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Finite Source Retrial Queue with Inventory Management: Semi MDP

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ABSTRACT

In this article, we address the problem of Ordering policy in retrial service facility system with inventory. We consider a finite source (N) demand generation system and unsatisfied demand enter an finite orbit for retrial. Arrival of demands to the system is assumed to follows a Poisson Process and service times are assumed to follows an exponential distribution. Let the maximum inventory S and (s,S) policy is adopted for replenishment. The system is formulated as Semi-Markov Decision Process and we find the controlling the inventory ordering policy implemented at each instant of time for a given inventory capacity S. Linear Programming method is implemented with the criterion minimizing the long-run expected cost rate. Numerical examples is provided to establish the result obtained.

Keywords: Inventory control, Service facility, Markov Decision Process, LPP and Policy iteration expect cost rate criteria.

1. INTRODUCTION

In most of the inventory models considered in the literature, the demanded items are directly issued from the stock, if available. The demands that occurred during stock—out period are either not satisfied (lost sales case) or satisfied only after the receipt of the ordered items (backlog case). In the latter, it is assumed either all (full backlog case) or only a prefixed number of demands (partial backlogging) that occurred during stock out period are satisfied. For review of these works see Nahmias(1982), Raafat (1991), Kalpakam and Arivarignan (1990, 1993), Elango and Arivarignan (2003), Liu and Yang (1999), Cakanyildirim *et al*. (2000), Goyal and Giri (2001), Duran *et al*. (2004), and Yadavalli *et al*. (2004) and the references therein.

But in the case of inventories maintained at service facilities, the demanded items are issued to the customers only after some service is performed on it. In this situation the items are issued not at the time of demand but after a random time of service. This forces the formation of queues in these models. This necessitates the study of both the inventory level and the queue length joint distributions. Study of such models is beneficial to organizations which

(i) Provide service to customers by using items from a stock.
(ii) Maintain stock of items each of which needs service such as assembly or initialization or installation, etc.

Examples for the first type include firms that are engaged in servicing consumer products such as.

Television sets, Computers, etc., and for the second type include firms that supply bicycles which need assembly of its parts, that supply food items which need heating or garnishing and that computers which need installation of basic services. Recently Berman *et al*. (1993) have considered an inventory management system at a service facility which uses one item of inventory for each service provided. They assumed that both demand and service rates are deterministic and constant as such queues can form only during stock outs. They determined optimal order quantity that minimizes the total cost rate.

Berman and Kim (1999) analyzed a problem in stochastic environment where customers arrive at service facilities according to a Poisson process and the service times are exponentially distributed with mean inter-arrival time assumed to be greater than the mean service time, and each service requires one item from inventory. A logically related model was studied by He *et al*. (1998), who analyzed a Markovian inventory—production system, where customer demands arrive at a workshop and are processed by a single machine in batch sizes of one. Berman and Sapna (2000) studied extensively an inventory control problem at a service facility which uses one item of inventory for each service. They assumed Poisson arrivals, arbitrarily distributed service times and zero lead times. They analyzed the system with the restriction that the waiting space is finite. Under a specified cost structure, they derived the optimal ordering quantity that minimizes the long-run expected cost rate.

Elango (2001) has considered a Markovian inventory system with instantaneous supply of orders at a service facility. The service time is assumed to have exponential distribution with parameter depending on the number of waiting customers. Arivarignan et al. (2002) have extended this model to include exponential lead time. Sivakumar and Arivarignan (2006) have considered a Markovian perishable inventory system in which the size of the space for the waiting customers is assumed to be infinite. Arivarignan and Sivakumar (2003) have considered an inventory system with arbitrarily distributed demand, exponential servicetime and exponential lead time.

Arivudainambi, Averbakh and Berman (2009) studied a single server retrial queue with Bernoulli vacations and a priority queue. A customer, who finds the server busy upon arrivals, either joins the priority queue or leaves the service area after some time he enters a retrial group (orbit). Using the supplementary variable technique, they find the joint probability generating function of the number of customers in the priority queue and of the number of customers in the retrial group in a closed form. Also find the explicit expressions for the mean queue length and the mean waiting time for both queues, drive steady –state performance measures for the system.

In 2000, Artalejo and Lopez- Herrero are concerned with the M/G/1 retrial queue with balking. The ergodicity condition is first investigated making use of classical mean and the limiting distribution of the number of customers in the system is determined with the help of a recursive approach based on the theory of regenerative processes. Many closed form expression are obtained when we reduce to the M/M/1 queue for some representative balking policies.

Artalejo, Rajagobalan and Sivasamy, (2000), are deals with the stochastic modeling of a wide class of finite retrial queueing systems in a markovian environment. Using Matrix method they obtained the stationary distribution and first passage times.

Krishnamoorthy and Jose,(2005), discuss an inventory system with positive service time and retrial customers. They assume arrival of customers to form a Poison process and lead time is exponentially distributed. Also they calculated the expected number of departures after receiving service, the expected number of customers lost without getting service and the expected total cost of the system.

Krishnamoorthy and Jose, (2007) analyze and compare three (s,S) inventory systems with positive service time and retrial of customers. In all these systems, arrivals of customers form a Poisson process and service times are exponentially distributed. When the inventory level depletes to s due to service, an order for replenishment is placed with lead time. The problem considered is LDQBQ. They investigate these systems to obtain performance measures and construct suitable cost functions for the three cases with numerical example.

The main contribution of this article is to derive the optimum inventory replenishment control in retrial service facility system. For the given formulated as a Semi-Markov Decision process and the optimum inventory policy employed using Policy-iteration method

And LPP method so that the long-run expected cost rate is minimized.

The rest of the paper is organized as follows. We provide a formulation of our Markov Decision model in the next section. Analysis part of the model is given in section 3. In section 4, we present a procedure to prove the existence of a stationary optimal control policy and solve it by employing Policy iteration technique.

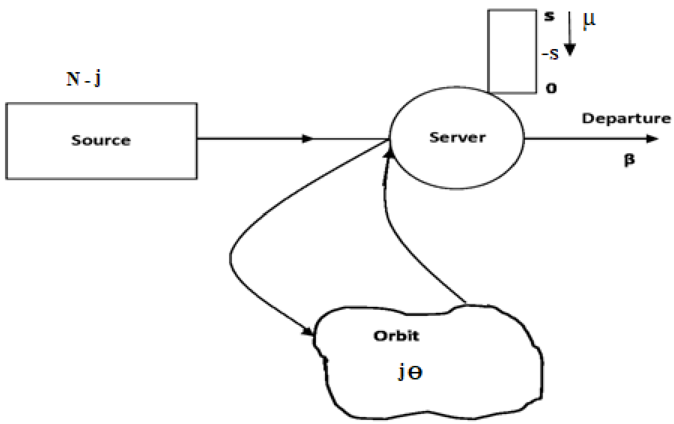
2. PROBLEM FORMULATION

We consider a service facility system holding inventory for service purpose. Arriving customers from a finite source enter into an orbit of finite Capacity when the server is busy. We assume the following for the smooth running of system

- Customers arrive the service facility according to a Poisson process with rate λ (>0).
- An arriving customer get service and leave the System, if the server is free and inventory is available.

- Whenever the server is busy, arriving customers enter into the orbit. After the exponential time the orbit customer retry for another chance with rate $j\theta$ where j is the number of the existing customers in the orbit
- The service times follow an exponential distribution with parameter $\beta(>0)$
- Inventory is maintained in the system to satisfy the customer during service
- The Maximum capacity of source is N say finite. Inventory with maximum level S is maintained with (s, S) policy Q=S-s is the ordering quantity such that Q>s.
- Lead time for inventory level is assumed to be exponentially distribution with parameter μ (>0)
- If the inventory level is zero the arriving customers or the retrial customers enter into the orbit.

Let X(t), Y(t) and I(t) denote the inventory level and the number of customers in the system at time t. Then $\{X(t), Y(t), I(t) : t \geq 0\}$ is a three dimensional stochastic process with state space, $E_1 \times E_2 \times E_3$, where E_1={0,1}, E_2= {0, 1, 2, … ,S} and E_3 = {0, 1, 2, …,N}. where {0,1} denote the status of the systems(0-server free, 1- server busy).



3. ANALYSIS

Let X(t) ,Y(t) and I(t) denotes the status of the server, number of customers in the orbit and inventory level at time t respectively.

Then {X(t), Y(t),I(t): t≥ 0 }is a three dimensional continuous time Markov process with state space E x E_2x $E_{3,}$ where E_1= $\{0,1\}$ (0 denotes the idle server and 1 denotes the busy server)

E_2= {0,1,2,…,N}and E $_3$= {0,1,2,…,S}

The infinitesimal generator A of the Markov process has entries of the form $(a_{(i,j,k)}^{(l,m,n)})$

Some of the state transitions are noted below:

(a) From state (0,j,k) only transitions into the following states are possible:

(i) (1,j,k) with rate $(\mathrm{N}-\mathrm{j})\lambda$ for $0 \le j \le N$; $1 \le k \le S$ (Primary customer arrival)

(ii) (1,j-1,k) with rate $j\theta$ for $1 \le j \le N$; $1 \le k \le S$ (orbit customer retrial).

(b) From state (1,j,k) only transitions into the following states are possible:

(i) (1,j+1,k)with rate $(\mathrm{N}-\mathrm{j})\lambda$ for $0 \le j \le N-1$; $1 \le k \le S$ (customer arrival).

(ii) (0,j,k-1) with rate β *for* $0 \le j \le N; 1 \le k \le S$ (Service completion)

Now, we have to convert this Markov process into continuous time MDP by considering the following five components,

(i) Decision epochs: The decision epochs occurs at random points on the time ie each service Completion times.

(ii) State space:

$$\mathrm{E} = \{ e = (\mathrm{i},\mathrm{j},\mathrm{k}) : \mathrm{i} = 0,1; 0 \le \mathrm{j} \le \mathrm{N}, \mathrm{N} \le \infty; 0 \le \mathrm{k} \le \mathrm{S} \}$$

(iii) Action set: The ordering decisions (0-no order; 1-order;2-compulsory order) taken at each State of the system (i,j,k) $\in E$ and the replenishment of inventory done at μ. The compulsory order for S items is made when inventory level is zero .

Let $A_{(\mathrm{i},\mathrm{j},\mathrm{k})}$ denote the action set for the MDP at state (i,j,k) and

$$A_{(\mathrm{i},\mathrm{j},\mathrm{k})} \subseteq A, where A = \bigcup_{(\mathrm{i},\mathrm{j},\mathrm{k}) \in \mathrm{E}} A_{(\mathrm{i},\mathrm{j},\mathrm{k})}$$

The action set a for the MDP can be expanded us

$$A = \begin{cases} 0 \;, & s+1 \le k \le S \\ 0,1 \;, & 1 \le k \le s \qquad (\mathrm{i},\mathrm{j},\mathrm{k}) \in \mathrm{E}. \\ 2 \;, & k = 0 \end{cases}$$

A decision rule from the class Π is equivalent to a function $\Pi : E \to A$ and is given by

$$\pi(\mathrm{i},\mathrm{j},\mathrm{k}) = \{ a : (\mathrm{i},\mathrm{j},\mathrm{k}) \in \mathrm{E}, \mathrm{a} \in \mathrm{A} \} \text{ , where } \Pi \subset MD \text{ (Markov Deterministic)}$$

$$Let \;\; E_1 = \{ (\mathrm{i},\mathrm{j},\mathrm{k}) \in E / \pi(\mathrm{i},\mathrm{j},\mathrm{k}) = 0 \} .$$

$$E_2 = \{ (\mathrm{i},\mathrm{j},\mathrm{k}) \in E / \pi(\mathrm{i},\mathrm{j},\mathrm{k}) = 0 \;\; or \;\; 1 \} .$$

$$E_3 = \{ (\mathrm{i},\mathrm{j},\mathrm{k}) \in E / \pi(\mathrm{i},\mathrm{j},\mathrm{k}) = 2 \} , then\, E = E_1 \cup E_2 \cup E_3.$$

(iv) Transition probability:

$p_{(i,j,k)}^{(\mathrm{l},\mathrm{m},\mathrm{n})}(a)$ -a transition probability from state (i,j,k) to the state (l,m,n) when decision ’a’ is made at state (i, j, k)

(v) Cost:

Let $C_{(i,j,k)}(a)$ denote the cost occurred in the system when action "a" is taken at state (i,j,k).

The long-run expected cost rate when policy π is adopted is given by

$$C^{\pi} = h\overline{I}^{\pi} + c_1\overline{W}^{\pi} + c_2\overline{R}^{\pi} + v\overline{\alpha_c}^{\pi} \quad (1)$$

$\overline{I}^{\pi}$ is the mean inventory level , $\overline{w}^{\pi}$ is the average waiting time for a customer , $\overline{R}^{\pi}$ is the inventory reorder rate, $\overline{\alpha_c}^{\pi}$ is the service completion rate , h denotes the holding cost / unit time/ unit item , c_1 denotes the waiting cost /customer / unit time , c_2 denotes the reordering cost/order and v denotes the service cost /customer .

3.1. Steady state Analysis

Let f denote the stationary policy, which is deterministic time invariant and Markovian Policy (MD). From our assumptions it can be seen that $\{(X(t), \mathrm{Y(t)}, \mathrm{I}(t)) : t \geq 0\}$ is denoted as the controlled process $\{(X^{\pi}(t), \mathrm{Y}^{\pi}(\mathrm{t}), \mathrm{I}^{\pi}(t)) : t \geq 0\}$ when policy π is adopted. Since the process $\{(X^{\pi}(t), \mathrm{Y}^{\pi}(\mathrm{t}), \mathrm{I}^{\pi}(t)) : t \geq 0\}$ is a Semi- Markov Decision Process with finite state space E. The process is completely Ergodic, if every stationary policy gives rise to an irreducible Markov chain. It can be seen that for every stationary policy π the Markov process is completely Ergodic and also the optimal stationary policy π^* exists, because the state and action spaces are finite

Our objective is to find an optimal policy Π^* for which $C^{\Pi^*} \leq C^{\Pi}$ for every MR policy in Π^{MR}

For any fixed MD policy Π^{MD} and (i,j,k),(l,m,n)$\in E$,define

$p^{\pi}_{ijk}(\mathrm{l,m,n,t}) = \mathrm{pr}\{\mathrm{X}^{\pi}(\mathrm{t}) = l, \mathrm{Y}^{\pi}(\mathrm{t}) = \mathrm{m}, \mathrm{I}^{\pi}(\mathrm{t}) = \mathrm{n} / \mathrm{X}^{\pi}(0) = \mathrm{i}, \mathrm{Y}^{\pi}(0) = \mathrm{j}, \mathrm{I}^{\pi}(0) = \mathrm{k}\}$; where $(\mathrm{i,j,k}),(\mathrm{l,m,n}) \in \mathrm{E}$

Let $P(\mathrm{t}) = (\mathrm{P}^{l,m,n}_{(\mathrm{i,j,k})})$ be probability transformation matrix for the given Markov Decision Process.

Now $p^{\pi}_{ijk}(\mathrm{l,m,n,t})$ satisfies the Kolmogorov forward differential equation. P'(t)=P(t)A, where A is an infinitesimal generator of the Markov process $\{(\mathrm{X}^{R}(\mathrm{t}), \mathrm{Y}^{R}(\mathrm{t}), \mathrm{I}^{R}(\mathrm{t})) : \mathrm{t} \geq 0\}$

For each MD policy π , we get a Markov chain with state space E and action set A which are finite, the steady state probability

$p^{\pi}(\mathrm{l,m,n}) = \lim_{t\to\infty} p^{\pi}_{(ijk)}(\mathrm{l,m,n;t})$ exists and is independent of initial state (i,j,k)conditions.

The balance equations are obtained by using the fact that transition out of a state is equal to transition into a state (PA=0)

$$((N-j)\lambda + j\theta)P^{\pi}(0,j,S) = \mu P^{\pi}(0,j,s), \quad 0 \le j \le N \tag{2}$$

$$(N-j)\lambda + j\theta)P^{\pi}(0,j,k) = \mu P^{\pi}(0,j,k-Q) + \beta P^{\pi}(1,j,k-Q+1),\ 0 \le j \le N,\ Q \le k \le S-1 \tag{3}$$

$$(N-j)\lambda + j\theta)P^{\pi}(0,j,k) = \beta P^{\pi}(1,j,k+1),\ 0 \le j \le N,\ s+1 \le k \le Q-1 \tag{4}$$

$$((N-j)\lambda + j\theta + \mu)P^{\pi}(0,j,k) = \beta P^{\pi}(1,j,k+1),\ 0 \le j \le N,\ 1 \le k \le s \tag{5}$$

$$((N-j)\lambda + \mu)P^{\pi}(0,j,0) = \beta P^{\pi}(1,j,1) + (N-j+1)\lambda P^{\pi}(0,j-1,0), \quad 1 \le j \le N \tag{6}$$

$$(N\lambda + \mu)P^{\pi}(0,0,0) = \beta P^{\pi}(1,0,1)$$

$$((N-1)\lambda + \beta)P^{\pi}(1,0,k) = \mu P^{\pi}(1,0,k-Q) + \theta P^{\pi}(0,1,k) + N\lambda P^{\pi}(0,0,k),\ Q+1 \le k \le S \tag{7}$$

$$\begin{aligned}(N-(j+1)\lambda + \beta)P^{\pi}(1,j,k) &= \mu P^{\pi}(1,j,k-Q) + (j+1)\theta P^{\pi}(0,j+1,k) \\ &+ (N-j)\lambda\left[P^{\pi}(0,j,k) + P^{\pi}(1,j-1,k)\right], \quad 1 \le j \le N-1, Q+1 \le k \le S\end{aligned} \tag{8}$$

$$((N-1)\lambda + \beta)P^{\pi}(1,0,k) = \theta P^{\pi}(0,1,k) + N\lambda P^{\pi}(0,0,k), \quad s+1 \le k \le Q \tag{9}$$

$$\begin{aligned}((N-(j+1)\lambda + \beta)P^{\pi}(1,j,k) &= (j+1)\theta P^{\pi}(0,j+1,k) + \\ &(N-j)\lambda\left[P^{\pi}(0,j,k) + P^{\pi}(1,j-1,k)\right], 1 \le j \le N-1,\ s+1 \le k \le Q\end{aligned} \tag{10}$$

$$(\beta + (N-1)\lambda + \mu)P^{\pi}(1,0,k) = N\lambda P^{\pi}(0,0,k) + \theta P^{\pi}(0,1,k),\ 1 \le k \le s \tag{11}$$

$$\begin{aligned}(\beta + (N-(j+1)\lambda) + \mu)P^{\pi}(1,j,k) &= (N-j)\lambda\left[P^{\pi}(0,j,k) + P^{\pi}(1,j-1,k)\right] + \\ &(j+1)\theta P^{\pi}(0,j+1,k),\ 1 \le j \le N-1, 1 \le k \le s\end{aligned} \tag{12}$$

Together with the above set of equations, the total probability condition

$$\sum_{(i,j)\in E} P^{\pi}(i,j,k) = 1, \textit{yield the} \tag{13}$$

steady state probabilities $\{P^{\pi}(i,j,k), (i,j,k) \in E\}$ uniquely.

3.2 System Performance Measures.

(i)The average inventory level in the system is given by

$$\bar{I}^{\pi} = \sum_{i=0}^{1}\sum_{j=0}^{N}\sum_{k=1}^{S} kP^{\pi}(i,j,k) \tag{14}$$

(ii)Mean waiting time for a customer in the system is given by

$$\bar{W}^{\pi} = \sum_{j=1}^{N}\sum_{k=0}^{S}\left(\frac{j\theta + \lambda}{\beta}\right)P^{\pi}(0,j,k) \tag{15}$$

(iii) The inventory reorder rate is given by

$$\bar{R}^{\pi} = \sum_{j=1}^{N}\sum_{k=1}^{s+1}\beta P^{\pi}(1,j,k) \tag{16}$$

(iv)The service completion rate is given by

$$\overline{\alpha_c}^{\pi} = \beta\sum_{j=1}^{N}\sum_{k=1}^{S} P^{\pi}(1,j,k) \tag{17}$$

Now the long run expected cost rate is given by

$$C^{\Pi} = h\sum_{i=0}^{1}\sum_{j=0}^{N}k\sum_{k=1}^{S}P^{\pi}(\mathrm{i,j,k}) + c_1\left(\sum_{j=1}^{N}\sum_{k=0}^{S}\left(\frac{\lambda + j\theta}{\beta}\right)P^{\pi}(0,\mathrm{j,k}) + c_2\sum_{j=1}^{N}\sum_{k=1}^{S+1}\beta P^{\pi}(1,\mathrm{j,k})\right. \left. + v\left(\beta\sum_{j=1}^{N}\sum_{k=1}^{S}P^{\pi}(1,\mathrm{j,k})\right)\right) \tag{18}$$

4. LINEAR PROGRAMMING PROBLEM

4.1 Formulation of LPP

In this section we propose a LPP model within a MDP framework.
First we define the variables, D(i,j,a) as a conditional probability expression

$$D(\mathrm{i,j,k,a}) = \Pr\ decisionis\,'a'/\,\mathrm{stateis}\,(\mathrm{i,j,k})$$

Since 0 ≤ D (i,j,k,a) ≤ 1, this is compatible with Randomized time invariant Markovian policies. Here, the Semi-Markovian decision problem can be formulated as a linear programming problem.

Hence

$$0 \le D(i,j,k,a) \le 1 \ \ and \sum_{a\in(0,1,2)} D(i,j,\mathrm{k},a) = 1,\ \mathrm{i} = 0,1;\ 0 \le j \le N;\ \ 0 \le k \le S.$$

For the reformulation of the MDP as LPP, we define another variable y(i,j,k,a) as follows.

$$\mathrm{y}(i,j,\mathrm{k},a) = D(\mathrm{i,j,k,a})P^{\pi}(\mathrm{i,j,k}) \tag{19}$$

From the above definition of the transition probabilities

$$P^{\pi}(i,j,k) = \sum_{a\in A} y(\mathrm{i,j,k,a}),\ \text{where } (\mathrm{i,j,k}) \in \mathrm{E}, \mathrm{a} \in \mathrm{A} \tag{20}$$

Expressing $P^{\pi}(\mathrm{i,j,k})$ in terms of y(i,j,k,a), the expected total cost rate function(18) is Obtained and the LPP formulation is of the form

Minimize

$$C^{\Pi} = h\sum_{a\in(0,1,2)}\sum_{i=0}^{1}\sum_{j=0}^{N}\ \sum_{k=1}^{S}kP^{\pi}(\mathrm{i,j,k}) + c_1\left(\sum_{a\in(0,1,2)}\sum_{j=1}^{N}\sum_{k=0}^{S}\left(\frac{\lambda + j\theta}{\beta}\right)P^{\pi}(0,\mathrm{j,k}) + c_2\sum_{a\in(0,1,2)}\sum_{j=1}^{N}\sum_{k=1}^{S+1}\beta P^{\pi}(1,\mathrm{j,k})\right.$$

$$\left. + v\left(\beta\sum_{a\in(0,1,2)}\sum_{j=1}^{N}\sum_{k=1}^{S}P^{\pi}(1,\mathrm{j,k})\right)\right) \tag{21}$$

Subject to the constraints,

$$\text{(i) } \mathrm{y(i,j,k,a)} \ge 0; (\mathrm{i,j,k}) \in \mathrm{E}, \mathrm{a} \in \mathrm{A}$$

$ii) \sum_{(i,j,k)\in E} \sum_{a\in A} y(i,j,k,a) = 1,$

And the balance equation (2)-(12) obtained by replacing
$P^{\pi}(i,j,k)$ by $\sum_{a\in A} y(i,j,k,a).$

4.3 Lemma:

The optimal solution of the above Linear Programming Problem yield a deterministic policy.

Proof:

From the equations (19) and (20),

$$y(i,j,k,a) = D(i,j,k,a)\,P^{\pi}(i,j,k) \quad (22)$$

and

$$P^{\pi}(i,j,k) = \sum_{a\in A} y(i,j,k,a), \text{ where } (i,j,k)\in E \quad (23)$$

We have,

$$D(i,j,k,a) = \frac{y(i,j,k,a)}{\sum_{a=0}^{2} y(i,j,k,a)}, \quad (24)$$

Since the decision process is completely ergodic every basic feasible solution to the above linear

Programming problem has the property that for each $(i,j,k)\in E,\ y(i,j,k,a) > 0$ for exactly one $a \in A$

Hence, for each $(i,j,k)\in E,$ D (i, j, k, a)=1, for a unique state and zero for other values of a Thus given the number of customers in the orbit, we have to choose the service rate β for which D (i,j,k,a)=1 . Hence the basic feasible solution of the LPP yields a optimal deterministic policy Π^{*}.

5. NUMERICAL ILLUSTRATION AND DISCUSSION

In this section we consider a service facility system to illustrate the method described in section 4, through numerical examples. We implemented TORA software to solve LPP by simplex algorithm.

Example 5.1 S=3, s=1, N=3**,** $\lambda = 2$ $,\beta = 3, \mu = 4, \theta = 2$, $c_j = 2j, j = 1,2$ $;v = 0.8$

Inventory ordering policy is (s,S) , ie., whenever the inventory reaches level s, an order for Q=S-s >s , items are placed. Lead time is exponential distributed with parameter μ =4>0.

The optimum cost for the system implementation is C^{π^*} =39.243

X(t),Y(t),I(t)	(0,0,3)	(0,1,3)	(0,2,3)	(0,3,3)	(1,0,3)	(1,1,3)
Action	0	0	0	0	0	0
X(t),Y(t),I(t)	(1,2,3)	(0,0,2)	(0,1,2)	(0,2,2)	(1,0,2)	(1,2,2)
Action	0	0	0	0	0	0
X(t),Y(t),I(t)	(0,0,1)	(0,1,1)	(0,2,1)	(0,3,1)	(1,0,1)	(1,1,1)
Action	1	1	1	0	1	1
X(t),Y(t),I(t)	(1,2,1)	(0,0,0)	(0,3,0)	(0,3,0)	(0,3,0)	
Action	1	2	2	2	2	

6. CONCLUSIONS AND FUTURE RESEARCH

Analysis of inventory control at service facility is fairly recent system study. Most of previous work determined optimal ordering policies or system performance measures. We approached the problem in a different way using Semi-Markov Decision Process to control optimally the inventory ordering policy. The optimum control policy to be employed is found using Linear Programming method and Policy iteration method so that the long – run expected cost rate is minimized. In future we may extend this model to multi server-retrial service Facility system with inventory maintenance.

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