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MHD Effects On Unsteady Free Convective Flow of a Fluid through a Porous Medium in a Channel with Adiabatic

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ABSTRACT

In this paper, we study the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.

Keywords: Free convection MHD flow, Porous medium, Adiabatic, Perturbation technique, Velocity and temperature fields.

1. INTRODUCTION

Flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. Examples of natural porous media are beach sand, stand stone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. Flow through a porous medium has been studied by a number of workers employing Darcy's law Scheidegger[8]. Some studies about this point have been made by Varshney[11] and Raptis and Perdikis[7]. Influence of MHD and radiation effects on oscillatory flow through a porous medium with constant suction velocity has been studied by El-Hakeem[4]. Makinde and Mhone[6] have investigated the heat transfer and MHD effects on an oscillatory flow in a channel filled with porous medium.

The unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic field between two horizontal plates, lower plate being a stretching sheet and upper being porous was studied by Sharma and Kumar[9]. Borkakati and

Chakrabarty[2] have investigated unsteady free convection MHD flow between two heated vertical plates. The unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account the viscous dissipative heat under the influence of a uniform transverse magnetic field was analyzed by Sreekant *et al*.[10]. Gourla and Katoch[5] have studied the unsteady free convection MHD flow between two heated vertical plates. Recently, Bhaskar[1] have studied the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate adiabatic.

In view of these, we studied the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.

2. MATHEMATICAL FORMULATION

We consider the free convective unsteady MHD flow of a viscous incompressible electrically conducting fluid between two heated vertical parallel plates filled with porous medium. Let x -axis be taken along the vertically upward direction through the central line of the channel and the y-axis is perpendicular to the x-axis. The plates of the channel are kept at $y = \pm h$ distance apart. A uniform magnetic field B_0 is applied in the plane of y-axis and perpendicular to the both x-axis and y-axis. u is the velocity in the direction of flow of fluid, along the x -axis and v is the velocity along the y-axis. Consequently, u is a function of y and t, but v is independent of y . The fluid is assumed to be of low conductivity, such that the induced magnetic field is negligible. In order to derive the equations of the problem, we assume that the fluid is finitely conducting and the viscous dissipation the Joule heats are neglected. The polarization effect is also neglected.

At time $t > 0$, the temperature of the plate at $y = h$ changes according to the temperature function: $T = T_0 + (T_w - T_0)(1 - e^{-n't'})$, where T_w and T_0 are the temperature at the plates $y = h$ and at $y = -h$ respectively, and $n(\geq 0)$ is a real number, denoting the decay factor. Hence the flow field is seen to be governed by the following equations:

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) - \left(\frac{\nu}{k} + \frac{\sigma B_0^2}{\rho}\right)u \tag{2.2}$$

$$\frac{\partial T}{\partial t}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{2.3}$$

here ρ is the density of the fluid, B_0 is the magnetic field strength, σ is the electrical conductivity of the fluid, ν is the co-efficient of kinematic viscosity, k is the permeability of the porous medium. K is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, β is the co-efficient of thermal expansion, g is the acceleration due to gravity and T' is the temperature of the fluid.

The initial and boundary conditions for the problem are:

$$t=0: \quad u=0, T=T_0 \quad \text{for all } -h\le y\le h$$

$$t>0: u=0, T=T_0+(T_w-T_0)(1-e^{-nt}) \quad \text{for} \quad y=h$$

$$u=0, \frac{\partial T}{\partial y}=0 \quad \text{for} \quad y=-h \tag{2.4}$$

The non-dimensional variables are

$$\bar{u}=\frac{\nu u}{\beta g h^2(T_w-T_0)}, \bar{y}=\frac{y}{h}, \bar{T}=\frac{T-T_0}{T_w-T_0},$$

$$\bar{t}=\frac{\nu t}{h^2}, P_r=\frac{\mu C_p}{K}, \bar{n}=\frac{h^2 n}{\nu}, M=B_0 h\sqrt{\frac{\sigma}{\mu}}$$

$$Da=\frac{k}{h^2} \tag{2.5}$$

in which Pr is the Prandtl number, Da is the Darcy number and M is the Hartmann number.

Using the non-dimensional variables (2.5) in to the equations (2.2) and (2.3), we obtain

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-N^2 u+T \tag{2.6}$$

$$\frac{\partial T}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \tag{2.7}$$

where

Under the above non-dimensional quantities, the corresponding boundary conditions reduce to

$$t=0: \quad u=0, T=0 \quad \text{for} \quad -1\le y\le 1$$

$$t>0: u=0, T=(1-e^{-nt}) \quad \text{for} \quad y=1$$

$$u=0, \frac{\partial T}{\partial y}=0 \quad \text{for} \quad y=-1 \tag{2.8}$$

3. SOLUTION OF THE PROBLEM

We look for a regular perturbation series solution to solve the Equations (2.6) and (2.6) of the form

$$u = u_o\left(y\right) + e^{-nt}u_1\left(y\right) \tag{3.1}$$

$$T = T_0\left(y\right) + e^{-nt}T_1\left(y\right) \tag{3.2}$$

Substituting Equations (3.1) and (3.2) into the Equations (2.6) – (2.8) and solving the resultant Equations, we obtain

$$u = \frac{1}{N^2}(1 - \frac{\cosh Ny}{\cosh N}) + \left(\begin{array}{l} \left(\dfrac{-1-\cosh 2m_1}{2\left(m_1^2 - m_2^2\right)\cosh m_2 \cosh 2m_1} \right)\cosh m_2 y \\ + \left(\dfrac{-\cosh 2m_1}{2\left(m_1^2 - m_2^2\right)\sinh m_2 \cosh 2m_1} \right)\sinh m_2 y \\ + \left(\dfrac{1}{m_1^2 - m_2^2} \right)\dfrac{\cosh m_1(1+y)}{\cosh 2m_1} \end{array} \right) e^{-nt} \tag{3.3}$$

and

$$T = 1 - \frac{\cosh m_2(1+y)}{\cosh 2m_1} e^{-nt} \tag{3.4}$$

Here $m_1 = i\sqrt{n\Pr}$ and $m_2 = \sqrt{N^2 - n}$.

4. RESULTS AND DISCUSSIONS

In order to see the effect of various physical parameters on the velocity and temperature, we plotted Figs. 1-6.

Fig. 1 depicts the variation of velocity u with Hartmann number M for $\Pr = 0.71$, $Da = 0.01$, $n = 1$ and $t = 1$. It is found that the velocity u decreases with increasing M .

The variation of velocity u with Darcy number Da for $\Pr = 0.71$, $n = 1$ and $t = 1$ is shown in Fig. 2. It is noticed that the velocity u increases with increasing Da .

Fig. 3 illustrates the variation of velocity u with decay parameter n for $\Pr = 0.71$, $Da = 0.01$, $M = 2$ and $t = 1$. It is observed that the velocity u decreases with an increase in n .

The variation of velocity u with Prandtl number $\Pr$ for $n = 1$, $Da = 0.01$, $M = 2$ and $t = 1$ is presented in Fig. 4. It is noted that the velocity u decreases with increasing $\Pr$.

Fig. 5 shows the variation of temperature T with decay parameter n for $\Pr = 0.71$ and $t = 1$. It is found that the temperature T decreases with increasing n .
The variation of temperature T with Prandtl number $\Pr$ for $n = 1$ and $t = 1$ is shown in Fig. 6. It is observed that the temperature T decreases with increasing Prandtl number $\Pr$.

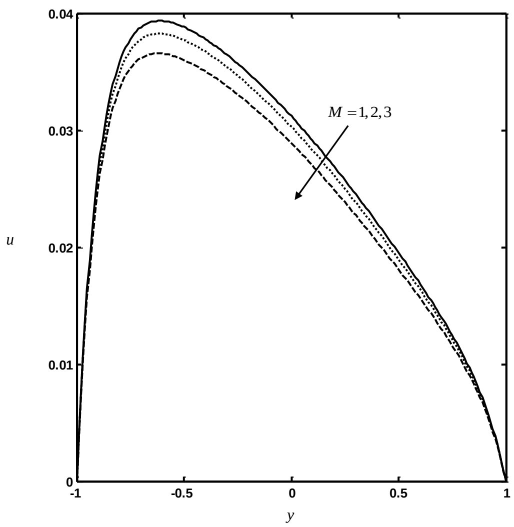

Fig. 1 The variation of velocity u with Hartmann number M for $\Pr = 0.71$, $Da = 0.01$, $n = 1$ and $t = 1$.

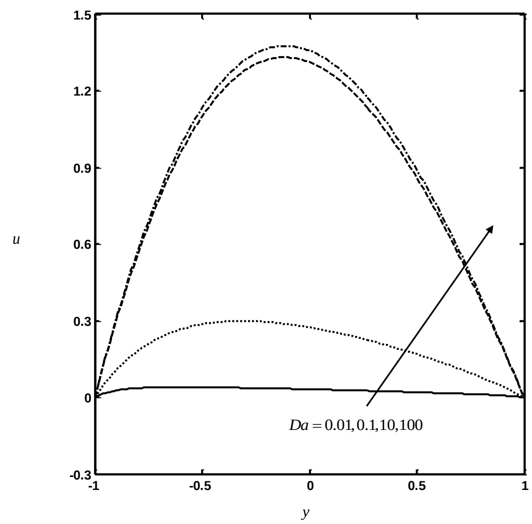

Fig. 2 The variation of velocity u with Darcy number Da for $\Pr = 0.71$, $M = 1$, $n = 1$ and $t = 1$.

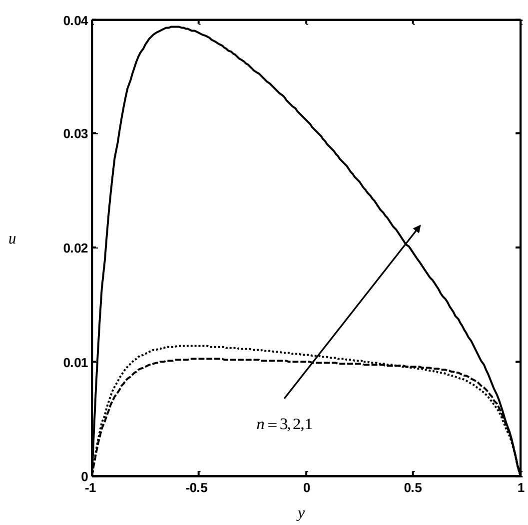

Fig. 3 The variation of velocity u with n for $\Pr = 0.71$, $Da = 0.01$, $M = 1$ and $t = 1$.

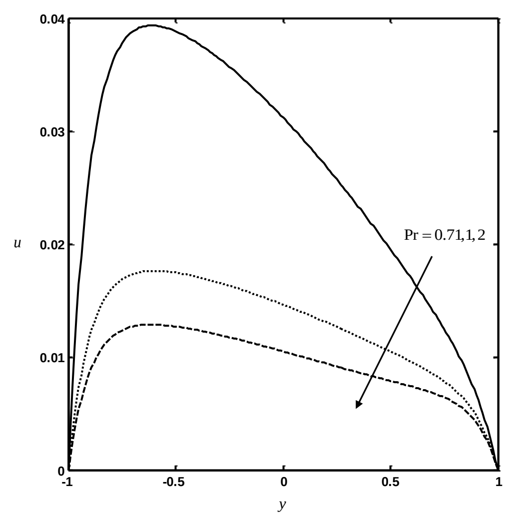

Fig. 4 The variation of velocity u with Prandtl number $\Pr$ for $M = 1$, $Da = 0.01$, $n = 1$ and $t = 1$.

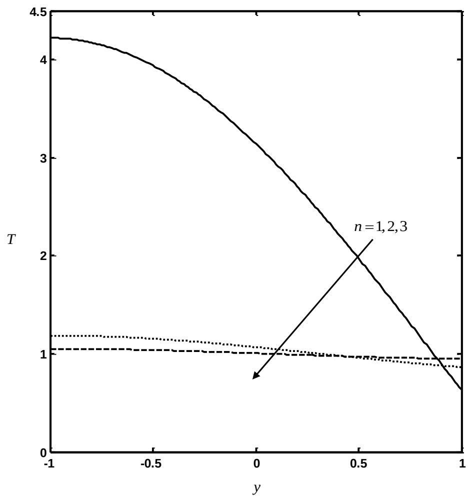

Fig. 5 The variation of temperature T with n for $\Pr = 0.71$ and $t = 1$.

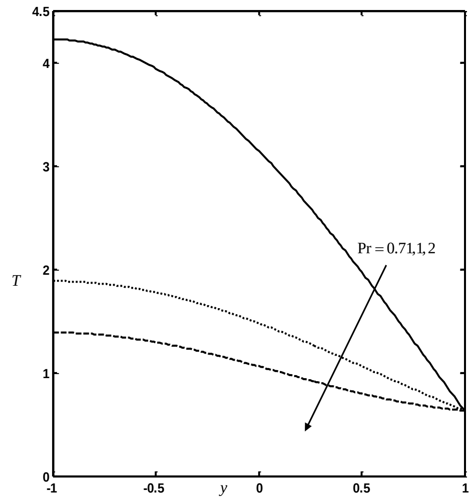

Fig. 6 The variation of temperature T with Prandtl number $\Pr$ for $n = 1$ and $t = 1$.

5. CONCLUSIONS

In this paper, we studied the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate is adiabatic. The expressions for velocity and temperature are obtained by using regular perturbation technique. It is found that the velocity increases with increasing Darcy number Da , while it decreases with increasing $M, \Pr$ and n and the temperature decreases with increasing $\Pr$ and n .

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