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Fixed Point Theorems for Symmetric Hausdorff Function

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ABSTRACT

The main objective of this paper we prove the fixed point theorem for continuous mapping in Symmetric Hausdorff space.

Keywords: Fixed Point, Symmetric Hausdorff function, Continuous mapping.

INTRODUCTION

Banach contraction principle in 1922 several authors, extended, improved and generalized the result of Banach in different types. In the past few years, a number of authors such as Singh and Zarzitto[1], Ray and Chatterjee[2], Chatterjee and Ghoshal[3], Fisher and Khan[4] and Popa[5], Bondar[6], Shrivastava[7] etc. have established several interesting results on different types in Hausdorff Space.

MAIN RESULTS

Definition: Let X be a Hausdorff space and $H: X \times X \to [0,+\infty)$ be a continuous mapping such that for all $x,y \in X$

$H(x,y)=0$ if $x=y$
then H is called symmetric Hausdorff function.
In this paper we prove the following theorem

Theorem: Let T be a continuous mapping of a Hausdorff space X and let H is a symmetric Hausdorff function satisfied following conditions

$$H(Tx,Ty) \leq \alpha\,(M(x,y)) + \beta(H(x,y)) \qquad \text{(i)}$$

for all $x, y \in X$ where $\alpha, \beta \geq 0$ are constant such that $(\alpha + \beta) < 1$ where

$$M(x,y) = max \left\{ \begin{array}{c} \left[\frac{H(y,Tx)\sqrt{H(x,Ty)}+H(x,Tx)\sqrt{H(y,Ty)}}{H(x,y)}\right]^2, \\ \left[\frac{H(x,Tx)\sqrt{H(y,Ty)}+H(x,Ty)\sqrt{H(y,Tx)}}{H(x,y)}\right]^2, \\ \left[\frac{\sqrt{H(x,y)d(x,Ty)}+\sqrt{H(x,y)H(y,Tx)}}{\sqrt{H(x,Ty)+H(y,Tx)}}\right]^2 \end{array} \right\} \quad \text{(ii)}$$

If for some $x_0 \in X$, the sequence $x_n = T^n x_0$ has a convergent subsequence then T has a fixed point . If $2\alpha + \beta < 1$ then T has unique fixed point.

Proof : Let we have choose x_0 ε X such tha $x_1 = Tx_0$. Now define a sequence $\{x_n\}$in X such that $x_{n+1} = Tx_n$.
Case 1 : We claim that $\{x_n\}$ has a convergent subsequence and converges to some real numbers λ
Now for each n≥0 consider

$H(x_n, x_{n+1}) = H(Tx_{n-1}, Tx_n)$
$\leq \alpha (M(x_{n-1}, x_n)) + \beta(H(x_{n-1}, x_n))$ (iii)

From equation(ii) we have

$$M(x,y) = max \left\{ \begin{array}{c} \left[\frac{H(x_n,Tx_{n-1})\sqrt{H(x_{n-1},Tx_n)}+H(x_{n-1},Tx_{n-1})\sqrt{H(x_n,Tx_n)}}{H(x_{n-1},x_n)}\right]^2, \\ \left[\frac{H(x_{n-1},Tx_{n-1})\sqrt{H(x_n,Tx_n)}+H(x_{n-1},Tx_n)\sqrt{H(x_n,Tx_{n-1})}}{H(x_{n-1},x_n)}\right]^2, \\ \left[\frac{\sqrt{H(x_{n-1},x_n)d(x_{n-1},Tx_n)}+\sqrt{H(x_{n-1},x_n)H(x_n,Tx_{n-1})}}{\sqrt{H(x_{n-1},Tx_n)+H(x_n,Tx_{n-1})}}\right]^2 \end{array} \right\}$$

$$= max \left\{ \begin{array}{c} \left[\frac{H(x_n,x_n)\sqrt{H(x_{n-1},x_{n+1})}+H(x_{n-1},x_n)\sqrt{H(x_n,x_{n+1})}}{H(x_{n-1},x_n)}\right]^2, \\ \left[\frac{H(x_{n-1},x_n)\sqrt{H(x_n,x_{n+1})}+H(x_{n-1},x_n)\sqrt{H(x_n,x_n)}}{H(x_{n-1},x_n)}\right]^2, \\ \left[\frac{\sqrt{H(x_{n-1},x_n)d(x_{n-1},Tx_n)}+\sqrt{H(x_{n-1},x_n)H(x_n,Tx_{n-1})}}{\sqrt{H(x_{n-1},Tx_n)+H(x_n,Tx_{n-1})}}\right]^2 \end{array} \right\}$$

$= max\{ H(x_n, x_{n+1}), H(x_n, x_{n+1}), H(x_n, x_{n-1})\}$ (iv)

If $H(x_n, x_{n+1}) > H(x_n, x_{n-1})$
then by (iii) we have

$H(x_n, x_{n+1}) = H(Tx_{n-1}, Tx_n)$
$\leq \alpha (H(x_n, x_{n+1})) + \beta(H(x_{n-1}, x_n)$
$\leq [\frac{\beta}{(1-\alpha)}] H(x_{n-1}, x_n)$
$< H(x_{n-1}, x_n)$ (v)

And if $H(x_n, x_{n+1}) \leq H(x_n, x_{n-1})$ by (iii) then
$H(x_n, x_{n+1}) = H(Tx_{n-1}, Tx_n)$
$\leq \alpha\,(H(x_n, x_{n-1})) + \beta(H(x_{n-1}, x_n)$
$\leq (\alpha+\beta)\,(H(x_{n-1}, x_n)$
$< H(x_{n-1}, x_n)$ (vi)
Continuing In this way we have
$H(x_n, x_{n+1}) < \cdots \ldots\ldots < H(x_0, x_1)$ (vii)
Thus we get a monotone sequence of non negative real numbers which must converge with all its subsequence to some real number say z.
Case (ii) we claim that u belong to X is a fixed point of T. Now suppose If $z \neq Tz$
Consider
$H(z,Tz) = H(Tz, T^2z) \leq \alpha\,(M(z,Tz)) + \beta(H(z,Tz))$ (viii)
Now we have

$$M(z,Tz) = max \left\{ \begin{array}{c} \left[\frac{H(Tz,Tz)\sqrt{H(z,T^2z)}+H(z,Tz)\sqrt{H(Tz,T^2z)}}{H(z,Tz)}\right]^2, \\ \left[\frac{H(z,Tz)\sqrt{H(Tz,T^2z)}+H(z,T^2z)\sqrt{H(Tz,Tz)}}{H(z,Tz)}\right]^2, \\ \left[\frac{\sqrt{H(z,Tz)d(z,T^2z)}+\sqrt{H(z,Tz)H(Tz,Tz)}}{\sqrt{H(z,T^2z)+H(Tz,Tz)}}\right]^2 \end{array} \right\}$$

$= max\{\ H(Tz, T^2z), H(Tz, T^2z), H(z,Tz)\}$
$= max\{\ H(z, Tz), H(z, Tz), H(z,Tz)\}$
We get from (viii)
$H(z,Tz) = H(Tz, T^2z)$
$\leq \alpha\,(M(z,Tz)) + \beta(H(z,Tz))$
$\leq (\alpha+\beta)\ \ H(z,Tz)$
Which is contradiction. Therefore $z = Tz$. i.e. z is a fixed point of T.
Case (iii) For uniqueness let u is a another fixed point belongs to X such that Tu=u from (i) we get
$H(Tz,Tu) = H(z, u)$
$\leq \quad \alpha\,(M(z,u)) + \beta(H(z,u))$ (ix)

$$\text{Where } M(z,u) = max \left\{ \begin{array}{c} \left[\frac{H(u,Tz)\sqrt{H(z,Tu)}+H(z,Tz)\sqrt{H(u,Tu)}}{H(z,u)}\right]^2, \\ \left[\frac{H(z,Tz)\sqrt{H(u,Tu)}+H(z,Tu)\sqrt{H(u,Tz)}}{H(z,u)}\right]^2, \\ \left[\frac{\sqrt{H(z,u)H(z,Tu)}+\sqrt{H(z,u)H(u,Tz)}}{\sqrt{H(z,Tu)+H(u,Tz)}}\right]^2 \end{array} \right\}$$

$= max\{ H(u,z), H(u,z), 2H(u,z)\}$
$= 2H(u,z)$ (x)
From equation (ix) and (x) we have

H(Tz,Tu)= H(z, u)
$\leq \alpha \left(2\mathrm{H}(\mathrm{z},\mathrm{u})\right) + \beta\left(\mathrm{H}(\mathrm{z},\mathrm{u})\right)$
$< (2\alpha + \beta)\ \mathrm{H}(\mathrm{z},\mathrm{u})$
which is contradiction . Therefore $z = \mathrm{u}$. This complete of the proof.

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