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Unsteady Micropolar Boundary Layer Three Dimensional Fluid Flow and Heat Transfer Over a Stretching Flat Sheet with the Effects of Variable Viscosity and Thermal Conductivity

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ABSTRACT

Effects of variable viscosity and thermal conductivity for unsteady boundary layer micropolar three dimensional fluid flow with heat transfer over a continuous stretching sheet is consider for numerical solution and analysis. Nonlinear partial differential equations representing the flow changed to ordinary differential equations with the help of similarity transformation. A very efficient fourth order Runge Kutta method is used to solve the non linear differential equations .The important feature of the boundary layer micropolar fluid flow are presented graphically also the influence two most important physical quantities of interest Skin friction and Nusselts number have been analyzed through tabular forms.

Keywords: Unsteady flow, Micropolar fluid, Boundary layer, Skin friction.

INTRODUCTION

The thin layer near the solid surface is known as boundary layer, which play a vital role in analyzing the complex behavior of fluid which was introduce by Ludwig Prandtl. Recently the boundary layer fluid problem due to stretching sheet has attracted the interest of researchers due to its applications in manufacturing process such as paper production, polymer sheet synthesis, continuous stretching of plastic films, cable coating ,artificial fibers etc. Crane[11] was one of them who studied two dimensional boundary layer fluid flow due to stretching of a flat sheet. Peddiesen and Mc Nitt[12], Gorla[16] consider the boundary layer micro-polar fluid flow near the two dimensional stagnation point in steady state. Eringen[1,2] was known as the first introducer of Micro-polar fluid which can't be explained by classical hydrodynamics. Micro-polar fluid have two new variables of velocity one is micro-rotations variables which represent spin and another is micro-inertia which describe the distribution of atoms and molecule inside the fluid. Eringen[1,2], Lukaszewicz[9] has extended the theory and application of micro-polar fluid. The body fluid and biological fluid flow are some of the example of micro-polar fluid model. Many more research work has been done on stretching sheet considering different environment. Steady state three dimensional fluid flow was studied by Wang[6], Chamkha *et al.*[4] also Borthakur *et al.*[15] three dimensional micropolar fluid due to a rotating fluid. Three dimensional unsteady flow analyzed by Lakshmisha *et al.*[14], Hayat *et al.*[17].

Physical properties may change with temperature. Variation of viscosity and temperature has been taken for consideration for accurately analyzed the fluid flow. The aim of this paper also to consider the effect of variable viscosity and thermal conductivity on three dimensional unsteady micropolar fluid flow over stretching sheet with heat transfer in absence of magnetic field.

MATHEMATICAL FORMULATION

Considering three dimensional viscous incompressible unsteady micropolar flow with heat transfer over a plane sheet located at z=0, due to the sheet is stretched in two lateral direction x and y with the effects of time dependent viscosity and thermal conductivity. The velocities in x and y direction as consider as $U_w = \frac{ax}{1-ct}$, $V_w = \frac{bx}{1-ct}$ respectively where a , b are positive constant. Due to the stretching, thermal boundary layer exist hence thermal boundary layer equations with viscous dissipation term also consider here.

Basic equations governing the flow are:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

Equation of Linear momentum:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)+\kappa\left(\frac{\partial^2 u}{\partial z^2}-\frac{\partial N_2}{\partial z}\right) \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\frac{\partial}{\partial x}\left(\mu\frac{\partial w}{\partial z}\right)+\kappa\left(\frac{\partial^2 v}{\partial z^2}+\frac{\partial N_1}{\partial z}\right) \tag{3}$$

Equation of Angular momentum:

$$\rho j\left(\frac{\partial N_1}{\partial t}+u\frac{\partial N_1}{\partial x}+w\frac{\partial N_1}{\partial z}\right)=\gamma\frac{\partial^2 N_1}{\partial z^2}-\kappa\left(2N_1+\frac{\partial v}{\partial z}\right) \tag{4}$$

$$\rho j\left(\frac{\partial N_2}{\partial t}+u\frac{\partial N_2}{\partial x}+w\frac{\partial N_2}{\partial z}\right)=\gamma\frac{\partial^2 N_2}{\partial z^2}-\kappa\left(2N_2-\frac{\partial u}{\partial z}\right) \quad (5)$$

Energy Equation:

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\left(\mu+\kappa\right)\left(\left(\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right) \quad (6)$$

Where u,v and w be the velocity ,(N_1,N_2,0) are microrotation components along x,y and z respectively ,j micro-inertia per unit mass, μ-dynamic viscosity, κ-vortex viscosity, λ the thermal conductivity, γ Spin gradient viscosity consider as by Rees and Pop[7]:T temperature and C_p is the specific heat.

$$\gamma=\left(\mu+\frac{\kappa}{2}\right)j=\mu\left(1+\frac{1}{2}\Delta\right)j \quad \text{where} \quad \Delta=\frac{\kappa}{\mu} \quad (7)$$

As discussed by Ahmadi[11] in limiting case when micro-rotation reduces to the angular velocity the above relation (7) invoked to allow eqs. (1)-(6) to predict the correct behavior of the fluid. With boundary conditions:

$$\left.\begin{aligned}&u=\frac{ax}{1-ct}=U_w\ ,v=\frac{by}{1-ct}=V_w\ ,w=0\ ,\ N_1=-n\frac{\partial u}{\partial y}\ ,\ N_2=-n\frac{\partial v}{\partial y}\ \ and\ T=T_w\ at\ z=0\\&u=0\ \ ,v=0,\ w=0\ \ ,N_1\to 0\ \ ,\ N_2\to 0\ \ and\ T=T_\infty\ at\ z\to\infty\end{aligned}\right] \quad (8)$$

In this case fluid viscosity and thermal conductivity consider as inverse linear function of temperature as according as Lai and Kularki[8].

$$\left.\begin{aligned}&\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\alpha(T-T_\infty)\right]=b^*(T-T_c)\\&where\quad b^*=\frac{\alpha}{\mu_\infty}\quad ,T_c=T_\infty-\frac{1}{\alpha}\end{aligned}\right)\ \text{and}\ \left.\begin{aligned}&\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\varepsilon(T-T_\infty)\right]=c^*(T-T_r)\\&where\quad c^*=\frac{\varepsilon}{\lambda_\infty}\quad ,T_r=T_\infty-\frac{1}{\varepsilon}\end{aligned}\right) \quad (9)$$

Using similarity variables in equations (1) to (6)

$$\left.\begin{aligned}&u=\frac{ax}{1-ct}\ f'(\eta)\ ,v=\frac{by}{1-ct}\ g'(\eta)\ ,w=-\sqrt{\frac{a\nu}{1-ct}}\left\{f(\eta)+g(\eta)\right\},\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\\&\eta=\sqrt{\frac{a}{\nu(1-ct)}}z\ ,\ N_1=\left(\frac{a}{1-ct}\right)^{\frac{3}{2}}\frac{y}{\sqrt{\nu}}G_1(\eta)\ ,\ N_2=\left(\frac{a}{1-ct}\right)^{\frac{3}{2}}\frac{x}{\sqrt{\nu}}G_2(\eta)\end{aligned}\right] \quad (10)$$

The equations (1) automatically satisfied by the similarity transformations the other equations (2)-(6) using similarity variables reduced to ordinary differential equations(11)-(15) as follows:

$$\left(1+\Delta\right)f'''(\eta)+\left(f+g-\frac{\theta'}{\theta-\theta_c}\right)f''-\frac{1}{2}A\eta f''-(A+f')f'-\Delta G_2'(\eta)=0 \quad (11)$$

$$(1+\Delta)g'''(\eta)+\left(f+g-\frac{\theta'}{\theta-\theta_c}-\frac{1}{2}A\eta\right)g''-(A+g')g'-\Delta G_1'(\eta)=0 \tag{12}$$

$$(1+\Delta/2)G_1''-\frac{1}{2}A\eta G_1'+\left(f+g-2\Delta B-\frac{3}{2}A\right)G_1-\Delta Bg''=0 \tag{13}$$

$$(1+\Delta/2)G_2''+\left(f+g-\frac{1}{2}A\eta\right)G_2'-(f'+2\Delta B)G_2+\Delta Bf''=0 \tag{14}$$

$$\theta''+\frac{1}{\theta-\theta_r}\theta'^2+(f+g-\frac{1}{2}AP_r\eta)\theta'+(1+\Delta)P_rE_c\left\{(g'')^2+(f'')^2\right\}=0 \tag{15}$$

Boundary conditions become:

$$\left.\begin{array}{l} f'(\eta)=1 \ , g'(\eta)=\frac{b}{a}=\beta,\ -1<\beta\le 1,\ f(\eta)=0\ , g(\eta)=0, \\ G_1(\eta)=-nf''\ , G_2(\eta)=-nf''\ , \theta(\eta)=1\ as\ \eta\to 0 \\ f'(\eta)=0\ , g'(\eta)=0\ , \theta(\eta)=0, G_1(\eta)=0\ , G_2(\eta)=0\ as\ \eta\to\infty \end{array}\right\} \tag{16}$$

Where β stand for stretching rate parameter whose value lie between $-1\le\beta\le 1$. The problem becomes axisymmetric for β =1 and it becomes two dimensional flow when β=0. Similarly when β takes the negative value the forces in x and y direction are opposite to each other [ref. Chamkha[4]]. Also η the similarity variable and the prime denoted the differentiation with respect to η.

$\Delta=\frac{\kappa}{\mu}$ Micropolar parameter, $P_r=\frac{\mu c_p}{\lambda}$ Prandtls number, $A=\frac{a}{c}$ unsteady parameter and $B=\frac{\nu(1-ct)}{jb}$ non dimensional parameter [ref. Aziz[]] $Ec=\frac{U_w^2}{C_p(T_w-T_\infty)}$ Eckert No.

Important physical quantities skin friction along x ,y axes in terms of wall shear stresses and Nusselt's number are given by:

$$C_{fx}=\frac{\tau_{zx}}{\rho U_w^2}\ , \tau_{zx}=\left\{(\mu+\kappa)\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+\frac{\kappa}{\rho}N_2\right\} \tag{17}$$

$$C_{fy}=\frac{\tau_{yz}}{\rho V_w^2}\ ,\ \tau_{yz}=\left\{(\mu+\kappa)\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)-\frac{\kappa}{\rho}N_1\right\}_{z=0} \tag{18}$$

And $$N_u=\frac{xq_w}{\lambda_\infty(T_w-T_\infty)}\ ,\ \ q_w=-k\left(\frac{\partial T}{\partial z}\right)_{z=0} \tag{19}$$

After simplification it becomes:

$$\left.\begin{aligned} C_{fx}R_{ex}^{1/2} &= \left(\Delta - \frac{\theta_c}{\theta-\theta_c}\right) f^{//}(\eta) \\ C_{fy}R_{ey}^{1/2} &= \left(\Delta - \frac{\theta_c}{\theta-\theta_c}\right) g^{//}(\eta) \end{aligned}\right]_{\eta=0} \tag{20}$$

$$N_u R_{ex}^{1/2}(\theta-\theta_r)+\theta\theta^{/}=0\,,\, as\ \ \eta=0 \tag{21}$$

RESULTS AND DISCUSSIONS

The equations (11)-(15) along with the boundary conditions (16) are solved with the help of Runge Kutta method with shooting technique constructing a Mat-lab program. For solving these initially we consider
$P_r = 0.72$, $E_c = 0.05$, $\theta_c = 2$, $\theta_r = 3$, $\Delta = 1$, $B = 0.1$, $A = 0.25$, $N = 0.5$ and $\beta = 0.25$ as the value of the parameters. The physical interpretation of the parameter involved in the equations presented graphically in the fig:2-17.

The velocity distribution for various values of different parameters are presented by the fig: 1-6. In all these cases it is seen that the velocity components $g^{/}$ increases for unsteady parameter, micro polar parameter and due to increasing value of stretching rate. The other velocity components $f^{/}$ decreases for both the increasing value of stretching rate and micro polar parameters but it becomes opposite for unsteady parameter.

The temperature profile of the problems for different values of the parameter are presents in fig: 7-12. From fig: 9-12 temperature profile increases for Prandt's , Eckert number increases. Due to increasing value of thermal conductivity and variable viscosity temperature profile increases but it is not so significant changes occur in those cases. It can be say that small changes in temperature take place for different values of the parameters thermal conductivity and variable viscosity.

The micro polar distribution profile presented in fig :13-14. A significant change occurs with the increasing value of the micro polar and unsteady parameters. For both the cases changes are opposite for unsteady parameter temperature increases but it becomes decreases for micro polar parameter.

The effects of different parameters Δ , β , P_r , θ_r on surface characteristic i.e the skin friction along x and y axis Cf_x , Cf_y , rate of heat transfer i.e. Nusselt's number N_u and the missing values $f^{//}(0)$, $g^{//}(0)$, $\theta^{/}(0)$, $G_1^{/}(0)$, $G_2^{/}(0)$ are presented in table:1-4. Tables are self sufficient for analyzing the behavior of these parameters so a brief analysis of the tables given here. It is seen that the two missing values $G_1^{/}(0)$ and $G_2^{/}(0)$ are remain same with the changing values of the parameters in all those tables. In table:1 and 2 due to change of Δ ,B and β the values of $f^{//}(0), g^{//}(0), Cf_x, Cf_y$ are decreasing in 1st table, N_u slightly increasing and in table:2 it is decreasing . It is observed that the value of the Nusselt's number increases due to increasing value of the thermal conductivity and micro polar parameter. [table:6] .The skin friction coefficient for both the axes are different and of opposite to each other with the variation of P_r and θ_r illustrate by table:3 on the other hand in table:4 both values and the Nusselt's number decreases due to increasing value of θ_r and micro polar parameter. The different missing values are presented for various parameter in all those tables.

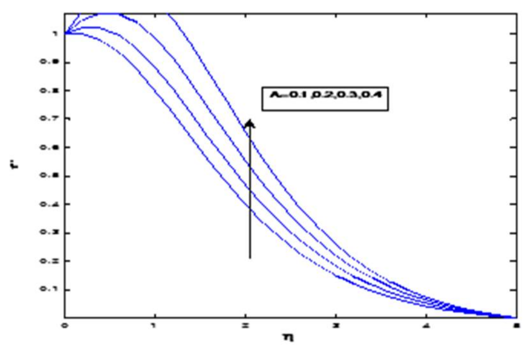

fig:1

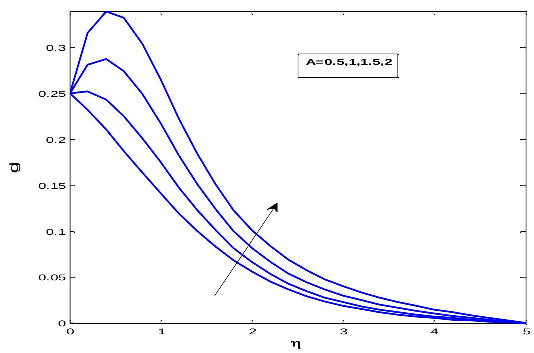

fig:2

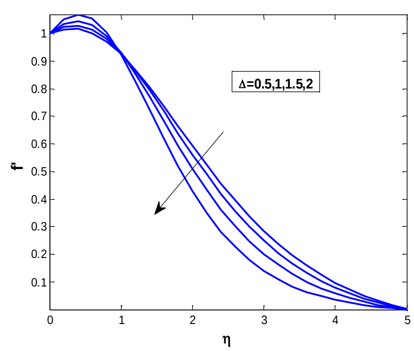

fig:3

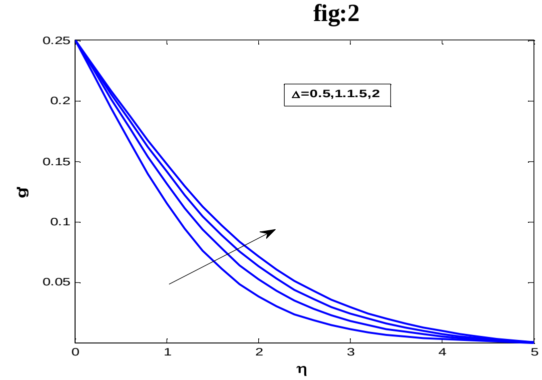

fig:4

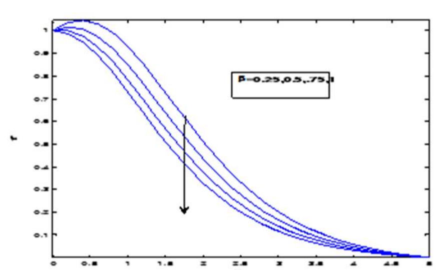

fig:5

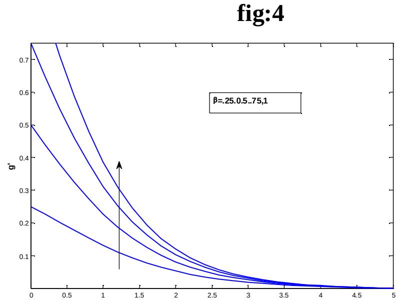

fig:6

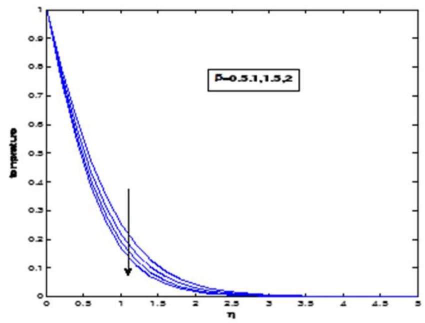

fig:7

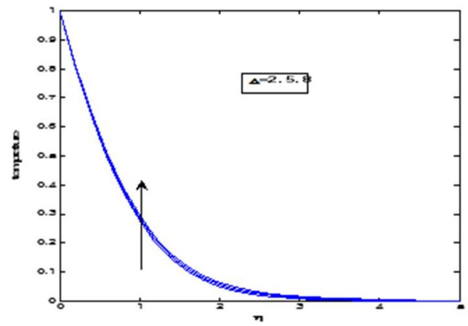

fig:8

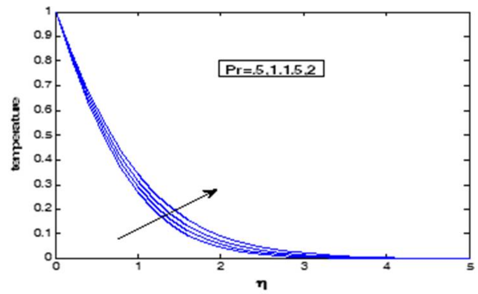

fig:9

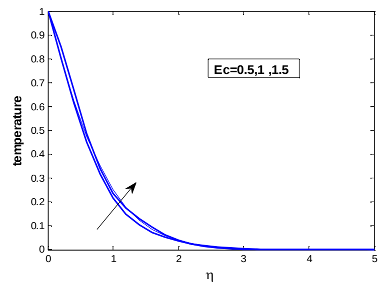

fig:10

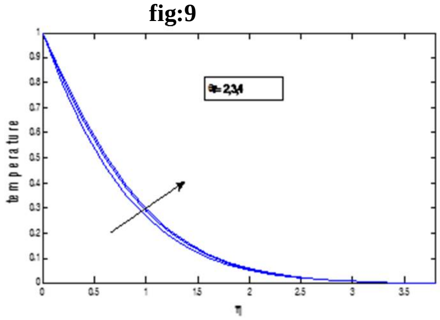

fig:11

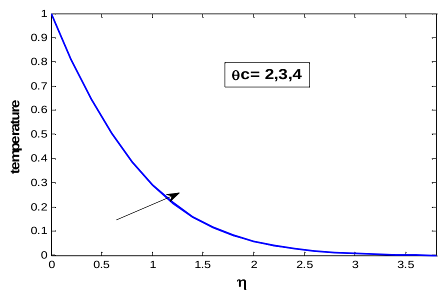

fig:12

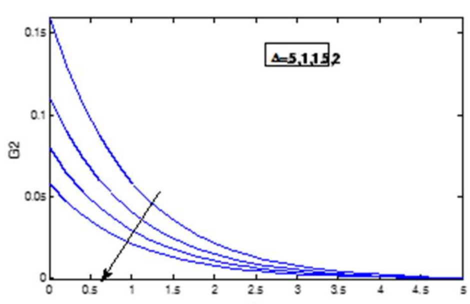

fig:13

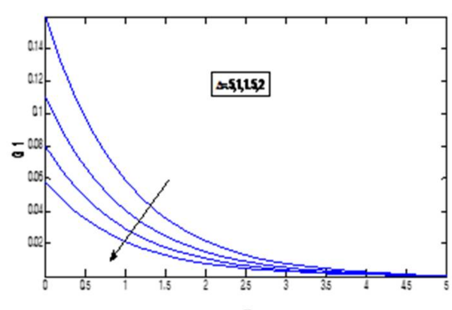

fig:14

Table:1

Δ	B	f''(0)	g''(0)	θ′(0)	$G_1{}'$	$G_2{}'(0)$	Cf_x	Cf_y	N_u
2	0.2	0.033143	-0.11789	-0.98644	-0.01659	-0.01659	0.132574	-0.47154	-2.98644
	0.4	0.033143	-0.11789	-0.98644	-0.01659	-0.01659	0.132574	-0.47154	-2.98644
3	0.2	-0.02379	-0.11842	-0.97677	0.011906	0.011906	-0.11894	-0.59211	-2.97677
	0.4	-0.02379	-0.11842	-0.97677	0.011906	0.011906	-0.11894	-0.59211	-2.97677
4	0.2	-0.05794	-0.12003	-0.96812	0.029	0.029	-0.34765	-0.7202	-2.96812
	0.4	-0.05794	-0.12003	-0.96812	0.029	0.029	-0.34765	-0.7202	-2.96812

Table:2

Δ	β	f''(0)	g''(0)	θ′(0)	$G_1{}'(0)$	$G_2{}'(0$	Cf_x	Cf_y	Nu
2	0.25	0.033143	-0.11789	-0.98644	-0.01659	-0.01659	0.132574	-0.47154	-2.98644
	0.5	-0.00822	-0.28034	-1.06145	0.004113	0.004113	-0.03287	-1.12136	-3.06145
	0.75	-0.04504	-0.46733	-1.1264	0.022541	0.022541	-0.18014	-1.86933	-3.1264
	1	-0.07837	-0.67671	-1.18127	0.039226	0.039226	-0.3135	-2.70682	-3.18127
3	0.25	-0.02379	-0.11842	-0.97677	0.011906	0.011906	-0.11894	-0.59211	-2.97677
	0.5	-0.05264	-0.26306	-1.05712	0.026344	0.026344	-0.26317	-1.31528	-3.05712
	0.75	-0.07905	-0.42927	-1.12532	0.039564	0.039564	-0.39525	-2.14635	-3.12532
	1	-0.10352	-0.61515	-1.1817	0.05181	0.05181	-0.51759	-3.07577	-3.1817
4	0.25	-0.05794	-0.12003	-0.96812	0.029	0.029	-0.34765	-0.7202	-2.96812
	0.5	-0.07987	-0.2523	-1.05163	0.039975	0.039975	-0.47921	-1.51377	-3.05163
	0.75	-0.10031	-0.40386	-1.1215	0.050206	0.050206	-0.60187	-2.42314	-3.1215
	1	-0.11953	-0.57306	-1.17825	0.059824	0.059824	-0.71717	-3.43836	-3.17825

Table:3

θ_r	Pr	f''(0)	g''(0)	θ′(0)	$G1{}'(0)$	$G2{}'(0)$	Cf_x	Cf_y	N_u
2	0.2	0.140428	-0.12304	-1.25254	-0.07028	-0.07028	0.421283	-0.36912	-2.25254
	0.4	0.141089	-0.12317	-1.23036	-0.07062	-0.07062	0.423266	-0.36951	-2.23036
	0.6	0.141764	-0.12331	-1.20782	-0.07095	-0.07095	0.425292	-0.36993	-2.20782
	0.8	0.142454	-0.12346	-1.18489	-0.0713	-0.0713	0.427361	-0.37037	-2.18489
3	0.2	0.14423	-0.12341	-1.0448	-0.07219	-0.07219	0.432691	-0.37024	-3.0448
	0.4	0.144896	-0.12355	-1.02674	-0.07252	-0.07252	0.434689	-0.37065	-3.02674
	0.6	0.145576	-0.12369	-1.00838	-0.07286	-0.07286	0.436727	-0.37107	-3.00838
	0.8	0.146268	-0.12384	-0.9897	-0.07321	-0.07321	0.438805	-0.37152	-2.9897
4	0.2	0.145853	-0.12357	-0.97563	-0.073	-0.073	0.437558	-0.37072	-3.97563
	0.4	0.146519	-0.12371	-0.95894	-0.07333	-0.07333	0.439556	-0.37113	-3.95894
	0.6	0.147197	-0.12385	-0.94195	-0.07367	-0.07367	0.441591	-0.37156	-3.94195
	0.8	0.147888	-0.12401	-0.92467	-0.07402	-0.07402	0.443664	-0.37201	-3.92467

Table:4

θr	Δ	f''(0)	g''(0)	θ′(0)	$G_2{}'(0)$	$G_2{}'(0)$	Cf_x	Cf_y	N_u
	0.5	0.251088	-0.13635	-1.20023	-0.12567	-0.12567	0.627719	-0.34087	-2.20023
	1	0.142176	-0.1234	-1.19411	-0.07116	-0.07116	0.426528	-0.37019	-2.19411
	1.5	0.076156	-0.11895	-1.18755	-0.03812	-0.03812	0.266547	-0.41633	-2.18755
2	2	0.031602	-0.11766	-1.18112	-0.01582	-0.01582	0.126407	-0.47064	-2.18112
	0.5	0.258379	-0.13716	-1.0025	-0.12932	-0.12932	0.645947	-0.3429	-3.0025
	1	0.14599	-0.12378	-0.99721	-0.07307	-0.07307	0.437969	-0.37134	-2.99721
	1.5	0.078474	-0.11922	-0.99173	-0.03928	-0.03928	0.274659	-0.41726	-2.99173
3	2	0.033143	-0.11789	-0.98644	-0.01659	-0.01659	0.132574	-0.47154	-2.98644
	0.5	0.261489	-0.13751	-0.93665	-0.13088	-0.13088	0.653722	-0.34377	-3.93665
4	1	0.14761	-0.12394	-0.93162	-0.07388	-0.07388	0.44283	-0.37183	-3.93162
	1.5	0.079454	-0.11933	-0.92649	-0.03977	-0.03977	0.27809	-0.41766	-3.92649
	2	0.033792	-0.11798	-0.92157	-0.01691	-0.01691	0.135169	-0.47194	-3.92157

CONCLUSION

A numerical study of the micropolar fluid flow and heat transfer is presented to describe the effects of variable viscosity and thermal conductivity.The following conclusion have been made

1. Velocity boundary layer thickness increases by increasing value of the unsteady parameter A.
2. Due to increasing value of the micropolar fluid parameter velocity profile along x-axis decreases but it increases in y-direction.
3. Temperature decreases with the increasing value of the streatching parameter.
4. Increasing value of both the parameter variable viscosity and thermal conductivity temperature slightly increases.
5. Thermal boundary layer increses due to increasing value of the Prandtl number and Eckert number .
6. Micropolar profile significantly increases due to increasing value of unsteady paramter and micropolar paramter.
7. Both the missing values of $G_1(0)$, $G_2(0)$ are same for various parameter values.
8. Due to increasing value of the variable viscosity and micropolar paramter skin friction along x-axis decreases significantly also Nusselt’s number decreases significantly with the variation of thermal conductivity and Prandtls number.

REFERENCES

1. A. C Eringen, Theory of simple microfluids, *Intl, J. Sci.*, pp.205-217, (1964).
2. A. C. Eringen, Theory of micropolar fluids, *J. Math. Mech.* 16, pp.1-18, (1966).
3. A.R.Aurangzaib, M Kasim, N. F.Mohammad and S. Sharidan, Unsteady MHD mixed convection flow with heat and mass transfer over a vertical plate in a micropolar fluid-saturated porous medium.’’ *J. App. Sci. & Engg*. Vol.16, No.2, pp.141-150 (2013).

4. A.J.Chamkha, H.A.Jaradat & I.Pop. “Three dimensional micropolar flow due to a stretching flat surface.” *I.J F. Mech. Rsearch*, vol.30,No.4 (2013).
5. B.K.Dutta , P.Roy & A.S.Gupta. “ Temperature field in flow over a stretching sheet with uniform heat flux” *Int.Comm.in Heat and Mass Transfer*,Vol.12(1) pp.89-94 (1985).
6. C. Y. Wang. “The three dimensional flow due to stretching flat surface”. *Physics Fluid*. Vol.27, No. 8, pp.1915-1917 (1984).
7. D.A. S. Rees, and I. Pop, “Free convection boundary layer flow of a micropolar fluid from a vertical flat plate”. *IMAJ. Appl. Math.* Vol.61, pp.179-197 (1998).
8. F.C.Lai. & F.A.Kulacki, “The effect of variable viscosity and mass transfer along a vertical surface in a saturated porous medium”, *Int. J. Heat and Mass transfer*, Vol.33, pp.1028-1031, (1990).
9. G. Lukaszewicz ‘Micropolar fluids,theory and application’(1990).
10. G.Ahmadi “Self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite plate.” Int.J.Eng.Sci.Vol.14 .pp.639-646 (1976).
11. J. L. Crane “ Flow past a stretching plane” *ZAMP* Vol.21(4); pp.645-647 (1970).
12. J. Peddison and R.P. McNitt., “Boundary layer theory for micropolar fluid” *Recent Adv. Eng.Sci*.,Vol.5,pp.405-426 (1970).
13. M. A. El Aziz, “Unsteady fluid and heat flow induced by a stretching sheet with mass transfer and chemical reaction”. *Chemical Engineering Communications.* Vol. 197, pp. 1261- 1272 (1995).
14. N. Lakshmisha, S.Venkateswaran, G. Nath,. “Three dimensional unsteady flow with heat and mass transfer over continuous stretching surface ”. *J. Heat Transfer*. Vol-113, No.3, pp.590-595 (1988).
15. P.J. Borthakur, and G.C. Hazarika, “Effects of variable viscosity and thermal conductivity on flow and heat transfer of a stretching surface in a rotating micropolar fluid with suction and blowing”. *Bull. Pure and Appl.Sci.* Vol. 25, No.2, pp. 361-370 (2006).
16. R.S. Gorla,“Heat transfer in micropolar boundary layer flow over a flat plate”. *Int. J. Eng. Sci.* 21, pp.791-796 (1983).
17. T.Hayat, M.Awais, A.Safdar, & AA.Hendi. Unsteady three dimensional flow of couple stress fluid over a stretching surface with chemical reaction. *Non linear Anay. Modelling and Control*, vol.17 no.1 pp:47-59. (2012).