JOURNAL OF COMPUTER AND MATHEMATICAL SCIENCES
An International Open Free Access, Peer Reviewed Research Journal
www.compmath-journal.org

ISSN 0976-5727 (Print)
ISSN 2319-8133 (Online)
Abbr:J.Comp.&Math.Sci.
2014, Vol.5(3): Pg.310-318

Ranking Algorithm for Symmetric Octagonal Fuzzy Numbers

Dhanalakshmi V. and Felbin C. Kennedy

Research Scholar,
Stella Maris College (Autonomous), Chennai, INDIA.
Research Guide, Associate Professor,
Stella Maris College (Autonomous), Chennai, INDIA.

(Received on: June 27, 2014)

ABSTRACT

Ranking of fuzzy numbers play an important role in decision making, optimization, forecasting etc. Fuzzy numbers must be ranked before an action is taken by a decision maker. In this paper, we propose a ranking algorithm for symmetric octagonal fuzzy numbers, which results in equality only when the numbers coincide. Also the ranking algorithm is verified for some reasonable properties of ordering of fuzzy numbers

Keywords: Symmetric Octagonal fuzzy number, ranking, mode, deviation, spread.

1. INTRODUCTION

Ranking fuzzy numbers is an important task in analyzing fuzzy information in optimization, data mining, decision making and related areas. Unlike real numbers, fuzzy numbers have no natural order; also no ordering technique is conducive to all. Thus significant contributions have been made in ranking fuzzy numbers[1–7,10,12,13]. All the ranking procedures in literature defines $\tilde{A} \succ \tilde{B}$, $\tilde{A} \prec \tilde{B}$ and $\tilde{A} \approx \tilde{B}$, here $\tilde{A} \approx \tilde{B}$ implies the two fuzzy numbers are equivalent, not necessarily be equal. That is, two different fuzzy numbers depending on the ranking method are infered as equivalent. But in many applications the two fuzzy numbers need to be shown different. In this paper, we have proposed a ranking algorithm for symmetric octagonal fuzzy number, in which we obtain the equality only when the two symmetric octagonal fuzzy numbers coincide. Some of the reasonable properties for ranking fuzzy numbers proposed in[11] are verified for this algorithm.

After the introduction, this paper is arranged as follows. In Section 2, the concept of fuzzy number and octagonal fuzzy numbers are recalled. Section 3 introduces the ranking algorithm for symmetric octagonal fuzzy numbers and the properties of the proposed algorithm are discussed in Section 4. The concluding remarks are presented in Section 5.

2. SYMMETRIC OCTAGONAL FUZZY NUMBERS

For the sake of completeness we recall the required definitions and results.

Definition 2.1 [8]The characteristic function $\mu_{\tilde{A}}$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ is called a fuzzy set.

Definition 2.2 [8]A fuzzy set $\tilde{A}$,defined on the universal set of real numberR, is said to be a fuzzy number if its membership function has the following characteristics:

i. $\tilde{A}$ is convex i.e.

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$$
$$\forall x_1, x_2 \in \mathrm{R}, \forall \lambda \in [0,1]$$

ii. $\tilde{A}$ is normal i.e. $\exists x_0 \in \mathbb{R}$such that $\mu_{\tilde{A}}(x_0) = 1$

iii. $\mu_{\tilde{A}}$ is piecewise continuous

Definition 2.3[9]A fuzzy number $\tilde{A}$ is said to be a generalized octagonal fuzzy number denoted by

$$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$$

where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, k, w$ are real numbers such that

$$a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8$$

and $0 < k < w$ and its membership function $\mu_{\tilde{A}}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq a_1 \\ k\dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ k & a_2 \leq x \leq a_3 \\ \dfrac{k(a_4-x)+w(x-a_3)}{a_4-a_3} & a_3 \leq x \leq a_4 \\ w & a_4 \leq x \leq a_5 \\ \dfrac{k(x-a_5)+w(a_6-x)}{a_6-a_5} & a_5 \leq x \leq a_6 \\ k & a_6 \leq x \leq a_7 \\ k\dfrac{x-a_8}{a_8-a_7} & a_7 \leq x \leq a_8 \\ 0 & x \geq a_8 \end{cases}$$

Definition 2.5A fuzzy number $\tilde{A}$ is said to be a symmetric octagonal fuzzy number denoted by $\tilde{A} = (a_L, a_U, r, s, t; k, w)$ where $a_L \leq a_U$ and r, s, t are real numbers such that its representation as generalized octagonal fuzzy number is

$(a_L - r - s - t, a_L - s - t, a_L - t, a_L, a_U, a_U + t,$
$a_U + s + t, a_U + r + s + t; k, w).$

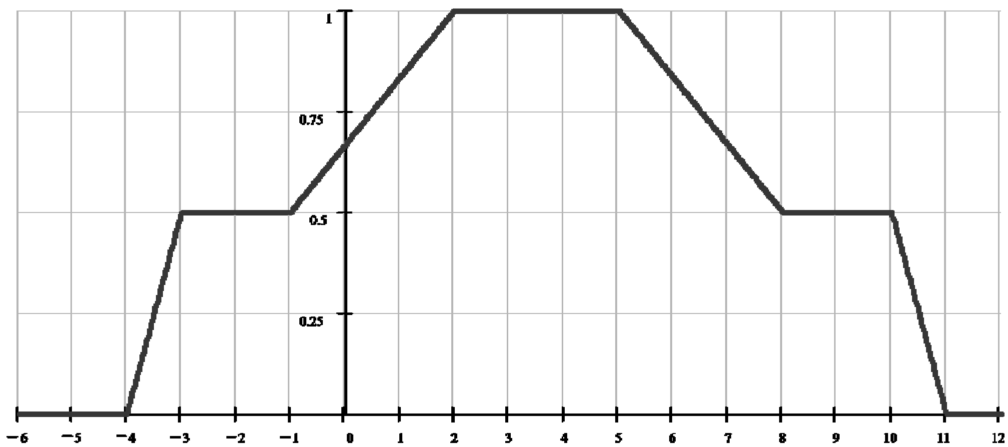

Figure 1: Graphical representation of the symmetric octagonal fuzzy number $(2,5,1,2,3;\frac{1}{2},1)$

Definition 2.6 The α-cut of the symmetric octagonal fuzzy number $\tilde{A} = (a_L, a_U, r, s, t; k, w)$ given in Definition 2.5 is

$$\tilde{A} = [A_\alpha^L, A_\alpha^R] = \begin{cases} (A_\alpha^L)_1, (A_\alpha^R)_1] & \alpha \in [0,k] \\ (A_\alpha^L)_2, (A_\alpha^R)_2] & \alpha \in (k,1] \end{cases}$$

where

$$(A_\alpha^L)_1 = a_L - s - t - r\frac{k-\alpha}{k}, \qquad (A_\alpha^L)_2 = a_L - t\frac{w-\alpha}{w-k},$$
$$(A_\alpha^R)_1 = a_U + s + t + r\frac{k-\alpha}{k}, \qquad (A_\alpha^R)_2 = a_U + t\frac{w-\alpha}{w-k}$$

Definition 2.7 Let $\tilde{A} = (a_L, a_U, r_1, s_1, t_1; k_1, w_1)$ and $\tilde{B} = (b_L, b_U, r_2, s_2, t_2; k_2, w_2)$ be two symmetric octagonal fuzzy numbers, then their sum is defined as

$$\tilde{A} + \tilde{B} = (a_L + b_L, a_U + b_U, r_1 + r_2, s_1 + s_2, t_1 + t_2; \min(k_1, k_2), \min(w_1, w_2))$$

3. RANKING OF SYMMETRIC OCTAGONAL FUZZY NUMBERS

Definition 3.1 For any symmetric octagonal fuzzy number $\tilde{A} = (a_L, a_U, r, s, t; k, w)$ mode, divergence & spread at *w* and *k* level are defined as :

i. $w - Mode(\tilde{A}) = \dfrac{w(a_L + a_U)}{2}$

ii. $w - Divergence(\tilde{A}) = w(a_U - a_L + 2t)$

iii. $Spread_{k,w}(\tilde{A}) = wt$

iv. $k - Mode(\tilde{A}) = \dfrac{k(a_L + a_U)}{2}$

v. $k - rightmode(\tilde{A}) = \dfrac{k(2a_U + 2t + s)}{2}$

vi. $k-leftmode(\tilde{A}) = \frac{k(2a_L - 2t - s)}{2}$

vii. $0-Divergence(\tilde{A}) = k(a_U - a_L + 2r + 2s + 2t)$

viii. $Spread_{0,k}(\tilde{A}) = k\,r$

Given any two symmetric octagonal fuzzy numbers we present here an algorithm to compare them.

ALGORITHM

Let $\tilde{A} = (a_L, a_U, r_1, s_1, t_1; k_1, w_1)$ and $\tilde{B} = (b_L, b_U, r_2, s_2, t_2; k_2, w_2)$ be two symmetric octagonal fuzzy numbers, then use the following steps to compare $\tilde{A}$ and $\tilde{B}$

Step1: Find $w-Mode(\tilde{A})$ and $w-Mode(\tilde{B})$

Case(i): If $w-Mode(\tilde{A}) > w-Mode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $w-Mode(\tilde{A}) < w-Mode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $w-Mode(\tilde{A}) = w-Mode(\tilde{B})$ then go to Step 2

Step2: Find $w-Divergence(\tilde{A})$ and $w-Divergence(\tilde{B})$

If $w-Mode(\tilde{A}) \geq 0$ then

Case(i): If $w-Divergence(\tilde{A}) > w-Divergence(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $w-Divergence(\tilde{A}) < w-Divergence(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $w-Divergence(\tilde{A}) = w-Divergence(\tilde{B})$ then go to Step 3

else

Case(i): If $w-Divergence(\tilde{A}) < w-Divergence(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii) : If $w-Divergence(\tilde{A}) > w-Divergence(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $w-Divergence(\tilde{A}) = w-Divergence(\tilde{B})$ then go to Step 3

Step 3: Find $Spread_{k,w}(\tilde{A})$ and $Spread_{k,w}(\tilde{B})$

If $w-Mode(\tilde{A}) \geq 0$ then

Case(i): If $Spread_{k,w}(\tilde{A}) < Spread_{k,w}(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $Spread_{k,w}(\tilde{A}) > Spread_{k,w}(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $Spread_{k,w}(\tilde{A}) = Spread_{k,w}(\tilde{B})$ then go to Step 4

else

Case(i): If $Spread_{k,w}(\tilde{A}) > Spread_{k,w}(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $Spread_{k,w}(\tilde{A}) < Spread_{k,w}(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $Spread_{k,w}(\tilde{A}) =$

$Spread_{k,w}(\tilde{B})$ then go to Step 4

Step 4: Find w_1 and w_2

Case (i): If $w_1 > w_2$ then $\tilde{A} \succ \tilde{B}$

Case (ii): If $w_1 < w_2$ then $\tilde{A} \prec \tilde{B}$

Case (iii): If $w_1 = w_2$ then go to Step 6

Step 5: Find $k - Mode(\tilde{A})$ and $k - Mode(\tilde{B})$

Case(i): If $k - Mode(\tilde{A}) > k - Mode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $k - Mode(\tilde{A}) < k - Mode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $k - Mode(\tilde{A}) = k - Mode(\tilde{B})$ then go to Step 6

Step 6: Find $k - rightmode(\tilde{A})$ and $k - rightmode(\tilde{B})$

Case(i): If $k - rightmode(\tilde{A}) > k - rightmode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $k - rightmode(\tilde{A}) < k - rightmode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $k - rightmode(\tilde{A}) = k - rightmode(\tilde{B})$ then go to Step 7

Step7: Find $k - leftmode(\tilde{A})$ and $k - leftmode(\tilde{B})$

Case(i): If $k - leftmode(\tilde{A}) > k - leftmode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $k - leftmode(\tilde{A}) < k - leftmode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $k - leftmode(\tilde{A}) = k - leftmode(\tilde{B})$ then go to Step 8

Step 8: Find $0 - Divergence(\tilde{A})$ and $0 - Divergence(\tilde{B})$

Case(i): If $0 - Divergence(\tilde{A}) > 0 - Divergence(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case(ii): If $0 - Divergence(\tilde{A}) < 0 - Divergence(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case(iii): If $0 - Divergence(\tilde{A}) = 0 - Divergence(\tilde{B})$ then go to Step 9

Step 9: Find k_1 and k_2

Case (i): If $k_1 > k_2$ then $\tilde{A} \succ \tilde{B}$

Case (ii): If $k_1 < k_2$ then $\tilde{A} \prec \tilde{B}$

Case (iii): If $k_1 = k_2$ then $\tilde{A} \approx \tilde{B}$

4. PROPERTIES OF THE RANKING ALGORITHM

Proposition 4.1: If equality holds in all the nine steps of the above algorithm, then the two symmetric octagonal fuzzy numbers are equal.

Proof:

Let $\tilde{A} = (a_L, a_U, r_1, s_1, t_1; k_1, w_1)$ and $\tilde{B} = (b_L, b_U, r_2, s_2, t_2; k_2, w_2)$ be two symmetric octagonal fuzzy numbers, such that all the nine steps of the above algorithm

leads equality. To prove that $\tilde{A}$ and $\tilde{B}$ are equal, i.e. to prove that
$a_L = b_L$, $a_U = b_U$, $r_1 = r_2$, $s_1 = s_2$, $t_1 = t_2$,
$k_1 = k_2$, $w_1 = w_2$.

From step 4,

$$w_1 = w_2 \qquad (1)$$

From step 9,

$$k_1 = k_2 \qquad (2)$$

From step 3 and equation (1),

$$t_1 = t_2 \qquad (3)$$

From step 1 and equation (1),

$$w-Mode(\tilde{A}) = w-Mode(\tilde{B})$$
$$\Rightarrow \frac{w_1(a_L + a_U)}{2} = \frac{w_2(b_L + b_U)}{2} \qquad (4)$$
$$\Rightarrow a_L + a_U = b_L + b_U$$

From step 2 and equation (1) and (3),

$$w-Divergence(\tilde{A}) = w-Divergence(\tilde{B})$$
$$\Rightarrow w_1(a_U - a_L + 2t_1) = w_2(b_U - b_L + 2t_2) \qquad (5)$$
$$\Rightarrow a_U - a_L = b_U - b_L$$

From equations (4) and (5),

$$a_U = b_U,\ a_L = b_L \qquad (6)$$

From step 6 and equations (3) and (6),

$$k-rightmode(\tilde{A}) = k-rightmode(\tilde{B})$$
$$\Rightarrow \frac{k_1(2a_U + 2t_1 + s_1)}{2} = \frac{k_2(2b_U + 2t_2 + s_2)}{2} \qquad (7)$$
$$\Rightarrow s_1 = s_2$$

From step 8 and equations (3), (6) and (7),

$$0-Divergence(\tilde{A}) = 0-Divergence(\tilde{B})$$
$$\Rightarrow k_1(a_U - a_L + 2r_1 + 2s_1 + 2t_1)$$
$$= k_2(b_U - b_L + 2r_2 + 2s_2 + 2t_2)$$
$$\Rightarrow r_1 = r_2$$

Hence the proof.

Remark 4.2: In the above theorem, we have proved that equality, not just the equivalence of symmetric octagonal fuzzy numbers is obtained by the proposed algorithm.

Proposition 4.3: Let
$\tilde{A} = (a_L, a_U, r_1, s_1, t_1; k_1, w_1)$,
$\tilde{B} = (b_L, b_U, r_2, s_2, t_2; k_2, w_2)$ and
$\tilde{C} = (c_L, c_U, r_3, s_3, t_3; k_3, w_3)$ be three symmetric octagonal fuzzy numbers such that $\tilde{A} \succ \tilde{B}$ and $\tilde{B} \succ \tilde{C}$, then $\tilde{A} \succ \tilde{C}$.

Proof:

Case (i):

Let the order be determined in step 1, i.e
$\tilde{A} \succ \tilde{B}$ by $w-Mode(\tilde{A}) > w-Mode(\tilde{B})$
and $\tilde{B} \succ \tilde{C}$ *by*
$w-Mode(\tilde{B}) > w-Mode(\tilde{C})$,
then

$$w-Mode(\tilde{A}) > w-Mode(\tilde{B}) > w-Mode(\tilde{C})$$
$$\Rightarrow w-Mode(\tilde{A}) > w-Mode(\tilde{C})$$
$$\Rightarrow \tilde{A} \succ \tilde{C}$$

Case (ii):

Suppose $\tilde{A} \succ \tilde{B}$ by step 1 and $\tilde{B} \succ \tilde{C}$ in step *i* say *i*=2,3,...,9
i.e. $w-Mode(\tilde{A}) > w-Mode(\tilde{B})$ and

$w-Mode(\widetilde{B})=w-Mode(\widetilde{C})$ and the inequality happens in the later steps, then

$w-Mode(\widetilde{A})>w-Mode(\widetilde{B})=w-Mode(\widetilde{C})$

$\Rightarrow w-Mode(\widetilde{A})>w-Mode(\widetilde{C})$

$\Rightarrow \widetilde{A}\succ\widetilde{C}$

Similarly, $\widetilde{A}\succ\widetilde{C}$ if

$w-Mode(\widetilde{A})=w-Mode(\widetilde{B})$ and

$w-Mode(\widetilde{B})>w-Mode(\widetilde{C})$

Case (iii):

Similar to *case (ii)*, it can be proved that $\widetilde{A}\succ\widetilde{C}$ if $\widetilde{A}\succ\widetilde{B}$ by step *i* and $\widetilde{B}\succ\widetilde{C}$ in step *j* where *j*=*i*+1,...,9or $\widetilde{B}\succ\widetilde{C}$ by step *i* and $\widetilde{A}\succ\widetilde{B}$ in step *j* where *j*=*i*+1,...,9.

Thus in all possibilities, $\widetilde{A}\succ\widetilde{B}$ *and* $\widetilde{B}\succ\widetilde{C}$ $\Rightarrow \widetilde{A}\succ\widetilde{C}$.

Hence the proof.

Proposition 4.4: Let $\widetilde{A}$ and $\widetilde{B}$ be two symmetric octagonal fuzzy numbers with same height and

inf supp($\widetilde{A}$) > sup supp($\widetilde{B}$), then $\widetilde{A}\succ\widetilde{B}$.

Proof:

Let $\widetilde{A}=(a_L,a_U,r_1,s_1,t_1;k_1,w_1)$ and $\widetilde{B}=(b_L,b_U,r_2,s_2,t_2;k_2,w_2)$ be the two given symmetric octagonal fuzzy numbers.

Since they have the same height, $w_1=w_2$.

inf supp($\widetilde{A}$) > sup supp($\widetilde{B}$)

$\Rightarrow a_L-r_1-s_1-t_1>b_U+r_2+s_2+t_2$

$\Rightarrow a_L>a_L-r_1-s_1-t_1>b_U+r_2+s_2+t_2>b_U$

$$\Rightarrow a_L>b_U \tag{8}$$

$\Rightarrow a_U>a_L>b_U>b_L$, by definition of $\widetilde{A}$ and $\widetilde{B}$

$$\Rightarrow a_U>b_L \tag{9}$$

Adding (8) and (9),

$a_U+a_L>b_L+b_U$

$\Rightarrow w-Mode(\widetilde{A})>w-Mode(\widetilde{B})$

$\Rightarrow \widetilde{A}\succ\widetilde{B}.$

Hence the proof.

Proposition 4.5: Let $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$ be three symmetric octagonal fuzzy numbers with same k and w then $\widetilde{A}\succ\widetilde{B}$

$\Rightarrow \widetilde{A}+\widetilde{C}\succ\widetilde{B}+\widetilde{C}.$

Proof: Let $\widetilde{A}=(a_L,a_U,r_1,s_1,t_1;k_1,w_1)$, $\widetilde{B}=(b_L,b_U,r_2,s_2,t_2;k_2,w_2)$ and $\widetilde{C}=(c_L,c_U,r_3,s_3,t_3;k_3,w_3)$ be the three given symmetric octagonal fuzzy numbers. By hypothesis, $k_1=k_2=k_3$ and $w_1=w_2=w_3$ Suppose $\widetilde{A}\succ\widetilde{B}$ happens in step 1, then

$w-Mode(\widetilde{A})>w-Mode(\widetilde{B})$

$\Rightarrow \frac{w_1(a_L + a_U)}{2} > \frac{w_2(b_L + b_U)}{2}$

$\Rightarrow a_L + a_U > b_L + b_U$

$\Rightarrow a_L + a_U + c_L + c_U > b_L + b_U + c_L + c_U$, adding $c_L + c_U$ both sides

$\Rightarrow \frac{\min(w_1, w_3)(a_L + a_U + c_L + c_U)}{2} > \frac{\min(w_2, w_3)(b_L + b_U + c_L + c_U)}{2}$

$\Rightarrow w - Mode(\tilde{A} + \tilde{C}) > w - Mode(\tilde{B} + \tilde{C})$

$\Rightarrow \tilde{A} + \tilde{C} \succ \tilde{B} + \tilde{C}$.

Suppose $\tilde{A} \succ \tilde{B}$ happens in step 2, then $w - Mode(\tilde{A}) = w - Mode(\tilde{B})$ and $w - Divergence(\tilde{A}) > w - Divergence(\tilde{B})$

$w - Mode(\tilde{A}) = w - Mode(\tilde{B})$

$\Rightarrow \frac{w_1(a_L + a_U)}{2} = \frac{w_2(b_L + b_U)}{2}$

$\Rightarrow a_L + a_U = b_L + b_U$

$\Rightarrow a_L + a_U + c_L + c_U = b_L + b_U + c_L + c_U$

$\Rightarrow w - Mode(\tilde{A} + \tilde{C}) = w - Mode(\tilde{B} + \tilde{C})$

and

$w - Divergence(\tilde{A}) > w - Divergence(\tilde{B})$

$\Rightarrow a_U - a_L + 2t_1 > b_U - b_L + 2t_2$

$\Rightarrow a_U - a_L + 2t_1 + c_U - c_L + 2t_3 > b_U - b_L + 2t_2 + c_U - c_L + 2t_3$

$\Rightarrow w - Divergence(\tilde{A} + \tilde{C}) > w - Divergence(\tilde{B} + \tilde{C})$

$\Rightarrow \tilde{A} + \tilde{C} \succ \tilde{B} + \tilde{C}$.

In a similar manner, the result can be proved for all cases.

Remark 4.6: The above Proposition may not hold for arbitrary *k* and *w*, for

Let $\tilde{A} = (1,4,1,1,1;\frac{1}{2},1)$, $\tilde{B} = (2,4,1,1,1;\frac{1}{4},\frac{1}{2})$ and $\tilde{C} = (1,2,1,1,1;\frac{1}{4},\frac{1}{2})$, then $\tilde{A} \succ \tilde{B}$ as

$w - Mode(\tilde{A}) = \frac{5}{2} > \frac{5}{4} = w - Mode(\tilde{B})$,

but $\tilde{A} + \tilde{C} = (2,6,2,2,2;\frac{1}{4},\frac{1}{2})$ and

$\tilde{B} + \tilde{C} = (3,6,2,2,2;\frac{1}{4},\frac{1}{2})$

$\Rightarrow w - Mode(\tilde{A} + \tilde{C}) = 2 < \frac{9}{4} = w - Mode(\tilde{B} + \tilde{C})$

$\Rightarrow \tilde{A} + \tilde{C} \prec \tilde{B} + \tilde{C}$.

5. CONCLUSION

In this paper, an algorithm for pairwise comparison of symmetric octagonal fuzzy numbers is introduced. A strong property of uniqueness of a symmetric octagonal fuzzy numbers is established in Proposition 4.1. Also some of the reasonable properties of ranking fuzzy numbers were verified.

REFERENCES

1. Abbasbandy S. and B. Asady, Ranking of fuzzy numbers by sign distance, *Information Sciences* 176(16), 2405-2416, (2006).
2. Asady, B. and Zendehnam, A. Ranking fuzzy numbers by distance minimization.*Applied Mathematical Modelling*.31: 2589-2598 (2007).

3. Bortolan G. and R. Degani, A review of some methods for ranking fuzzy numbers, *Fuzzy Sets and Systems* 15, 1-19, (1985).
4. Chen, L-H., Lu, H-W., An approximate approach for ranking fuzzy numbers based on left and right dominance, *Computers and Mathematics with Applications*, Vol. 41, pp.1589-1602 (2001).
5. Chu T.C. and C. T. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, *Computers and Mathematics with Applications* 43, 111-117, (2002).
6. Deng, Y., Z. Zhenfu, L. Qi, Ranking fuzzy numbers with an area method using radius of gyration, *Computers and Mathematics with Applications*, 51: 1127-1136 (2006).
7. Dubois D. and H. Prade, Ranking fuzzy numbers in the setting of possibility theory, *Information Sciences* 30, 183-224, (1983).
8. Klir George J and Bo Yuan, Fuzzysets and Fuzzylogic: Theory and Applications, *Prentice Hallof India*, (1997).
9. Malini S. U., Kennedy Felbin C., An Approach for Solving Fuzzy Transportation Problem Using Octagonal Fuzzy Numbers, *Applied Mathematical Sciences*, Vol. 7, pp. 2661-2673 (2013).
10. S. H. Nasseri and M. Sohrabi, Hadi's method and its advantage in ranking fuzzy numbers, *Australian Journal of Basic Applied Sciences* 4(10), 4630-4637, (2010).
11. X. Wang and E. E. Kerre, Reasonable properties for the ordering of fuzzy quantities , *Fuzzy Sets and Systems*, 118, 378-405, (2001).
12. Z.X. Wang, Y.J. Liu, Z.P. Fan and B. Feng, Ranking L-R fuzzy number based on deviation degree, *Information Sciences*, 179, 2070-2077 (2009).
13. Y. -M. Wang, J. -B. Yang, D. -L. Xu, and K. S. Chin, On the centroid of fuzzy numbers, *Fuzzy Sets and Systems*, 157, 919-926, (2006).