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Contra Harmonic Mean Labeling On H-Graphs

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ABSTRACT

Mean labeling of graphs was introduced by S.Somasundram and R.Ponraj[8]. S.Somasundram and S.S. Sandhya introduced Harmonic mean labeling of graphs[10]. Similar to their concept Contra Harmonic mean labeling of graphs are introduced[6]. This paper deals with the Contra Harmonic mean labeling on H-graphs. Super Contra Harmonic mean labeling on H-graphs are also discussed.

Keywords: Contra Harmonic mean graph, Super Contra Harmonic mean graph, H-graph.

1. INTRODUCTION

A graph is a finite set V = V(G), called the vertices of G together with a set E = E(G) of unordered pairs of vertices of G, called the edges . A labeling or numbering of a simple graph is a one-to-one function from its vertex set into a set of non –negative integers which induces an assignment of labels to the edges of G. Labeled graphs serve as useful mathematical models for a broad range of applications such as Coding theory, Electrical network analysis, Circuit design etc. We have referred to Gallian J. A (2016)[2] for detailed survey of graph labeling. For standard terminology and notation we refer to Bondy and Murthy[1]. Also we have referred to Harary[3] . Some basic concepts of mean and super mean labelings are taken from[4,5,8-10].

Definition: 1.1

A graph G = (V,E) with p vertices and q edges is called a Contra Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1,…,q in such a way that when each edge e = uv is labeled with f(uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels, where f is a Contra Harmonic mean labeling of G.

Definition 1.2 An injective function f: V(G)→{1,2,…,p+q} is called a Super Contra Harmonic mean labeling if the induced edge labeling f(e = uv) is defined by f(e) = $\left\lceil \frac{(f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{(f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$, so that{f(V(G))}∪{f(e)/e∈E(G)} ={1,2,…,p+q}. A graph which allows Super Contra Harmonic mean labeling is called Super Contra Harmonic mean graph.

Definition 1.3

The H- graph of a path P_n is the graph obtained from two copies of P_n with vertices $u_1,u_2,\ldots,u_n$ and $v_1,v_2,\ldots,v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$,if n is odd and the vertices $u_{\frac{n}{2}+1}$and $v_{\frac{n}{2}}$, if n is even.

2. MAIN RESULTS

Theorem :2.1 H-graphs are Contra Harmonic mean graphs, for n≥ 3 if n is odd and n≥4 if n is even.

Proof: Let G be a H-graph with vertices $u_1,u_2,\ldots,u_n$ and $v_1,v_2,\ldots,v_n$.

Define f: V(G)→{0,1,2,...,q},assign labels to the vertices as

$$f(u_i) = \begin{cases} i, & 1 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ & 1 \le i \le \frac{n}{2}, \text{ if } n \text{ is even} \\ 0, & i = \frac{n+1}{2} \text{ if } n \text{ is odd} \\ & i = \frac{n}{2}+1, \text{ if } n \text{ is even} \\ i-1, & \frac{n+3}{2} \le i \le n, \text{ if } n \text{ is odd} \\ & \frac{n}{2}+2 \le i \le n, \text{ if } n \text{ even} \end{cases}$$

$f(v_i) = n+i-1,\ 1 \le i \le n.$

Then by the definition 1.1 the edges get distinct labels. Hence H_n is a Contra Harmonic mean graph.

Illustration 2.2 The labeling pattern for H_7 and H_{10} are shown in the following figure.

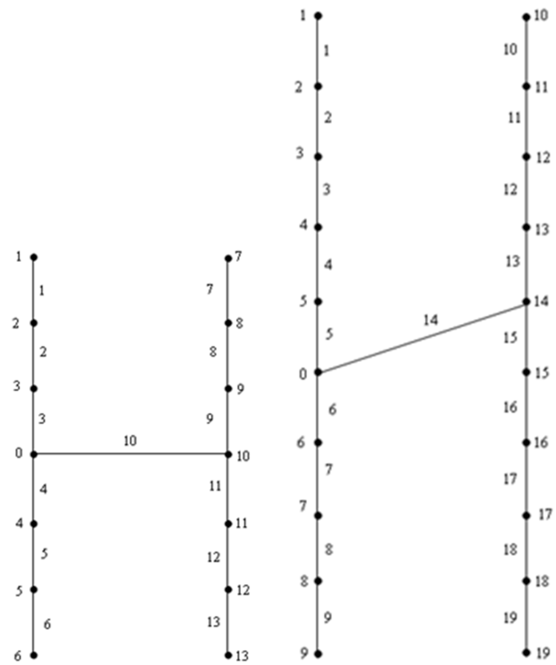

Figure.1

Theorem : 2.3 The graph $H_n \odot K_1$ is a Contra Harmonic mean graph for $n \geq 5$, if n is odd and $n \geq 4$, if n is even.

Proof: Let $G = H_n \odot K_1$ where H_n is a H-graph with vertices $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$. Let x_i, y_i be the pendant vertices at u_i and v_i respectively for $1 \leq i \leq n$.

Define f: $V(G) \rightarrow \{0,1,2,...,q\}$ so that the vertices are labeled as

$$f(x_i) = \begin{cases} 1, i = 1 \\ 2i, \begin{array}{l} 2 \leq i \leq \frac{n-1}{2}, if\ n\ is\ odd \\ 2 \leq i \leq \frac{n}{2}, if\ n\ is\ even \end{array} \\ n \quad, i = \frac{n+1}{2}, if\ n\ is\ odd \\ n+1, i = \frac{n}{2}+1, if\ n\ is\ even \\ 2i-2, \begin{array}{l} \frac{n+3}{2} \leq i \leq n, if\ n\ is\ odd \\ \frac{n}{2}+2 \leq i \leq n, if\ n\ is\ even \end{array} \end{cases}$$

$$f(u_i) = \begin{cases} 2, & i = 1 \\ 2i-1, & 2 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ & 2 \le i \le \frac{n}{2}, \text{ if } n \text{ is even} \\ 0, & i = \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ & i = \frac{n}{2}+1, \text{ if } n \text{ is even} \\ 2i-1, & \frac{n+3}{2} \le i \le n, \text{ if } n \text{ is odd} \\ & \frac{n}{2}+2 \le i \le n, \text{ if } n \text{ is even} \end{cases}$$

$f(v_i) = 2n+2i-2,\ 1 \le i \le n$, $f(y_i) = 2n+(2i-1),\ 1 \le i \le n$

Obviously, the edges are distinct. The labeling pattern for $H_n \odot K_1$ is illustrated in figure 2. Hence G is a Contra Harmonic mean graph.

llustration 2.4
The Contra Harmonic mean labeling of $H_9 \odot K_{1,2}$ and $H_8 \odot K_{1,2}$ are given below.

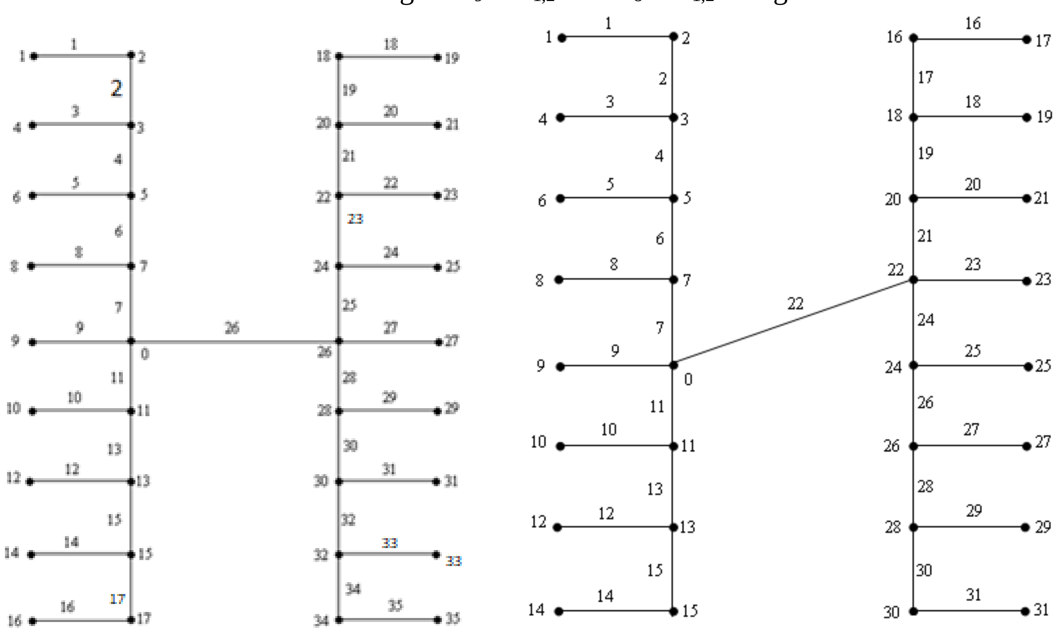

Figure 2

Theorem :2.5 The graph $H_n \odot K_{1,2}$ is a Contra Harmonic mean graph.
Proof: Let G = $H_n \odot K_{1,2}$ where H_n is a H-graph with vertices $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$. For $1 \le i \le n$, let t_i, s_i be the vertices of $K_{1,2}$ attached at u_i, and x_i, y_i be the vertices of $K_{1,2}$ joined at v_i.

Define f:V(G))→{0,1,2,...,q}so that the vertices get labels as follows:

$$f(u_i) = \begin{cases} 2, & i = 1 \\ 3i-2, & 2 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ & 2 \le i \le \frac{n}{2}, \text{ if } n \text{ is even} \\ 0 & i = \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ & i = \frac{n+2}{2}, \text{ if } n \text{ is even} \\ 3i-2, & \frac{n+3}{2} \le i \le n, \text{ if } n \text{ is odd} \\ & \frac{n}{2}+2 \le i \le n, \text{ if } n \text{ is even} \end{cases}$$

$$f(t_i) = \begin{cases} 1, & i = 1 \\ 3i-1, & 2 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ & 2 \le i \le \frac{n}{2}, \text{ if } n \text{ is even} \\ \frac{3n-1}{2}, & i = \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{3n}{2}+1, & i = \frac{n}{2}+1, \text{ if } n \text{ is even} \\ \frac{3n+3}{2}, & i = \frac{n+3}{2}, \text{ if } n \text{ is odd} \\ 3i-3, & \frac{n+5}{2} \le i \le n, \text{ if } n \text{ is odd} \\ & \frac{n}{2}+2 \le i \le n \text{ if } n \text{ is even} \end{cases}$$

$$f(s_i) = \begin{cases} 3i, & 1 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ & 1 \le i \le \frac{n}{2}, \text{ if } n \text{ is even} \\ \frac{3n+1}{2} & , i = \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ 3i-1, & \frac{n+1}{2} \le i \le n, \text{ if } n \text{ is odd} \\ & \frac{n}{2}+1 \le i \le n, \text{ if } n \text{ is even} \end{cases}$$

$f(v_i) = 3n+3i-2, \ 1 \le i \le n$

$f(x_i) = 3n+3i-3, \ 1 \le i \le n$

$f(y_i) = 3n+3i-1, \ 1 \le i \le n$.

The above vertex labeling gives a Contra Harmonic mean labeling to G.

Hence $H_n \odot K_{1,2}$ is a Contra Harmonic mean graph. The labeling pattern is shown in figure 3.20.

Illustration 2.6

The Contra Harmonic mean labeling of $H_{11} \odot K_{1,2}$ and $H_{10} \odot K_{1,2}$ are shown below.

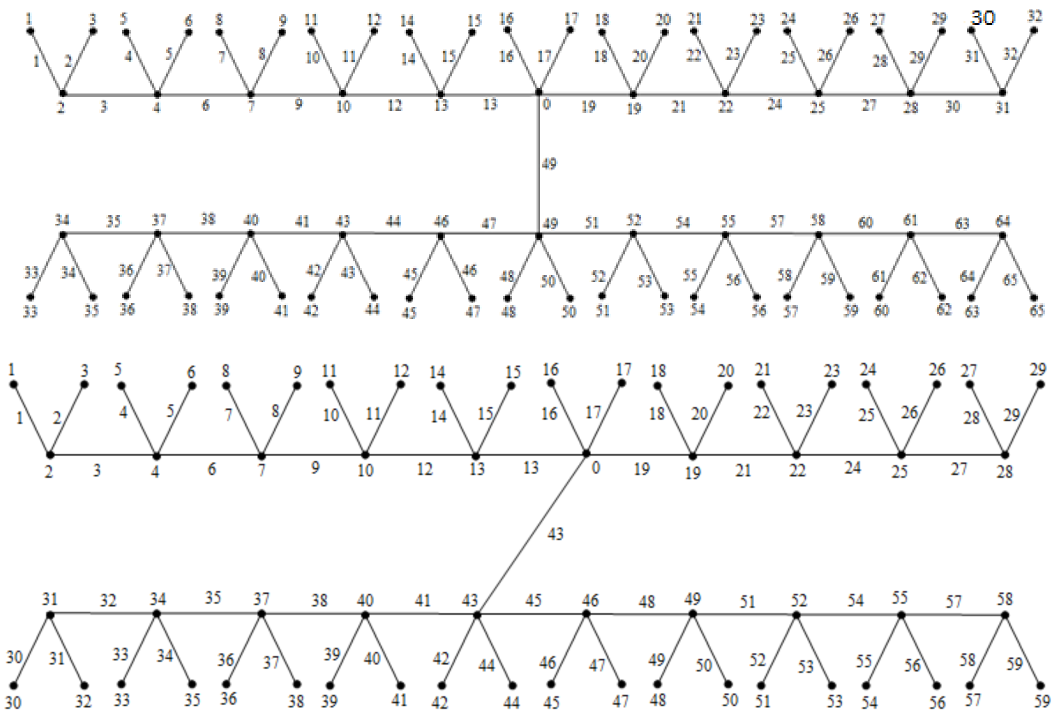

Figure 3

Theorem : 2.7 H-graphs are Super Contra Harmonic mean graphs, for n≥ 3 if n is odd and n≥4 if n is even.

Proof: Let G be the given graph with vertices $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$.

Define f: V(G)→{1,2,...,p+q}so that the vertices are labeled as follows

$$f(u_i) = \begin{cases} 2i, & 1 \le i \le \frac{n+1}{2} - 2 \text{ if } n \text{ is odd} \\ & 1 \le i \le \frac{n}{2} - 1, \text{ if } n \text{ is even} \\ n, & i = \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ & i = \frac{n}{2} \text{ if } n \text{ is even} \\ 1, & i = \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ & i = \frac{n}{2} + 1, \text{ if } n \text{ is even} \\ 2i - 1, & \frac{n+1}{2} + 1 \le i \le n, \text{ if } n \text{ is odd} \\ & \frac{n}{2} + 1 \le i \le n, \text{ if } n \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 2n + 2i - 2, & 1 \leq i \leq \frac{n-1}{2}, \ if\ n\ is\ odd \\ & \frac{n}{2} \leq i \leq n, \ if\ n\ is\ even \\ 2n + 2i - 1, & \frac{n+1}{2} \leq i \leq n, \ if\ n\ is\ odd \\ & \frac{n}{2} \leq i \leq n, \ if\ n\ is\ even \end{cases}$$

Then the edge labels are distinct and satisfies the condition $\{f(V(G))\} \cup \{f(E(G))\} = \{1,2,\ldots,p+q\}$, hence G is a Super Contra Harmonic mean graph.

Illustration 2.8 The labeling pattern of H_9 and H_6 is shown in the following figure.

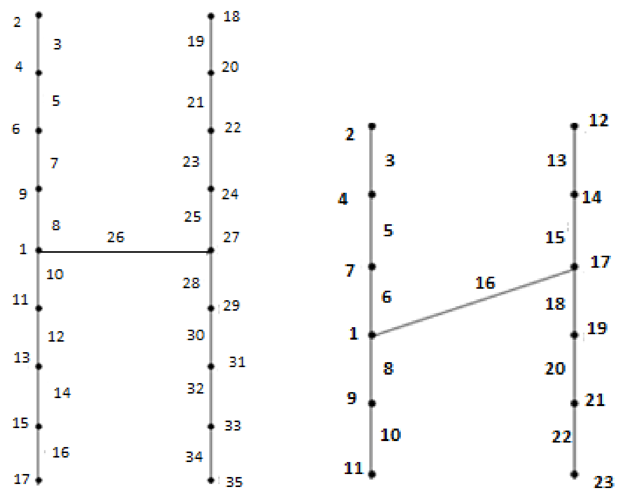

Figure. 4

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