Journal of Computer and Mathematical Sciences, *Vol.10(5),1016-1020 May 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

On g*ω Closed Sets in Topological Spaces

J. Therasa[1], V. Senthilkumaran[2] and Y. Palaniappan[3]

[1]M.Phil Scholar, Department of Mathematics,
Arignar Anna Government Arts College Musiri, Tamilnadu, INDIA.
[2]Associate Professor, Department of Mathematics,
Arignar Anna Government Arts College Musiri, Tamilnadu, INDIA.
[3]Associate Professor (Retd), Department of Mathematics,
Arignar Anna Government Arts College Musiri, Tamilnadu, INDIA.
email: therasa1923@gmail.com, palaniappany48@gmail.com, vsenthil1966@gmail.com

(Received on: April 27, 2019)

ABSTRACT

In this paper, we introduce a new closed set called g*ω closed set and study its properties. Also we introduce and investigate the concept of g*ω continuous function.

AMS Subject Classification: 54A05, 54C05.

Keywords: g*ω closed set and g*ω continuous function.

1. INTRODUCTION

The notion of ω-closed sets are introduced by Sundaram and Sheik John[10] and Benchalli *et al*.[1] studied ωα-closed sets in topological spaces. M. Parimala *et al*.[8] investigated αω closed sets in topological spaces. Levine[5] introduced the concept of generalized closed sets in topological spaces. Here we investigate generalized ω closed sets (briefly g*ω closed sets) in topological spaces. Further we study g*ω continuous function.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) represent topological spaces. They will be simply denoted by X and Y. For a subset A of a space X, cl (A) and int(A) denote the closure and interior of A respectively.
The following definitions will be useful on the sequel:

Definition 2.1. A subset A of a space X is called
1. a semi open set [4] if $A \subset cl\ int\ (A)$
2. a α open set [7] if $A \subset int\ cl\ int\ (A)$

Definition 2.2.
1. a αg closed [6] set if $\alpha\ cl\ A \subset U$ whenever $A \subset U$ and U is open.
2. a generalized closed [5] (briefly g-closed) if $cl(A) \subset U$ whenever $A \subset U$ and U is open in X.
3. a $\omega(=\hat{g})$-closed set [13] if $cl(A) \subset U$ whenever $A \subset U$ and U is semi open in X.
4. a $g^{\#}$ closed set [12] if $cl(A) \subset U$ whenever $A \subset U$ and U is αg open in X.

The complements of the above closed sets are respective open sets.

Definition 2.3. A function $f : X \rightarrow Y$ is called
1. $\omega(\hat{g})$ continuous [13] if $f^{-1}(V)$ is ω closed in X, for every closed set V of Y.
2. $g^{\#}$ -continuous [12], if $f^{-1}(V)$ is $g^{\#}$ closed in X for every closed set V of Y.

3. g*ω-CLOSED SETS

Definition 3.1. A subset A of X is called a $g^*\omega$-closed set if $\omega cl\ A \subset U$ whenever $A \subset U$ and U is g open in X.
The complement of $g^*\omega$ closed set is a $g^*\omega$ open set.

Theorem 3.2. Every closed set is a $g^*\omega$ closed set.

Proof. Let A be closed in X, then A = cl(A). Let $A \subset U$, where U is g-open in X. Since A is closed $\omega cl(A) \subset cl(A) = A \subset U$. This implies A is $g^*\omega$ closed set.
The converse of the above theorem need not be true can be seen from the following example.

Example 3.3. Let X = {a,b,c}and τ = {φ,{a},{a,b},X}, C(X,τ) = {φ, {c}, {b,c}, X} and $g^*\omega$C(X,τ) = {φ,{c},{a,c}{b,c},X}. Here A = {a,c} is $g^*\omega$ closed but not closed.

Theorem 3.4. Every $g^{\#}$-closed set is a $g^*\omega$-closed.

Proof. Let A be $g^{\#}$ closed and $A \subset U$, where g is open. So U is αg-open. A is $g^{\#}$ closed. Hence $cl\ A \subset U$ $\omega cl(A) \subset cl(A) \subset U$, A is $g^*\omega$ closed.
The converse of the above theorem need not be true can be seen from the following example.

Example 3.5. Let X = {a,b,c}and τ = {φ,{a},{a,b},X}, C(X,τ) = {φ, {c}, {b,c}, X} and $g^{\#}$C(X,τ) = {φ,{c},{b,c},X}. Here A = {a,c} is $g^*\omega$ closed but not $g^{\#}$ closed.

Theorem 3.6. Union of two g*ω-closed sets is g*ω-closed set.

Proof. Let A and B be two g*ω closed sets. Let A ∪ B ⊂ U, where U is g-open. Then A ⊂ U, B ⊂ U. As A and B are g*ω closed set. ωcl(A) ⊂ U, ωcl(B) ⊂ U. Hence ωcl(A∪B) = ωcl(A) ∪ ωcl(B) ⊂ U. So A ∪ B is g*ω closed.

Theorem.3.7. If a subset A of X is g*ω-closed set in X, then ωcl(A)-A does not contain any non empty g closed set in X.

Proof. Let A be g*ω closed. Let F be a g-closed set. Such that F ⊂ ωcl(A) - A, Then F ⊂ X-A. A ⊂ X-F and X-F is g-open. As A is g*ω closed set. ωcl(A) ⊂ X-F. That is F⊂ X-ωcl(A). Hence F ⊂ ωcl(A) ∩ (X-ωcl(A)) = φ. So, F = φ.

Theorem.38. If A is g*ω –closed set in X and A ⊂ B ⊂ ωcl(A), then B is also g*ω-closed set in X.

Proof. Let A be g*ω-closed set in X. Let B ⊂ U, where U is g-open set in X. Hence A ⊂ U, ωclB ⊂ ωcl(ωclA) = ωclA ⊂ U, as A is g*ω-closed. Hence B is g*ω-closed.

The converses of the above theorem need not be true can be seen from the following example.

Example.3.9. Let X = {a,b,c}and τ = {φ,{a},{a,b},X}, C(X,τ) = {φ, {c}, {b,c}, X} and A = {c} which is g*ω-closed, B = {b,c} which is g*ω-closed. Such that A ⊂ B but B⊄ωclA = {c}.

Theorem3.10. Let A be g*ω closed in X. Then A is ω closed iff ωclA-A is g-closed.

Proof. Let A be ω-closed. Then ωclA = A. Hence ωcl A-A = φ, which is g-closed. Conversely, let ωcl(A) - A be g-closed. Then ωcl(A)-A = φ . By theorem 3.9 that is ωcl(A)=A. So A is ω-closed.

Theorem3.11. If A is g-open and g*ω-closed set, then A is ω-closed.

Proof. A ⊂ A, A is g-open .A is g*ω-closed. Hence ωclA ⊂ A. So ωclA = A. So A is ω closed.

Theorem.3.12. A set A is g*ω-open in X iff F ⊂ ωint A, whenever F is g closed in X and F⊂A.

Proof. Let F ⊂ ωint A, where F is g closed and F ⊂ A. Let X-A ⊂ U, where U is g open in X. Then X-U is g closed and X-U ⊂ A. Hence X-U ⊂ ωint A. That is X-ωint A ⊂ U. That is ωcl(X-A) ⊂ U. Hence (X-A) is g*ω closed and so A is g*ω open.

Conversely, let A be g*ω open, F ⊂ A and F is g closed. Then X-F is g open X-A ⊂ X-F. X-A is g*ω closed. Hence ωcl(X-A) ⊂ X-F. That is X-ωint A ⊂ X-F. Hence F ⊂ ωint A.

Theorem.3.13. A subset A is g*ω open in X then G=X whenever G is g open and ωint A∪ (X-A) ⊂ G.

Proof. Let A be g*ω open. G is g open and ωint(A)∪(X-A)⊂G. This gives X-G⊂(X-ωintA)∩(X-(X-A)) = (X-ωintA)-(X-A) = ωcl(X-A)-(X-A). Since (X-A) is g*ω closed and X-G is g closed, Then by theorem3.9 it follows X-G = φ.

4. g*ω- CONTINUITY

Definition 4.1. A function f: (X,τ) →(Y,σ) is called g*ω continuous if $f^{-1}(V)$ is g*ω closed set of (X,τ) for every closed set V of (Y,σ).

Theorem.4.2. Every continuous map is g*ω-continuous.

Proof. Let V be a closed set of (Y, σ). Since f is continuous and $f^{-1}(V)$ is closed in (X, τ). But every every closed set is g*ω-closed set. Hence $f^{-1}(V)$ is g*ω-closd set in (X, τ). Thus f is g*ω-continuous.

The converse need not be true can be seen from the following example.

Example 4.3. Let X={a,b,c}, τ ={φ,{a},{b,c},X}. Let Y={a,b,c}, σ={φ,{a},{a,b},Y}. Let f: (X,τ)→(Y,σ) be the identity map f is g*ω continuous . Let V={c}, $f^{-1}(V)$={c} which is not closed in X. Hence f is not continuous.

Theorem 4.4. Every ω-continuous map is g*ω-continuous .

Proof. Let V be a closed set of (Y, σ). Since $f^{-1}(V)$ is ω closed in (X, τ). But every ω closed set is g*ω closed set in (X, τ). Thus f is g*ω continuous.

The converse need not be true can be seen from the following example.

Example: 4.5. Let X={a,b,c}, τ={φ,{a},{b,c},X} and Y={a,b,c}, σ={φ,{a},{a,b},Y}. Define f: (X,τ)→(Y,σ) by f(a)=b, f(b)=a, f(c)=c. f is g*ω continuous but not ω continuous, as f^{-1}{b,c}={a,c} is not ω closed in X.

Theorem 4.6. Every $g^{\#}$ continuous map is g*ω continuous.

Proof. Let V be a closed set of (Y, σ). Since $f^{-1}(V)$ is $g^{\#}$ closed in (X,τ). But every $g^{\#}$ closed set is g*ω-closed set. Hence $f^{-1}(V)$ is gω-closed set in (X,τ). Thus f is g*ω continuous.

The converse need not be true can be seen from the following example.

Example 4.7. Let X={a,b,c}, τ={φ,{a},{b,c},X} and Y={a,b,c}, σ={φ,{a},{a,b},Y}. Let f: (X, τ))→(Y,σ) be defined by f(a)=b, f(b)=a, f(c)=c. Hence f is g*ω continuous but not $g^{\#}$ continuous, as $f^{-1}(\{b,c\})=\{a,c\}$ is not $g^{\#}$ closed.

Theorem 4.8. A bijective mapping f: X→Y is g*ω closed iff for a subset S of Y and for each open set U containing $f^{-1}(S)$, there is g*ω open set V of Y such that S⊂V and $f^{-1}(V)\subset U$.

Proof. Let f be g*ω closed. Let S⊂Y and U be an open set of X such that $f^{-1}(S)\subset U$, then V=Y-f(X-U) ≤ f(U) is an g*ω open set containing S such that S⊂V and $f^{-1}(V) \subset U$.

Conversely, let F be a closed set of X. Then $f^{-1}(Y-f(F))=X-F$ and X-F is open. By hypothesis there is a g*ω open set V of Y such that Y-f(F) ⊂V and $f^{-1}(V) \subset X-F$. Therefore, $F\subset X-f^{-1}(V)$. Hence $Y-V \subset f(F) \subset f(X-f^{-1}(V)) \subset Y-V$ which implies f(F)=Y-V. As Y-V is g*ω closed, f(F) is g*ω closed and f is g*ω closed function.

REFERENCES

1. S.S. Benchalli, P.G.Patil and P.M.Nalwad, Generalized ωα-closed sets in Topological spaces , *Journal of New Results in Science*, 7, 7-19 (2014).
2. K. Balachandran, P.Sundaram and H.Maki, On generalized continuous maps in topological spaces, *Mem. Fac. Kochi Univ. Ser. A Math*., 12, 5-13 (1991).
3. R.Devi, K.Balachandran and H.Maki, Generalized α-closed maps and α generalized closed maps, *Indian J. Pure. Appl. Math*., 29(1), 37-49 (1998).
4. N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70: 36-41 (1963).
5. N.Levine, Generalized closed sets in topology, *Rend. Circ. Math. Palermo*, 19(2), 89-96 (1970).
6. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α-closed Sets and α-generalized closed sets, *Mem. Fac. Sci. Kochi. Univ. Ser. A Math*., 15, 51-63 (1994).
7. O. Njastad, On some classes of nearly open sets. *Pacific J. Math*., 15, 961-970 (1965).
8. M. Parimala, R. Udhayakumar, R. Jeevitha, V. Biju, On αω-Closed Sets in Topological Spaces, *International J. Pure. App. Math*, 115(5), 1049-1056 (2017).
9. P.G. Patil, S.S.Benchalli and Pallavi S. Mirajkar. Generalized star ωα-closed sets in Topological spaces. *Journal of New Results in Science*, 9, 37-45 (2015).
10. P. Sundaram and M. Shrik John. On ω-closed sets in topology. *Acta Ciencia Indica*, 4, 389-392 (2000).
11. M.K.R.S. Veera Kumar, Between closed sets and g-closed sets, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math*., 21, 1-19 (2000).
12. M.K.R.S. Veera Kumar, $g^{\#}$-closed sets in topological spaces, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math*., 24, 1-13 (2003).
13. M.K.R.S. Veera Kumar, On ĝ-closed sets in topological spaces, *Allahabad Math. Soc*., 18, 99-112 (2003).