat *et al*. (2009).

The objective of this study is to investigate numerically using Adomian decomposition method (ADM) together with two dimensional laminar forced convection flow of viscous fluid over a flat plate with uniform wall temperature. Also we comparing the numerical method (NM) and Adomian decomposition method (ADM) to determining the accuracy of them. In Pr=1, the ADM has a high accuracy than NM. Since fluid is a constant properties. So we applying the Adomian Decomposition Method (ADM) to obtain an approximate numerical solution of this problem. Finally, we have compared the analytic results with numerical results.

2. GOVERNING EQUATIONS

Consider steady force convection and heat transfer flow of a viscous, incompressible fluid past a flat plate under the influence of the thermal and concentration buoyancy with constant free stream velocity u_∞. All fluid properties are considered to be constant. At time t >0, the plate temperature and concentration are instantly raised and which are thereafter maintained constant, where T_∞ is the temperature outside the boundary layer. The combination of continuity, Navier-Stokes and energy equations in 2-dimensional and steady form was used as the essential governing equations. These equations and their boundary conditions are listed through numbers of equations 1 to 4.

The equation of continuity, Navier-Stokes equation and energy equation are

$$\frac{\partial \mathrm{u}}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial \mathrm{u}}{\partial x} + v\frac{\partial \mathrm{u}}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}+\frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

The following boundary conditions are

$\mathrm{T} = 0$ at $y = 0$; $\mathrm{T} = \mathrm{T}_\infty$ at $x = 0$; $\mathrm{T} \to \mathrm{T}_\infty$ at $y \to \infty$. (4a)

And $u = 0$, $v = 0$ at $y = 0$; $u = u_\infty$ at $x = 0$; $u \to u_\infty$ at $y \to \infty$. (4b)

This equations have been transformed so that, in effect, the grid grows along with the boundary layer in the direction normal to the plate. This strategy allows better resolution near the leading edge and reduces the number of extraneous grid points in the undisturbed free stream.

3. MATHEMATICAL ANALYSIS OF THE PROBLEM

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables. The solution to the momentum equation is decoupled from the energy solution. However, the solution of the energy equation is still linked to the momentum solution. We employ the following dimensionless variables:

$$\eta(x,y) = \frac{y}{x}Re^{\frac{1}{2}} \quad (5)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \quad (6)$$

We define the Reynolds number as

$$Re_x = \frac{u_\infty\, x}{\nu}. \quad (7)$$

A numerical analysis has been carried out to the forced convection heat transfer on laminar flow of viscous fluid over a flat plate. The boundary condition has been used for solid walls at constant temperature and the air was used as a functioning fluid. Initially it is assumed that the fluid and the plate are at the same temperature T and the concentration level everywhere in the fluid is same. The Reynolds number shows that the inclination angle and the length of unheated region of plate play significant role on heat transfer from the plate.

From equation (5), we get

$$\eta(x,y) = \frac{y}{x}\left(\frac{u_\infty\, x}{\nu}\right)^{\frac{1}{2}} = y\sqrt{\frac{u_\infty}{\upsilon x}}. \quad (8)$$

We define the stream function

$$u = \frac{\partial\Psi}{\partial y}, \quad v = -\frac{\partial\Psi}{\partial x}. \quad (9)$$

From equation (9), we get

$$d\Psi = u\, dy \Rightarrow \Psi = u_\infty\sqrt{\frac{\upsilon x}{u_\infty}}\, f(\eta) \quad (10)$$

Since $\eta(x,y) = y\sqrt{\frac{u_\infty}{\upsilon x}} \Rightarrow dy = \sqrt{\frac{\upsilon x}{u_\infty}}\, d\eta.$ (11)

Again from equation (9), we get

$$u = \frac{\partial\Psi}{\partial y} = u_\infty\sqrt{\frac{\upsilon x}{u_\infty}}\ f'(\eta)\ \frac{\partial\eta}{\partial y} = u_\infty\sqrt{\frac{\upsilon x}{u_\infty}}\ f'(\eta)\sqrt{\frac{u_\infty}{\upsilon x}}\ = u_\infty\ f'(\eta).$$

$$v = -\frac{\partial\Psi}{\partial x} = -\ u_\infty\sqrt{\frac{\upsilon}{u_\infty}}\ \left\{\sqrt{x}\, f'(\eta)\frac{\partial\eta}{\partial x} + f(\eta)\frac{1}{2\sqrt{x}}\right\} = \ \frac{1}{2}\sqrt{\frac{\upsilon u_\infty}{x}}\{\eta f'(\eta) - f(\eta)\}.$$

Differentiating with respect to x and y, respectively

$$\frac{\partial u}{\partial x} = u_\infty f''(\eta)\ \mathrm{y}\sqrt{\frac{u_\infty}{\upsilon}}\left(-\frac{1}{2}\right)\ \frac{1}{x\sqrt{x}} = -\frac{\eta}{2x}u_\infty f''(\eta)\ .$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\{u_\infty f'(\eta)\} = u_\infty f''(\eta)\sqrt{\frac{u_\infty}{\upsilon x}}\ .$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left\{u_\infty f''(\eta)\sqrt{\frac{u_0}{\upsilon x}}\right\} = \frac{{u_\infty}^2}{\upsilon x}f'''(\eta).$$

Putting the these values in equation (2), we get

$$u_\infty f'(\eta)\left\{-\frac{u_\infty\eta}{2x}f''(\eta)\ \right\} + \left[\frac{u_\infty}{2}\ \sqrt{\frac{\upsilon u_\infty}{x}}\{\eta\, f'(\eta) - f(\eta)\}\right]f''(\eta)\sqrt{\frac{u_\infty}{\upsilon x}} = \frac{{u_\infty}^2}{x}f'''(\eta)$$

$$\Rightarrow -\frac{\eta}{2x}\ {u_\infty}^2\ f'(\eta)\ f''(\eta) + \frac{1}{2x}\ {u_\infty}^2 f''(\eta)\{\eta\, f'(\eta) - f(\eta)\ \} = \frac{{u_\infty}^2}{x}f'''(\eta)$$

$$\therefore\ \ f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) = 0. \tag{12}$$

From equation (6), we get

$$T = \ T_\infty + \theta(\eta)\ (T_w\ -\ T_\infty\ ).$$

Differentiating with respect to x and y, respectively

$$\frac{\partial T}{\partial x} = \theta'(\eta)(T_w -\ T_\infty)\ \mathrm{y}\sqrt{\frac{u_\infty}{\upsilon}}\left(-\frac{\mathrm{x}^{-3/2}}{2}\right) = -\frac{\eta}{2x}\theta'(\eta)(T_w -\ T_\infty)\ .$$

$$\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\{T_\infty + \theta(\eta)\ (T_w - T_\infty)\} = \theta'(\eta)\sqrt{\frac{u_\infty}{\upsilon x}}(T_w - T_\infty).$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\{\theta'\ (\eta)\sqrt{\frac{u_\infty}{\upsilon x}}(T_w - T_\infty)\} = (T_w - T_\infty) + \frac{u_\infty}{\upsilon x}\theta''(\eta).$$

Putting the these values in equation (3), we get

$$u_\infty\ f'(\eta)\ \{-\frac{1}{2x}\eta\ \}\theta'(\eta)(T_w - T_\infty\ ) + \frac{1}{2}\sqrt{\frac{\upsilon u_\infty}{x}}\{\ \eta\ f'(\eta) - f(\eta)\}\ \theta'(\eta)\sqrt{\frac{u_\infty}{\upsilon x}}\ \ (T_w - T_\infty\ )$$
$$= \alpha\ (T_w - T_\infty\ )\frac{u_\infty}{\upsilon x}\theta''(\eta)$$

$$\Rightarrow -\frac{u_\infty}{2x}\eta\ f'(\eta)\theta'(\eta)\ (T_w - T_\infty\ ) + \frac{u_\infty}{2x}\theta'(\eta)(T_w - T_\infty\ )\{\eta\ f'(\eta) -\ f(\eta)\ \} = \alpha\ (T_w - T_\infty\ )\frac{u_\infty}{\upsilon x}\theta''(\eta)$$

$$\Rightarrow -\frac{1}{2}\eta\ f'(\eta)\theta'(\eta) + \frac{1}{2}\theta'(\eta)\{\eta\ f'(\eta) - f(\eta)\} = \frac{\alpha}{\upsilon}\theta''(\eta)$$

$$\therefore\ \theta''(\eta) + \frac{Pr}{2}\ f(\eta)\theta'(\eta) = 0\ . \tag{13}$$

Using Equations (1) through (6), the governing equations can be reduced to two equations where f *is* a function of the similarity variable (η):

$$f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) = 0$$

$$\theta''(\eta) + \frac{Pr}{2} f(\eta)\theta'(\eta) = 0$$

Where,

$$Pr = \frac{\upsilon}{\alpha}.$$

The reference velocity is the free stream velocity of forced convection. The boundary conditions are obtained from the similarity variables. For the forced convection case (Incropera *et al*. 1996), the boundary conditions reduces to

$$f(0) = 0, f'(0) = 1, \theta'(0) = 1, f'(\infty) = 1, \theta(\infty) = 0. \quad (14)$$

Where primes denote differentiation with respect to similarity variable η.

4. ADOMIAN DECOMPOSITION METHOD

We follow the standard procedure of ADM (Adomian et al. 1994) by introducing two linear differential operators $L_1 = {d^3}/{d\eta^3}$ and $L_2 = {d^2}/{d\eta^2}$ with inverse operators are

$$L_1^{-1}(\cdot) = \int_0^\eta \int_0^\eta \int_0^\eta (\cdot)\, dtdtdt$$

$$L_2^{-1}(\cdot) = \int_0^\eta \int_0^\eta (\cdot)\, dtdt.$$

Thus, Equations (12) and (13) in operator form reduces to

$$L_1(f) = -\frac{1}{2} f(\eta)\, f''(\eta) \quad (15)$$

$$L_2(f) = \frac{Pr}{2} f(\eta)\theta'(\eta) \quad (16)$$

Applying the inverse operators on Equations (15) and (16) and let $f''(0) = \propto_1$, $\theta'(0) = \propto_2$ we obtain,

$$f = \frac{1}{2} \propto_1 \eta^2 + \frac{1}{2} L_1^{-1}\big(P(f)\big) \quad (17)$$

$$\theta = 1 + \propto_2 \eta + \frac{Pr}{2} L_2^{-1}\big(Q(f,\theta)\big) \quad (18)$$

And their respective decompositions,

$$P(f) = \sum_{i=0}^{\infty} A_i \quad (19)$$

$$Q(f,\theta) = \sum_{i=0}^{\infty} E_i \quad (20)$$

A_i , E_i are so-called Adomian polynomials (Adomian et al. 1994), given by

$$A_i = \frac{1}{i!}\left[\frac{d^i}{d\lambda^i} P\left(\sum_{j=0}^{\infty} \lambda^j f_j\right)\right]_{\lambda=0}, \; i \geq 0\,. \quad (21)$$

In ADM (Wazwaz 2006), f and θ are defined as infinite series,

$$f = \sum_{i=0}^{\infty} f_i(\eta) \quad (22)$$

$$\theta = \sum_{i=0}^{\infty} \theta_i(\eta). \quad (23)$$

Substituting Equations (19) and (20) and Equations (22) and (23) into Equations (17) and (18), we obtain

$$\sum_{i=0}^{\infty} f_i(\eta) = \frac{1}{2} \propto_1 \eta^2 - \frac{1}{2} L_1^{-1}\left(\sum_{i=0}^{\infty} A_i\right) \quad (24)$$

$$\sum_{i=0}^{\infty} \theta_i(\eta) = 1 + \propto_2 \eta + \frac{Pr}{2} L_2^{-1}\left(\sum_{i=0}^{\infty} E_i\right) \quad (25)$$

This represents the individual terms for f and θ are obtained from the recursive relations. For practical numerical computation, we will compute the j-term approximation of $f(\eta), \theta(\eta)$ which are converge to the true series as j approaches infinity.

5. RESULTS AND DISCUSSION

A representative set of numerical results is shown graphically in Figure 1 to 3 to illustrate the comparison of the resulted by ADM and NM for $f(\eta)$, $\theta(\eta)$ and $f'(\eta)$ respectively. The Adomian polynomials in equation (21) and the recursive relations (24–25) are then coded in the Maple environment computer package with the controlling significant digits set to 11. In the flow field, Reynolds number is one of the most important parameter. The velocity profile near the solid wall will be increased and causes the departure phenomenon happened with delay if the value of Reynolds number is increased.

Table 1. Numerical values of $f''(0)$, $\theta'(0)$ for Pr = 1

	[8.8]	[9.9]	[10.10]	[11.11]	Exact
$\propto_1$	0.268	0.449	0.329	0.329	0.329
$\propto_2$	-0.279	-0.426	-0.348	-0.349	-0.349

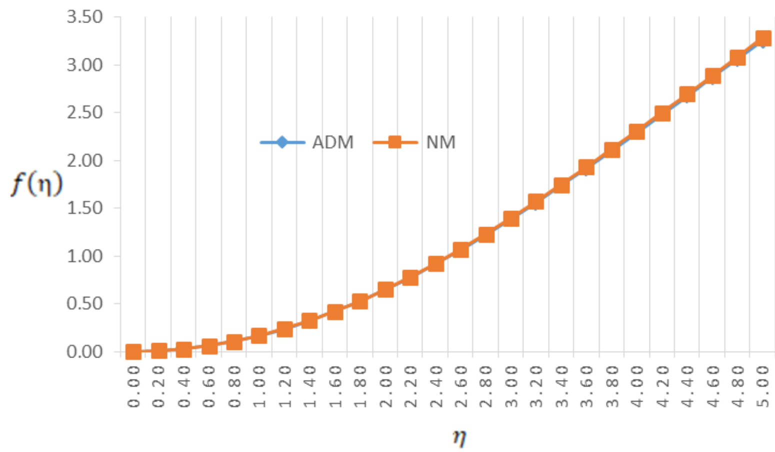

Figure 1. The comparison of the result between ADM and NM for $f(\eta)$

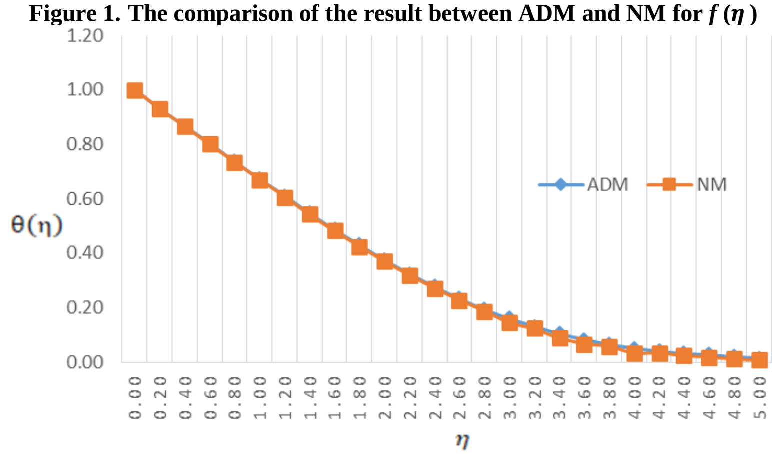

Figure 2. The comparison of the result between ADM and NM for $\theta(\eta)$

The flow field has been studied steadily and the vortexes were not allowed to enter the flow. If the Reynolds number is increasing then the value of dimensionless heat transfer coefficient will be raised which is better heat transfer mechanism. The numerical results of $\propto_1$ *and* $\propto_2$ for selected m in the range from 8 to 11 are presented in Table 1 for Pr =1. Since Equation (12) and (13) cannot be easily solved by the analytical method. Equation (12) and (13) are solved by the numerical method using the software MAPLE whose results are given in Tables 2. As we can see in Pr = 1, the ADM has a high accuracy. In Figure 1, the distribution of $f(\eta)$ is increasing along the wall with respect to similarity variable η. This figure represents the results of ADM and NM has the same trend. But in Figure 2, the distribution of θ(η) is decreasing along the wall with respect to similarity variable η with the same trend. Also the distribution of $f'(\eta)$ is increases with increase of this parameter as shown in Figure 3.

Table 2. The results of ADM and NM for $f(\eta)$, $f'(\eta)$ and θ(η) if Pr=1

	$f(\eta)$		$f'(\eta)$		θ (η)	
η	ADM	NM	ADM	NM	ADM	NM
0.0	0.000	0.000	0.000	0.000	1.000	1.000
0.2	0.007	0.007	0.066	0.066	0. 934	0.934
0.4	0.026	0.027	0.132	0.133	0.868	0.867
0.6	0.059	0.059	0.197	0.199	0.803	0.801
0.8	0.105	0.106	0.262	0.265	0.737	0.735
1.0	0.164	0.165	0.327	0.329	0.673	0.670
1.2	0.236	0.238	0.391	0.394	0.609	0.606
1.4	0.320	0.323	0.452	0.456	0.547	0.544
1.6	0.417	0.420	0.513	0.517	0.487	0.483
1.8	0.525	0.529	0.570	0.575	0.430	0.425
2.0	0.645	0.650	0.625	0.629	0.375	0.370
2.2	0.775	0.781	0.676	0.681	0.324	0.319
2.4	0.915	0.922	0.723	0.729	0.277	0.271
2.6	1.064	1.072	0.767	0.772	0.233	0.228
2.8	1.221	1.231	0.805	0.811	0.194	0.188
3.0	1.386	1.397	0.840	0.846	0.160	0.144
3.2	1.557	1.569	0.870	0.876	0.130	0.124
3.4	1.734	1.747	0.896	0.902	0.104	0.088
3.6	1.915	1.929	0.917	0.923	0.083	0.067
3.8	2.100	2.116	0.935	0.941	0.065	0.059
4.0	2.289	2.306	0.949	0.956	0.051	0.031
4.2	2.479	2.498	0.961	0.967	0.039	0.033
4.4	2.673	2.692	0.969	0.976	0.031	0.024
4.6	2.867	2.888	0.972	0.983	0.028	0.017
4.8	3.061	3.085	0.982	0.988	0.019	0.012
5.0	3.248	3.283	0.990	0.992	0.013	0.008

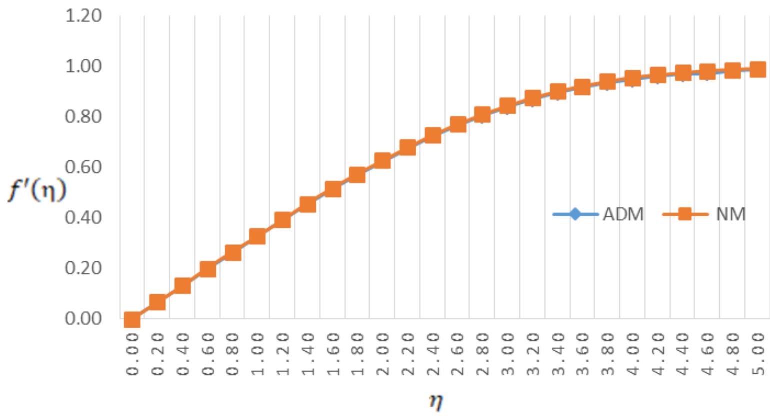

Figure 3. The comparison of the result between ADM and NM for $f'(\eta)$

We obtain 10-term approximation to both f and θ, but for lack of space, only the first 3 terms produced from (24-25) are given below:

$$f_0(\eta) = \frac{1}{2} \propto_1 \eta^2 \quad (26)$$

$$f_1(\eta) = -\frac{1}{240} {\propto_1}^2 \eta^5 \quad (27)$$

$$f_2(\eta) = -\frac{11}{161280} {\propto_1}^3 \eta^8 \quad (28)$$

$$\theta_0(\eta) = 1 + \propto_2 \eta \quad (29)$$

$$\theta_1(\eta) = -\frac{Pr}{48} \propto_1 \propto_2 \eta^4 \quad (30)$$

$$\theta_2(\eta) = -\frac{11}{20160} Pr \, {\propto_1}^2 \propto_2 \eta^7. \quad (31)$$

The undetermined values of $\propto_1$ and $\propto_2$ are calculated from the boundary conditions at infinity in equation (15). The difficulty at infinity is overcome by employing the diagonal Pade approximates (Baker et al. 1975) that approximates $f'(\eta)$ and $\theta(\eta)$ using $\varphi_{10}(\eta)$ and $\Psi_{10}(\eta)$ respectively. We observed that the temperature diminishes with increasing generalized Prandtl number (Pr) which reduces the thermal boundary layer thickness. The variation of $f(\eta)$ with η for different values are increasing exponentially but the variation of $\theta(\eta)$ with η are decreasing. Also the variation of $f'(\eta)$ with η for different values are increasing dramatically. Therefore, the numerical results of the ADM has a high accuracy than NM when Pr=1. These method gives the clear expression of the series approximation for nonlinear problem solving.

6. CONCLUSION

Numerical analysis of flow field and heat transfer of two dimensional laminar forced convective flow has been studied in this paper. The governing equations are transformed using the Adomian Decomposition Method (ADM) and then solved numerically by using the software MAPLE. The ADM has been successfully applied to forced convection heat transfer problem with specified boundary conditions. The obtained solutions of ADM are compared with numerical method (NM). The results relating to the present study have shown that all the parameters affect the flow field and heat transfer significantly. The excellent agreement of the ADM solutions and exact solutions shows the reliability and the efficiency of the method. Additionally, a graphical comparison has showed that the numerical results matched well with the NM results. The ADM provides efficient alternative tools in solving nonlinear problem.

NOMENCLATURE

v → velocity component in the *y* direction
Pr → Prandtl number
T_W → temperature imposed on the plate
ν → kinematic viscosity
T_∞ → local ambient temperature
α → thermal diffusivity
u → velocity component in the *x* direction
θ → dimensionless temperature
ADM → Adomian Decomposition Method
NM → numerical method
g → gravitational force
ρ → density

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