Journal of Computer and Mathematical Sciences, *Vol.10(3),605-607 March 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Fixed Point Theorem in Orbitally Complete 2- Metric space

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(Received on: March 24, 2019)

ABSTRACT

The main objective of this paper is, to prove the fixed point theorem in orbitally 2-complete metric space.

Keywords: Orbitally Countinuous, Fixed Point, Complete metric space, T-Orbitally complete metric space.

1. INTRODUCTION

A number of authors generalized, extended and improved fixed point theorems to different types. In 1974 Circic[1] proved some non-unique fixed point theorem for orbitally continuous self maps. Sessa[2], Dhage[3] Fisher and Sessa[4] Jain and Bohre[5] Turkoglu,Ozer and Fisher[6] etc. proved fixed point theorems.

Definitions: (1) Orbitally continuous mapping: Let (X,d) be a metric space .A mapping T on X is orbitally continuous if lim T^{n_i} x=u ⇒lim TT^{n_i} x=Tu for each x εX.

(2) T-orbitally complete metric space:- A space X is T-orbitally complete if every Cauchy sequence of the form { T^{n_i} x$\}_{i=1}^{\infty}$, x εX converges in X.

2. OUR RESULTS

Theorem:- Let X be a Orbitally complete 2- metric space and T be on orbitally continuous self mapping of X and satisfying following condition:

$$d(Tx,Ty,a) \leq \alpha \left[\frac{d(y,Tx,a)\sqrt{d(x,Ty,a)}+d(x,Tx,a)\sqrt{d(y,Ty,a)}}{d(x,y,a)}\right]^2$$
$$+\beta\left[\frac{d(x,Tx,a)\sqrt{d(y,Ty,a)}+d(x,Ty,a)\sqrt{d(y,Tx,a)}}{d(x,y,a)}\right]^2 + \gamma\left[\frac{\sqrt{d(x,y,a)d(x,Ty,a)}+\sqrt{d(x,y,a)d(y,Tx,a)}}{\sqrt{d(x,Ty,a)+d(y,Tx,a)}}\right]^2$$

For all x,y,a in X , $d(x, Tx, a) \neq 0$ and $d(y, Ty, a) \neq 0$ where α , β and γ are real numbers with α + β + γ < 1.Then for each x ε X, the sequence$\{T^n x\}_{n=1}^{\infty}$ converges to a fixed point of T.

Proof:- Let x_0 belongs to X be an arbitrary and define a sequence $x_{n+1} = Tx_n$ n=0,1,2, ……………If for some n, $x_n = x_{n+1}$ then immediately follows that $\{x_n\}$ is a Cauchy sequence. So we may suppose that $x_n \neq x_{n+1}$ for each n=0,1,2, ………………. Applying above condition for $x_n = x_{n-1}$ and $y = x_n$ we have

$$d(Tx_{n-1}, Tx_n, a) \leq \alpha\left[\frac{d(x_n, Tx_{n-1}, a)\sqrt{d(x_{n-1}, Tx_n, a)} + d(x_{n-1}, Tx_{n-1}, a)\sqrt{d(x_n, Tx_n, a)}}{d(x_{n-1},, x_n, a)}\right]^2$$

$$+\beta\left[\frac{d(x_{n-1}, Tx_{n-1}, a)\sqrt{d(x_n, Tx_n, a)} + d(x_{n-1}, Tx_n, a)\sqrt{d(x_n, Tx_{n-1}, a)}}{d(x_{n-1},, x_n, a)}\right]^2$$

$$+\gamma\left[\frac{\sqrt{d(x_{n-1},, x_n, a)d(x_{n-1}, Tx_n, a)} + \sqrt{d(x_{n-1},, x_n, a)d(x_n, Tx_{n-1}, a)}}{\sqrt{d(x_{n-1}, Tx_n, a) + d(x_n, Tx_{n-1}, a)}}\right]^2$$

$$d(x_n, x_{n+1}, a) \leq \alpha\left[\frac{d(x_n, x_n, a)\sqrt{d(x_{n-1}, x_{n+1}, a)} + d(x_{n-1}, x_n, a)\sqrt{d(x_n, x_{n+1}, a)}}{d(x_{n-1},, x_n, a)}\right]^2$$

$$+\beta\left[\frac{d(x_{n-1}, x_n, a)\sqrt{d(x_n, x_{n+1}, a)} + d(x_{n-1}, x_{n+1}, a)\sqrt{d(x_n, x_n, a)}}{d(x_{n-1},, x_n, a)}\right]^2$$

$$+\gamma\left[\frac{\sqrt{d(x_{n-1},, x_n, a)d(x_{n-1}, x_{n+1}, a)} + \sqrt{d(x_{n-1},, x_n, a)d(x_n, x_n, a)}}{\sqrt{d(x_{n-1}, x_{n+1}, a) + d(x_n, x_n, a)}}\right]^2$$

$$d(x_n, x_{n+1}, a) \leq \alpha\, d(x_n, x_{n+1}, a) + \beta\, d(x_n, x_{n+1}, a) + \gamma\, d(x_{n-1},, x_n, a)$$

$$(1-\alpha-\beta)\, d(x_n, x_{n+1}, a) \leq \gamma\, d(x_{n-1},, x_n, a)$$

$$d(x_n, x_{n+1}, a) \leq \left(\frac{\gamma}{1-\alpha-\beta}\right) d(x_{n-1}, x_n, a)$$

Which is contradiction, since α + β + γ < 1.So we obtain $d(x_n, x_{n+1}, a) \leq h\ d(x_{n-1}, x_n, a)$ where $h = (\frac{\gamma}{1-\alpha-\beta})$. In this manner ,this yields

$d(x_n, x_{n+1}, a) \leq h\ d(x_{n-1}, x_n, a) \leq h^2 d(x_{n-2}, x_{n-1}, a) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \leq h^n\, d(x_0, x_1, a)$

Hence for any integer p εI^+ one has $d(x_n, x_{n+p}, a) \leq \sum_{j=n}^{n+p-1} h\ d(x_j, x_{j+1}, a) \leq \sum_{j=n}^{n+p-1} h^j\ d(x_0, x_1, a) \leq (\frac{h^n}{1-h})\ d(x_0, x_1, a)$

Since $\lim_n h^n = 0$ it follows that sequence $\{x_n\}$ *is* a Cauchy sequence, X being

T- orbitally complete there is some u εX such that u=$\lim_n T^n x$ by orbital continuity of T.

T u= $\lim_{n} TT^{n}$ x=u

i.e. u is a fixed point of T .This is complete of the proof of the theorem.

CONCLUSION

In this paper we study fixed point In Orbitally complete 2- metric space.

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