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The New Algebraic Integers from Reciprocal Polynomial of Cyclotomic Graphs

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ABSTRACT

The simple graphs of order up to 5 classify through Perron number and N–graph, α- number(s) and αβ- numbers is defined. The new algebraic integers get by the transformation *z + (1/z) + n***,** where n = 0, 1, 2, 3, 4, 5 in the characteristic equation of the simple graphs and find the Mahler measure of the Reciprocal polynomials and fit the comparison diagram between transformations.

MSC Code: 11R04, 11R06, 15A18

Keywords: algebraic integer, Mahler measure, Perron number.

1. INTRODUCTION

Mahler[5] introduced Mahler measure M(A). It was applied by Mckee and Smyth[2]. Smith[4] classified of some graphs having maximum Eigen value atmost 2. cyclotomic matrices were worked in Mckee and Smyth[3] who classified all cyclotomic matrices over the integer. Salem number is a real algebraic integer greater than 1 whose conjugate roots all have modulus less than or equal to 1, and at least one of which has modulus exactly 1. A Reciprocal polynomial is a polynomial of degree n such that $x^n P\left(\frac{1}{x}\right) = \pm P(x)$ .Mckee and Smyth[1] applied the transformation $z + \frac{1}{z} + 2$ in the minimal polynomial of totally positive algebraic integers and got the reciprocal polynomial. Salem numbers can get from it. A graph whose adjacency matrix has only integral Eigen values is called the integral graph. The Perron number of a graph is defined to be the spectral radius of its adjacency matrix. In 1907, Perron proved the theorem in which the largest Eigen value is called Perron Number. Thereafter in 1912, Frobenius proved the same theorem named as Perron– Frobenious theorem. Pisot number is a real algebraic integer greater than 1 all of whose conjugate roots are less than 1 in modulus. Rameshkumar and Nagarajan[6] classified the simple graphs into cyclotomic graphs and Non–cyclotomic graphs. They named the special graphs for the cyclotomic graph whose Perron number is 2. The square root of any integer is subset of a cyclotomic field, but other nth roots of any integer are not in cyclotomic extension. Rameshkumar and Nagarajan[7] found the Mahler measure of the charged graphs with edge label cube root of 2. Nagarajan and Rameshkumar[8] introduced mixed graphs with edge label from the cubic field.

Here the N–graphs of order up to 6 is determined and get the new algebraic numbers from the reciprocal polynomial by the transformation $z+\frac{1}{z}+n$, where n = 0,1,2,3,4,5 **.** The α-number(s) and $\alpha\beta$-numbers is defined and plot the comparison diagrams between transformations.

2. PRELIMINARIES

Definition 2.1

For any positive integer n, the n^{th} cyclotomic polynomial, is the unique irreducible polynomial with integral coefficients, which is a divisor of $x^n - 1$ and is not a divisor of $x^r - 1$ for any $r < n$.

Definition 2.2

Let $P(z) = z^d + ... + a_d = \prod_{i=1}^{d}(z - \alpha_i)$ be a monic non–constant polynomial. The Mahler measure is $M(P) = \prod_{i=1}^{d}\max(1, |\alpha_i|)$. M (P) = 1 iff P is cyclotomic.

Definition 2.3

If A is an n × n integer symmetric matrix, then its associated polynomial is defined as $R_A(z) = z^n\chi_A(z + (1/z))$. If A has all Eigen values in [–2,2], then R_A is a cyclotomic polynomial. We call A as a cyclotomic matrix.

Definition 2.4

A Symmetric matrix A is called cyclotomic if its associated reciprocal polynomial R_A has integer coefficients and Mahler measure $M(R_A) = 1$. The adjacency graph of the cyclotomic matrix is called cyclotomic graph. The name cyclotomic come from Kronecker's theorem, R_A is a product of cyclotomic polynomials.

3. N GRAPHS

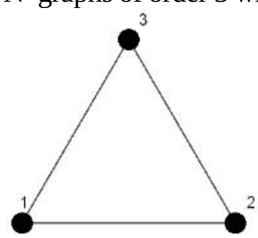
3.1 N graphs of order 3

Definition 3.1.1

Let A be set of all {0,1}symmetric matrices with all diagonal entries are zero and Let A_r, where r = 1, be the simple graph of order 2 for the adjacency matrices of order 2 in A. Let B_r , where r = 1,2,…,7 , be the simple graphs of order 3 for the adjacency matrix A. Define C_r, where r = 1,2,…,62 , be the simple graphs of order 4 for the adjacency matrices of order 4 in A. The N–graph is defined as the cyclotomic graph whose Perron number is 2.

A_1 is the connected cyclotomic integral graph having Perron number 1. There is no N–graphs of order 2.

The graphs (B_1,B_2,B_3,B_4,B_5,B_6,B_7) having Perron numbers (1,1,1, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$,2). The graphs B_1 to B_3 are disconnected cyclotomic graphs. The graphs B_4, B_5, B_6, B_7 are connected cyclotomic graphs. There is only one N–graphs of order 3 which is integral graph as shown below.



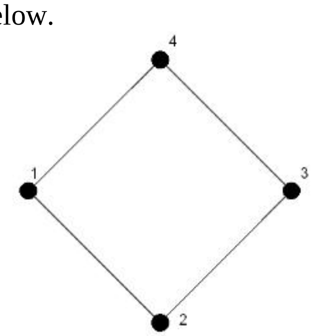
B_7

Here B_1, B_2, B_3 are integral graphs.

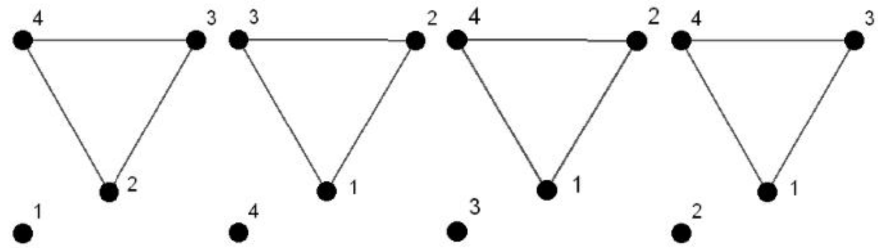
3.2 N graphs of order 4

Definition 3.2.1

The graphs ($C_{1–6}$, $C_{7–18}$, $C_{19–21}$, $C_{22–25}$, $C_{26–29}$, $C_{30–41}$, $C_{42–53}$, $C_{54–55}$, $C_{56–61}$, C_{62}) having Perron numbers (1, $\sqrt{2}$, 1, 2, $\sqrt{3}$, 1.61803, 2.17009, 2, 2.56155, 3) approximately , where $C_{i–j}$ represents the graphs from C_i to C_j and i > j . The graphs C_1 to C_{25} are disconnected cyclotomic graphs. The graphs $C_{26–41}$ and $C_{54–55}$ are connected cyclotomic graphs. The graphs $C_{42–53}$ and $C_{56–62}$ are connected non–cyclotomic graphs. The graphs C_{54} and C_{55} are same. There are five N–graphs of order 4 as shown below.



C_{55}



C_{22} C_{23} C_{24} C_{25}

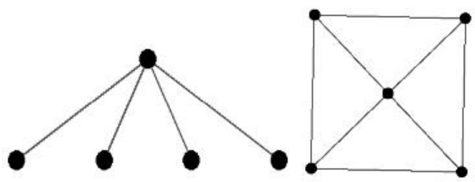
Here C_{1-6} , C_{19-21} , C_{22-25} , C_{55} and C_{62} are integral graphs. The Perron number of the graphs C_{30-41} are golden ratio.

3.3 N graphs of order 5

Definition 3.3.1

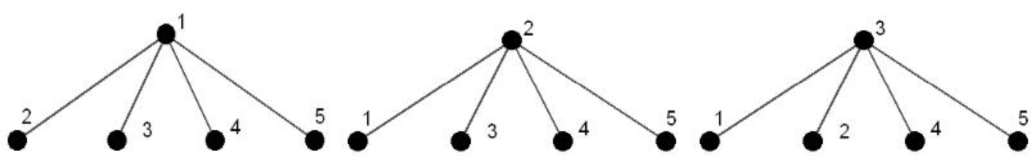
Let $D^{j}{}_{r}$, r = 1,2,…,36 be the Simple graphs of order 5 for the adjacency matrices of order 5 in A. The number of ‘j’ values for the particular graph for the corresponding value r = 1,2,…36 depending upon how many of that graphs got when representing the numbers 1,2,3,4,5 for the five vertices.

Examples

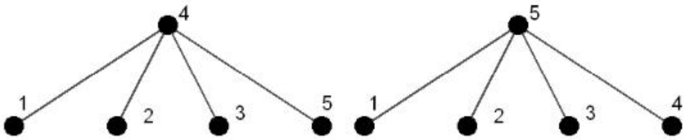


$D^{1}{}_{3}, j = 1\ to\ 5$ $D^{j}{}_{12}, j = 1\ to\ 13$

The above first graph represents following five graphs.



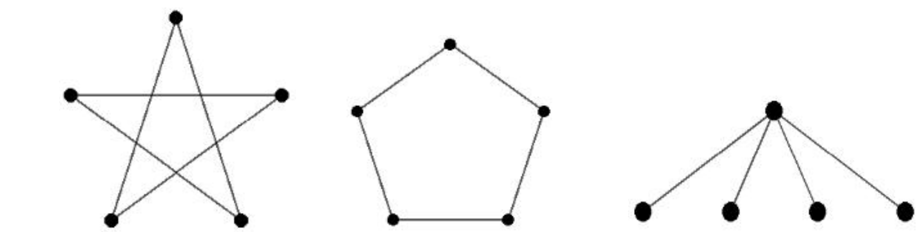
$D^{1}{}_{3}$ $D^{2}{}_{3}$ $D^{3}{}_{3}$



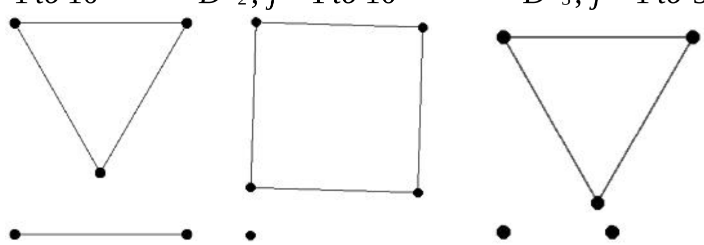
$D^{4}{}_{3}$ $D^{5}{}_{3}$

The simple graphs of order 5 can be classify with Perron number as follows as ($D^{j}{}_{1-3}$, $D^{j}{}_{4-6}$, $D^{j}{}_{7}$, $D^{j}{}_{8}$, $D^{j}{}_{9-10}$, $D^{j}{}_{11}$, $D^{j}{}_{12}$, $D^{j}{}_{13}$, $D^{j}{}_{14}$, $D^{j}{}_{15}$, $D^{j}{}_{16}$, $D^{j}{}_{17}$, $D^{j}{}_{18}$, $D^{j}{}_{19}$, $D^{j}{}_{20}$, $D^{j}{}_{21}$, $D^{j}{}_{22}$, $D^{j}{}_{23}$, $D^{j}{}_{24}$, $D^{j}{}_{25-27}$, $D^{j}{}_{28}$, $D^{j}{}_{29}$, $D^{j}{}_{30-31}$, $D^{j}{}_{32-33}$, $D^{j}{}_{34}$ $D^{j}{}_{35}$, $D^{j}{}_{36}$) having

Perron numbers (2, 1.84776, 1.73205, 4, 3.64575, 3.3234, 3.23607, 3.08613, 3, 2.93543, 2.85577, 2.68554, 2.64119, 2.56155, 2.48119, 2.44949, 2.30278, 2.21432, 2.13578, 2, 1.73205,1.61803, 1.41412, 1, 3, 2.56155, 2.17009) approximately. The graphs $D^j{}_{1-7}$ are connected cyclotomic graphs. The graphs $D^j{}_{8-24}$ are connected non–cyclotomic graphs. The graphs $D^j{}_{25-33}$ are disconnected cyclotomic graphs. The graphs $D^j{}_{34-36}$ are disconnected non–cyclotomic graphs. There are six N–graphs of order 5 as shown below.



$D^1{}_1, j=1\ to\ 10$ $D^j{}_2, j=1\ to\ 10$ $D^j{}_3, j=1\ to\ 5$



$D^j{}_{25}, j=1\ to\ 7$ $D^j{}_{26}, j=1\ to\ 13$ $D^j{}_{27}, j=1\ to\ 8$

Here $D^j{}_{1-2}, D^j{}_8, D^j{}_{14}, D^j{}_{25}, D^j{}_{26}, D^j{}_{27}, D^j{}_{32}, D^j{}_{33}, D^j{}_{34}, D^j{}_{34}$ are integral graphs.

3.4 N graphs of order 6

Definition 3.4.1

Let $E^j{}_{r\,,}$ r = 1,2,…,n be the Simple graphs of order 6 for the matrix A. The number of ‘j’ values for the particular graph for the corresponding value r = 1,2,…n depending upon how many of that graphs got when represent the numbers 1,2,3,4,5,6 for the five vertices. There are five N–graphs of order 6 as shown below.

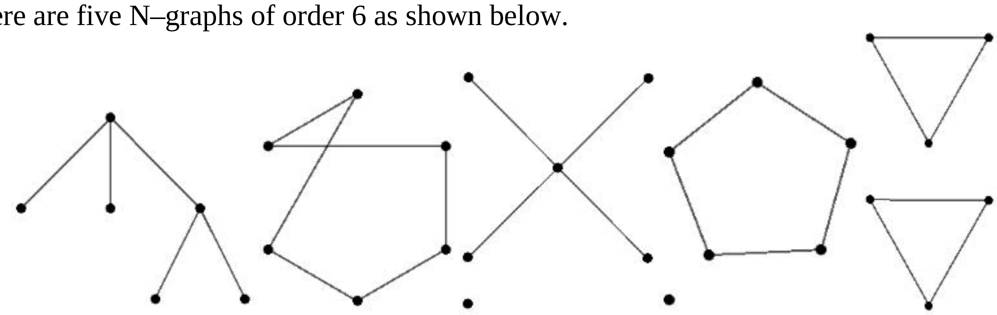


Fig 1

4. THE NEW ALGEBRAIC INTEGERS BY THE TRANSFORMATION $z+\frac{1}{z}+n$

A negative Salem numbers is a real algebraic integer α which is less than –1 whose conjugate roots all have modulus less than or equal to 1 , except for 1/α. The following definitions are developed by idea of this number.

Definition 4.1

The 'n' real algebraic integer(s) α's less than –1 whose conjugate roots all have modulus less than or equal to 1. They are denoted by $\langle \sim_1, \sim_2, \sim_3, ..., \sim_n \rangle$. It is called the α-number(s). $N = \min(\sim_1, \sim_2, \sim_3, ..., \sim_n)$

Definition 4.2

The 'n' real algebraic integer(s) α's (less than –1) and β (greater than 1) whose conjugate roots all have modulus less than or equal to 1 .They are denoted by $\langle \sim_1, \sim_2, \sim_3, ..., \sim_n, \quad \rangle$. It is called the αβ- numbers. $N = \min(\sim_1, \sim_2, \sim_3, ..., \sim_n, \quad)$.

4.1 Cyclotomic graphs

Mckee and Smyth[9] defined Mahler measure of the graph is the Mahler measure of R_A for the matrix A. Apply the transformation $z+\frac{1}{z}+n$ to the characteristic equation of the adjacency matrix of the simple graph and clear denominators. Then find the Mahler measure of this reciprocal polynomial.

For n = 0, this reciprocal polynomial R_A is the Mahler measure of the graph M(G) for the matrix A. For the values n = 1 to 5, this Mahler measure of the reciprocal polynomials is denoted by M.

~ - numbers from graph $D^{j}{}_{12}, j=1 \; to \; 13$

The characteristic equation of graphs $D^{j}{}_{12}, j=1 \; to \; 13$ is $a^3 - 8a^3 - 8a^2 = 0$. Substitute *z + (1/z) + 1* for λ and get the reciprocal polynomial $z^{10} + 5z^9 + 7z^8 - 2z^7 - 19z^6 - 29z^5 - 19z^4 - 2z^3 + 7z^2 + 5z + 1 = 0$. Its approximate roots are –2.61803, –1.61803, –0.61803, –0.5 ± 0.86602 i, –0.5 ± 0.86602 i , –0.38196, 0.61803, 1.61803. The algebraic integer $\langle \sim_1, \quad \rangle$ = ⟨ (1/2) (–3 – $\sqrt{5}$) , (1/2) (1 + $\sqrt{5}$) ⟩ ≈ ⟨–2.61803, 1.61803⟩ is αβ- number whose conjugates of modulus less than or equal to 1. Here N = min (–2.61803, –1.61803, 1.61803) = –2.61803 and M = 6.85405. see Table 2

Table 1

Cyclotomic Graph G	**M(G), when n = 0**	**After the transformation $z+\frac{1}{z}+n$, the values of N and M approximately**				
		When n=1	**When n= 2**	**When n= 3**	**When n= 4**	**When n= 5**
A_1	1	# & 1	–2.61803 & 2.61803	–3.73205 & 3.73205	–4.79129 & 12.54374	–5.82843 & 21.75199
$B_{4–6}$	1	–1.8832 & 1.8832	–3.09066 & 3.09066	–4.17467 & 10.92941	–5.22274 & 41.17370	–6.25432 & 98.31930
B_7	1	–2.61803 & 2.61803	–3.73205 & 3.73205	–4.79129 & 4.79129	–5.82843 & 39.94853	–6.8541 & 95.46525
$C_{26–29}$	1	–2.29663 & 2.29663	–3.44148 & 3.44148	–4.51034 & 30.91423	–5.55193 & 129.03554	–6.58008 & 442.01965
$C_{30–41}$	1	–2.15372 & 2.15372	–3.31651 & 7.14283	–4.39026 & 26.75972	–5.43401 & 133.92870	–6.46331 & 444.19727
$C_{54–55}$	1	–2.61803 & 2.61803	–3.73205 & 3.73205	–4.79129 & 32.83989	–5.82843 & 81.17952	–6.8541 & 411.93621
$B_{1–3}$	1	# & 1	–2.61803 & 2.61803	–3.73205 & 9.77061	–4.79129 & 46.81386	–5.82843 & 104.22010
$C_{1–6}$	1	# & 1	–2.61803 & 2.61803	–3.73205 & 25.57977	–4.79129 & 174.71169	–5.82843 & 499.34873
$C_{7–18}$	1	–1.8832 & 1.8832	–3.09066 & 3.09066	–4.17467 & 28.61352	–5.22274 & 153.66233	–6.25432 & 471.07628
$C_{19–21}$	1	# &1	–2.61803 & 6.85408	–3.73205 & 13.92819	–4.79129 & 157.34543	–5.82843 & 473.14916
$C_{22–25}$	1	# & 1	–2.61803 & 6.85408	–3.73205 & 36.46443	–4.79129 & 85.67465	–5.82843 & 426.11835
$D^j{}_{1-2}$	1	–2.15372 & 4.63850	–3.31651 & 10.99923	–4.39026 & 65.10293	–5.43401 & 275.51593	–6.46331 & 1874.90674
$D^j{}_3$	1	–2.61803 & 2.61803	–3.73205 & 3.76388	–4.79129 & 85.97581	–5.82843 & 302.96603	–6.8541 & 1973.70585
$D^j{}_{4-6}$	1	–2.4375 & 2.4375	–3.56745 & 8.33916	–4.63186 & 68.13994	–5.67144 & 409.51208	–6.69847 & 1995.80534
$D^j{}_7$	1	–2.24663 & 2.24663	–3.44148 & 9.00989	–4.51034 & 44.06881	–5.55193 & 433.69954	–6.58005 & 2006.71723

Where # denotes it has no value

Numbers are in bold denotes the minimum of the alpha numbers in the table 2.

RESULT

1. Mahler measure of the Non–cyclotomic graphs are Salem numbers.
2. The number(s) got from roots of the reciprocal polynomial after the transformation $z+\frac{1}{z}+n$, where n = 0,1,2,3,4,5. to the characteristic equation of the adjacency matrix of the simple graphs of order up to 5 are either Salem number or α-number(s) or αβ-numbers or all roots lie in a unit circle.

4.2 Non–cyclotomic graphs:

Table 2

Non–Cyclotomic Graph	M(G), n = 0	After the transformation z + (1/z) + n, the values of N and M				
		n = 1	n = 2	n = 3	n = 4	n = 5
$C_{42–53}$	1.50614	–1.9748 & 1.9748	–3.16526 & 8.28674	–4.24566 & 35.54163	–5.29224 & 86.06759	–6.32304 & 397.5483
$C_{56–61}$	2.08102	–2.08102 & 2.08102	–3.25426 & 8.51975	–4.33064 & 42.31303	–5.37552 & 96.12146	–6.40544 & 342.8559
C_{62}	2.61803	# & 1	–2.61803 & 17.94419	–3.73205 & 51.98072	–4.79129 & 109.99104	–5.82843 & 197.99526
$D^j{}_8$	3.73205	–1.01768 & 1.04390	–2.61803 & 46.97842	–3.73205 & 193.99467	–4.79129 & 526.99904	–5.82843 & 1154.00141
$D^j{}_{9–10}$	3.34697	**–2.1889** & 4.79128	–3.34697 & 22.94040	–4.41948 & 161.15385	–5.46269 & 468.01408	–6.49171 & 1056.61005
$D^j{}_{11}$	2.98882	**–2.23363** & 3.91539	–3.386 & 23.20791	–4.45696 & 135.59259	–5.4995 & 422.04300	–6.52815 & 979.22618
$D^j{}_{12}$	2.89005	**–2.61803** & 6.85405	–3.73205 & 10.78581	–4.79129 & 130.87155	–5.82843 & 408.94671	–6.8541 & 955.30343
$D^j{}_{13}$	2.71825	**–2.01879** & 2.70455	–3.20181 & 21.94546	–4.28052 & 124.87810	–5.32639 & 399.32466	–6.35683 & 937.56325
$D^j{}_{14}$	2.61803	–2.61803 & 2.61803	–3.73205 & 9.77061	–4.79129 & 122.56011	–5.82843 & 388.95463	–6.8541 & 917.07939
$D^j{}_{15}$	2.54205	–2.15372 & 4.22893	–3.31651 & 20.40713	–4.39026 & 107.49872	–5.43401 & 350.61861	–6.46331 & 891.03156
$D^j{}_{16}$	2.44713	–2.8232 & 2.8232	–3.92247 & 10.26914	–4.97646 & 108.42560	–6.01105 & 363.59581	–7.03527 & 1276.36966
$D^j{}_{17}$	2.2389	–2.31765 & 3.87960	–3.46011 & 10.13826	–4.52828 & 105.55193	–5.56957 & 355.25459	–6.59755 & 1491.17770
$D^j{}_{18}$	2.18313	–2.35022 & 2.35022	–3.48911 & 19.33620	–4.55623 & 93.92407	–5.59705 & 343.22708	–6.62476 & 1516.86741
$D^j{}_{19}$	2.08102	–2.08102 & 2.08102	–3.25426 & 22.30496	–4.33064 & 60.31800	–5.37552 & 323.07251	–6.40544 & 1556.53275
$D^j{}_{20}$	1.97482	–2.61803 & 3.94311	–3.73205 & 10.50504	–4.79129 & 85.17647	–5.82843 & 321.54056	–6.8541 & 1622.92986
$D^j{}_{21}$	1.93185	–3.13 & 3.13	–4.21208 & 4.24590	–5.25935 & 94.37477	–6.29052 & 326.98581	–7.31274 & 1662.26490
$D^j{}_{22}$	1.72208	–2.15372 & 3.70887	–3.31651 & 9.83534	–4.39026 & 85.68376	–5.43401 & 316.36123	–6.46331 & 1774.05405
$D^j{}_{23}$	1.58235	–2.22587 & 2.22587	–3.3792 & 18.15213	–4.45043 & 53.64032	–5.49308 & 296.76923	–6.5218 & 1793.74865
$D^j{}_{24}$	1.44257	–2.77548 & 2.77548	–3.87791 & 8.56855	–4.93307 & 77.10464	–5.96823 & 296.97164	–6.99277 & 1846.74778
$D^j{}_{34}$	2.61803	–1.00354 & 1.00649	–2.61803 & 17.94418	–3.73205 & 136.08710	–4.79129 & 410.49212	–5.82843 & 948.65262
$D^j{}_{35}$	2.08102	–2.08102 & 2.08102	–3.25426 & 8.51975	–4.33064 & 110.77678	–5.37552 & 358.73012	–6.40544 & 1642.72226
$D^j{}_{36}$	1.50614	–1.97482 & 1.97482	–3.16526 & 8.28674	–4.24566 & 93.04907	–5.29224 & 321.20855	–6.32304 & 1904.76929

4.3 Comparison diagrams

Comparison diagrams for each transformation $z + \frac{1}{z} + n$, n = 0,1,2,3,4,5 are shown as below

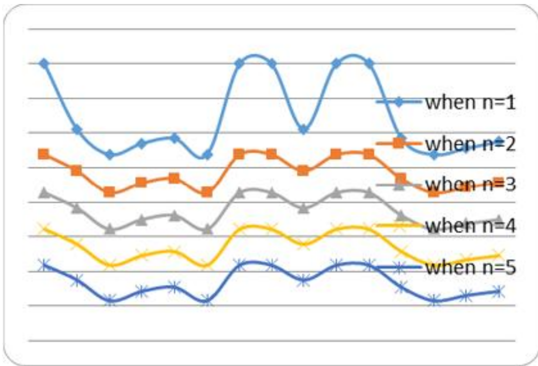


Fig 2 N values for the Cyclotomic graph

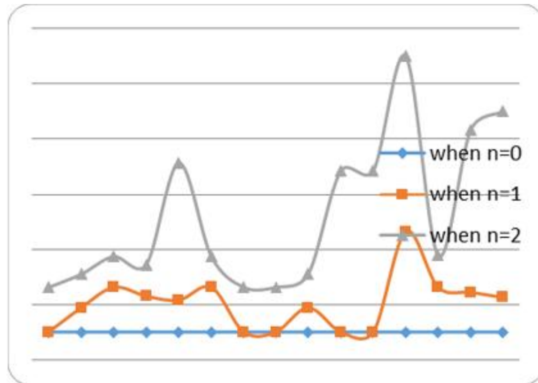


Fig 3 M(G) & M values for Cyclotomic graph

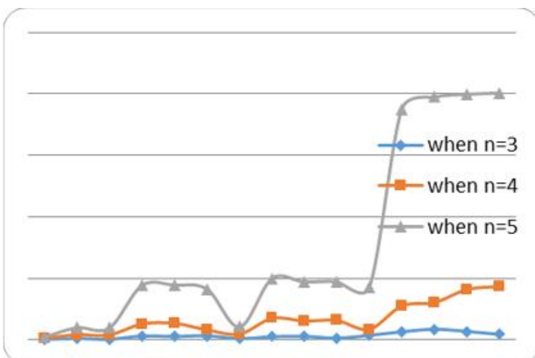


Fig 4 M values for the Cyclotomic graph

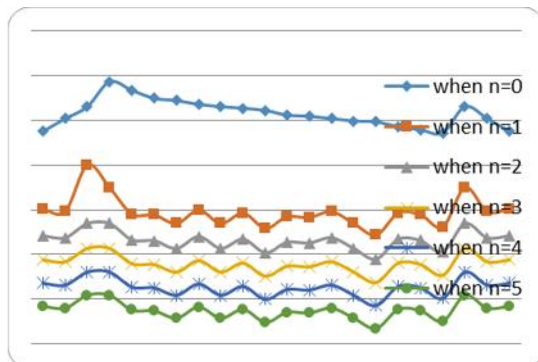


Fig 5 N values for the Non–Cyclotomic graph

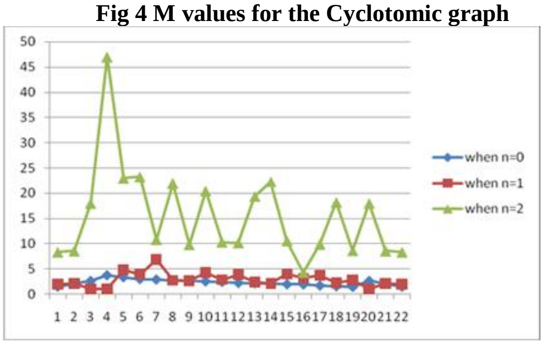


Fig 6 M(G) & M values for Non–Cyclotomic graph

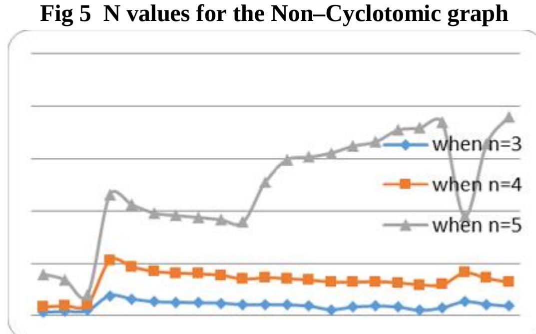


Fig 7 M values for the Non–Cyclotomic graph

4.4 ~- numbers from N- graph B_7

1. For the N graph B_7, The characteristic equation is $a^3 - 3a + 2 = 0$. Substitute $z + (1/z)$ for λ and get the reciprocal polynomial $z^6 + 2z^3 + 1 = 0$. Its approximate roots are –1, –

1, 0.5 ± 0.866025 i, 0.5 ± 0.866025 i. Mahler measure of this reciprocal polynomial M (R_A) is 1. It is the Mahler measure of the graph M (B_7).

2. Substitute *z + (1/z) + 1* for λ and get the reciprocal polynomial $z^6+3z^5+3z^4+6z^3+3z^2+3z+1=0$ .Its approximate roots are –2.61803, –0.381966, ± i, ± i. The algebraic integer $\langle \sim_1 \rangle$ = (1/2) (–3 – $\sqrt{5}$) ≈ –2.61803 (less than –1) is α- number whose conjugates of modulus less than or equal to 1. Here N = min (–2.61803) = –2.61803 and the Mahler measure M = 2.61803.
3. Substitute *z + (1/z) + 2* for λ and get the reciprocal polynomial $z^6+6z^5+12z^4+16z^3+12z^2+6z+1=0$. Its approximate roots are 0.5 ± 0.866025 i, 0.5 ± 0.866025 i , –3.73205, –0.267949. The algebraic integer $\langle \sim_1 \rangle$ = –2 – $\sqrt{3}$ ≈ -3.73205 is α- number and whose conjugates of modulus less than or equal to 1. Here N = min (–3.73205) = –3.73205 and M = 3.73205.
4. Substitute *z + (1/z) + 3* for λ and get the reciprocal polynomial $z^6+9z^5+27z^4+38z^3+27z^2+9z+1=0$. Its roots are ≈ –4.79129, –1, –1, –1, –1 , –0.208712. The algebraic integer $\langle \sim_1 \rangle$ = (1/2) (–5 – $\sqrt{21}$) ≈ –4.79129 is α- number and whose conjugates of modulus less than or equal to 1. Here N = min (–4.79129) = –4.79129 and M = 4.79129.
5. Substitute *z + (1/z) + 4* for λ and get the reciprocal polynomial $z^6+12z^5+48z^4+78z^3+48z^2+12z+1=0$. Its approximate roots are –5.82843, –2.61803, –2.61803, –0.381966, –0.381966, –0.171573. The algebraic integer $\langle \sim_1, \sim_2, \sim_3 \rangle$ = ⟨–3 – 2 $\sqrt{2}$, (1/2) (–3 – $\sqrt{5}$) , (1/2) (–3 – $\sqrt{5}$) ⟩ ≈ ⟨ –5.82843, –2.61803, –2.61803⟩ are α- numbers and whose conjugates of modulus less than or equal to 1. Here N = min (–5.82843, –2.61803, –2.61803) = –5.82843 and M = 39.94853.
6. Substitute *z + (1/z) + 5* for λ and get the reciprocal polynomial $z^6+15z^5+75z^4+142z^3+75z^2+15z+1=0$ .Its approximate roots –6.8541, –3.73205,–3.73205, –0.267949, –0.267949, –0.145898. The algebraic integer $\langle \sim_1, \sim_2, \sim_3 \rangle$ = ⟨(1/2) (–7 –3 $\sqrt{5}$),–2 – $\sqrt{3}$,–2 – $\sqrt{3}$ ⟩ ≈ ⟨ –6.8541, –3.73205, –3.73205⟩ are α- numbers and whose conjugates of modulus less than or equal to 1. Here N = min (–6.8541, –3.73205, –3.73205) = –6.8541 and M = 95.46525.

5. CONCLUSION

Hence the simple graphs of order up to 5 through Perron number were classified. N–graph, α– number(s) and αβ- numbers was defined. The new algebraic integers by the transformation were got. Plotted the comparison diagrams between transformations. The idea discussed can be extended for the simple graphs of higher order and for the non–simple graphs and to represent small negative Pisot and Salem numbers with Cyclotomic polynomials.

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