JOURNAL OF COMPUTER AND MATHEMATICAL SCIENCES
An International Open Free Access, Peer Reviewed Research Journal
www.compmath-journal.org

ISSN 0976-5727 (Print)
ISSN 2319-8133 (Online)
Abbr:J.Comp.&Math.Sci.
2014, Vol.5(2): Pg.205-208

Fixed Point Theorem in Dislocated Quasi Metric-Space

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(Received on: April 2, 2014)

ABSTRACT

The object of our paper, we have proved fixed point theorems in dislocated quasi metric spaces.

Keywords: Dislocated quasi-metric spaces, Fixed Point.

1. INTRODUCTION

S. Banach[1] proved the famous and well-known Banach contraction principle concerning the fixed point of contraction mapping in complete metric space. Hitzler and A.K. Seda[2] introduced the notion of dislocated metric spaces thereby extended the Banach contraction principle for such spaces. Zeyada *et al.*[3] generalized the work of Hitzler and Seda[2] in dislocated quasi metric spaces. Recently results on fixed point in dislocated and dislocated quasi metric spaces followed by Isuafti[4] Aage and Salunke[5]. The object of this paper we prove fixed point theorems in dislocated quasi metric space.

2. PRELIMINARIES

Before presenting the main result, we need some definitions:

Definition 2.1 [3]-:Let X be a non-empty and let $d: X \times X \rightarrow R^{+}$ be a function satisfying following conditions:

(i) d (x,y) = d (y,x) = 0 implies x = y
(ii) d (x,y) ≤ d (x,z) + d (z,y) for all x , y, z ε X

Then is called a dislocated quasi-metric on X, if d satisfies $d(x, y) = d(y, x)$ then it is called dislocated metric space.

Definition 2.2 [3]:- A sequence $\{x_n\}$ in dislocated quasi-metric space (X,d) is called Cauchy sequence if for given $\in > 0 \ \exists \ n_0 \ \varepsilon \ N$ such that for all m, n ≥ n_0 implies that $d(x_m, x_n) < \in \ or \ d(x_n, x_m) < \in$ i.e. $\min\{d(x_m, x_n),\ d(x_n, x_m)\} < \in$. If replace $d(x_m, x_n) < \in \ or \ d(x_n, x_m) < \in$ by $\max\{d(x_m, x_n),\ d(x_n, x_m)\} < \in$ the sequence $\{x_n\}$ is called bi- Cauchy and every bi- Cauchy sequence is Cauchy.

Definition 2.3 [3]:- A sequence $\{x_n\}$ dislocated quasi convergence to x if

$$\lim_{n\to\infty} d\,(x_n, x) = \lim_{n\to\infty} d\,(x, x_n) = 0$$

In this case x is called a dislocated quasilimitof $\{x_n\}$ and we write $x_n \to x$.

Lemma [2.1] [3]:- Dislocated quasi limits in a dislocated quasi-metric space are unique.

Definition 2.4 [3]:- A dislocated quasi metric space (X,d) is called complete if every Cauchy sequence in it is a dislocated quasi-convergent .

Definition 2.6 [3]: -Let (X, d) and (Y, d_2) be dislocated quasi-metric spaces and let $f : X \to Y$ be a function. Then f is continuous to $x_0 \; \varepsilon \; X$ if for each sequence $\{x_n\}$ which is d_1 - quasi convergent to x_0 the sequence $\{f\,(x_n)\}$ is d_2- quasi convergent to $f\,(x_0)$ in Y.

Definition 2.7 [3]:-Let (X,d) be a dislocated quasi metric space. A mapping $T : X \to X$ is called contraction if there exists a number λ where $0 \le \lambda < 1$ such that $d(Tx, Ty) \le \lambda\, d\,(x, y)$ for all $x, y \; \varepsilon \; X$.

MAIN RESULTS

Theorem (3.1):-Let (X, d) be a complete dislocated quasi - metric and $T : X \to X$ be a mapping satisfying the following conditions:-

$$\begin{aligned} d\,(Tx,Ty) \; &\le C_1 \max\,\{d\,(x,Tx),\, d\,(y,Ty),\, d\,(x,y)\} \\ &+ C_2 \; \max\{d\,(x,Tx) + d\,(x,y),\, d\,(y,Ty) + d\,(x,y)\} \\ &+ C_3 \frac{d(x,y)\;[1+\sqrt{d(x,y)\,d\,(x,Tx)} + \sqrt{d(x,y)\,d\,(y,Tx)}]^2}{[1+d(x,y)+d\,(x,Tx)d(x,Ty)d\,(y,Tx)\;d\,(y,Ty)]^2} \end{aligned} \tag{3.1}$$

For all $x, y \; \varepsilon \; X$, C_1, C_2 and C_3 are non- negative $C_1+2C_2 + C_3 < 1$, $C_1+C_2 < 1$ and $C_1+C_2 + C_3 < 1$.Then T has a unique fixed point.

Proof :- Let $\{x_n\}$ be a sequence in defined as follows,

Let $x_0 \in X$, $T\,(x_0) = x_1$, $T(x_1) = x_2 --- T\,(x_n) = x_{n+1}$

putting $x = x_{n-1}$ *and* $y = x_n$ in equation(3.1) we have

$$\begin{aligned} d\,(x_n, x_{n+1}) \le\; & C_1 \max\,\{d\,(x_{n-1}, Tx_{n-1}), d\,(x_n, Tx_n), d\,(x_{n-1}, x_n)\} \\ & + C_2 \max\,\{d\,(x_{n-1}, Tx_{n-1}) + d\,(x_{n-1}, x_n),\, d\,(x_n, Tx_n) + d(x_{n-1}, x_n)\} \end{aligned}$$

$$+ C_3 \frac{d(x_{n-1},x_n)\,[1+\sqrt{d(x_{n-1},x_n)\,d(x_{n-1},Tx_{n-1})} + \sqrt{d(x_{n-1},x_n)d(x_n,Tx_{n-1})}]^2}{[1+d(x_{n-1},x_n)+d(x_{n-1},Tx_{n-1})d(x_{n-1},Tx_n)\,d(x_n,Tx_{n-1})\;d(x_n,Tx_n)]^2}$$

$$\leq C_1 \max\{d(x_{n-1},x_n), d(x_n,x_{n+1}),\; d(x_{n-1},x_n)\}$$

$$+C_2 \max\{d(x_{n-1},x_n)+d(x_{n-1},x_n),\; d(x_n,x_{n+1})+d(x_{n-1},x_n)\}$$

$$+C_3 \frac{d(x_{n-1},x_n)\,[1+\sqrt{d(x_{n-1},x_n)\,d(x_{n-1},x_n)} + \sqrt{d(x_{n-1,}x_n)d(x_n,x_n)}]^2}{[1+d(x_{n-1},x_n)+d(x_{n-1},x_n)d(x_{n-1},x_{n+1})\,d(x_n,x_n)\;d(x_n,x_{n+1})]^2}$$

$$=C_1\; d(x_n,x_{n+1})+C_2\;[d(x_n,x_{n+1})+d(x_{n-1},x_n)]+C_3\; d(x_{n-1},x_n)$$

$$(1-C_1-C_2)\; d(x_n,x_{n+1}) \leq \left(C_2+C_3\right) d(x_{n-1},x_n)$$

$$d(x_n,x_{n+1}) \leq \left(\frac{C_2+C_3}{1-C_1-C_2}\right) d(x_{n-1},x_n)$$

$$d(x_n,x_{n+1}) \leq \lambda\; d\;(x_{n-1},x_n)\, where\; \lambda=\left(\frac{C_2+C_3}{1-C_1-C_2}\right) with\;\; 0\leq \lambda <1$$

Similarly ,we can show that

$$d(x_{n-1},x_n) \leq \lambda\; d\;(x_{n-2},x_{n-1})$$

and $$d(x_n,x_{n+1}) \leq \lambda^2\; d\;(x_{n-2},x_{n-1})$$

Thus,

$$d(x_n,x_{n+1}) \leq \lambda^n\; d\;(x_0,x_1)$$

Since $0\leq \lambda <1$, $\lambda^n \to 0$ as $n\to\infty$. Hence $\{x_n\}$ is a cauchy sequence in complete dislocated quasi metric space X, Thus $\{x_n\}$ dislocated quasi converges ,then we have

$$d(Tz,Tx_n)\leq C_1 \max\{d(z,Tz), d(x_n,Tx_n), d(z,x_n)\}$$

$$+ C_2 \max\;\{d(z,Tz)+d(z,x_n), d(x_n,Tx_n)+d(z,x_n)\}$$

$$+ C_3 \frac{d(z,x_n)\,[1+\sqrt{d(z,x_n)\,d(z,Tz)} + \sqrt{d(z,x_n)d(x_n,Tz)}]^2}{[1+d(z,x_n)+d(z,Tz)d(z,Tx_n)\,d(x_n,Tz)\;d(x_n,Tx_n)]^2}$$

Taking the point $n\to\infty$ we have,

$$d(Tz,z)\;\leq C_1 \max\{d(z,Tz),\; d(z,z), d(z,z)\}$$

$$+ C_2 \max\{d(z,Tz)+d(z,z), d(z,z)+d(z,z)\}$$

$$+C_3 \frac{d(z,z)\,[1+\sqrt{d(z,z)\,d(z,Tz)}+\sqrt{d(z,z)d(z,Tz)}]^2}{[1+d(z,z)+d(z,Tz)d(z,z)\,d(z,Tz)\;d(z,z)]^2}$$
$$\leq C_1\; d(z,Tz)+C_2\, d(z,Tz)$$
$$d(Tz,z)\leq(C_1+C_2)\,d(z,Tz)$$

Which is a contradiction, therefore Tz= z, i.e. z is a fixed point of Tz.
For uniqueness, let $w(z\neq w)$ be another fixed point of T. Then we have

$$d(z,w)=d(Tz,Tw)$$
$$\leq C_1\;\max\{d(z,Tz),\;d(w,Tw),d(z,w)\}$$
$$+\;C_2\;\max\{d(z,Tz)\;+d(z,w),d(w,Tw)+d(z,w)\}$$
$$+C_3 \frac{d(z,w)\,[1+\sqrt{d(z,w)d(z,Tz)}+\sqrt{d(z,w)d(w,Tz)}]^2}{[1+d(z,w)+d(z,Tz)d(z,Tw)d(w,Tz)\;d(w,Tw)]^2}$$
$$\leq C_1\;d(z,w)\;+\;C_2\;d(z,w)\;\;+\;C_3\;d(z,w)$$
$$d(z,w)\;\leq(C_1+C_2+C_3)d(z,w)$$

Which is a contradiction, since $0\leq\; C_1+C_2+C_3<1$ and therefore z = w. Thus the fixed point is unique.

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