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Non-Darcian Free and Forced Convection Flow Through a Porous Medium in a Coaxial Duct with Radiation

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ABSTRACT

In this paper we investigate Non-Darcian free and forced convection flow through a porous medium in a co-axial cylindrical duct with radiation effect, where the boundaries are maintained at constant temperatures. The Brainkmann-Forchheimer extended Darcy equations which takes into account the boundary and inertia effects are used in the governens linear momentum equations. The velocity and temperature enter flow field have been analytically evaluated using Gauss–Siedel Iteration method and their behaviour is discussed computationally for variations in the governing parameters viz G, the Grashoff number, D^{-1}, the inverse Darcy parameter α, heat source parameter, N_1, the radiation parameter and s, the non–dimensional gap of the coaxial duct. The shear stress on the inner and outer cylinder, and the Nusselt number on the boundaries have been analytically evaluated.

Keywords: Convection flow, Porous medium, Radiation.

1. INTRODUCTION

Convective flow through porous media is a branch of research undergoing rapid growth in fluid mechanics and heat transfer. This is quite natural because of its important applications in environmental, geophysical and energy related engineering problem. Prominent applications are utilization of geothermal energy, the control of pollutant spread in ground water, the design of nuclear reactors, compact heat exchangers, solar power collectors, heat transfer associated with the deep storage of nuclear waste and high performance insulations for buildings, as well as the heat transfer from stored agricultural products the release energy as a result of metabolism of the products.

Non–Darcy effects on natural convection in porous media have received a great deal of attention in recent years because of experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters which indicate that the experimental data for system other than glass water at low Rayleigh numbers, do not agree with theoretical predictions based on the Darcy Flow model. This divergence in the heat transfer results has been reviewed in detail in Cheng[6] and Prasad *et al.*[11] among others. Extensive effects are thus being made to include the inertia and is cous diffusion term in the flow equations and to examine their effects in order to develop a reasonable accurate mathematical model for convective transport in porous media. Detailed accounts of the recent efforts on non – Darcy convection have been recently reported in Tien and Hong[2] Chen[7] Prasad *et al.*[10], and Kladias and Prasad[5]. Here we will restrict our discussion to the vertical gravity only. Poulikakos and Bejan[3,4] investigated the inertia effects through the inclusion of Forchheimer's velocity squared term and presented the boundary layer analysis for tall cavities. They also obtained numerical results for a few cases in order to verify the accuracy of their boundary layer solutions. Later Prasad and Tuntomo[9] reported an extensive numerical work for a wide range of parameters, and demonstrated that effects of Prandtl number remain almost unaltered while the dependence on the modified Grashoff number G, changes significantly with an increase in the Forcheimer number.

Convective boundary layers are often controlled by injecting or withdrawing fluid through porous bounding surface. Thus can lead to enhanced heating or cooling of the system and can help to delay the transition from laminar to turbulent flow. Raptis[8] analyzed the thermal radiation and free convection flow through a porous medium by using perturbation technique. Bakier and Gorla[1] investigated the effect of thermal radiation on mixed convection from horizontal surfaces in saturated porous media.

Keeping in view the above mentioned facts, in this paper we investigate the Non-Darcian free and forced convection flow through a porous medium in a co-axial cylindrical duct with radiation effect, where the boundaries are maintained at constant temperatures.

The Brainkmann-Forchheimer extended Darcy equations which takes into account the boundary and inertia effects are used in the governing linear momentum equations. The velocity and temperature enter flow field have been analytically evaluated using Gauss– Siedel iteration method and their behaviour is discussed computationally for variations in the governing parameters viz G, the Grashoff number, D^{-1}, the inverse Darcy parameter α ,heat source parameter, N, the dimensionless temperature gradient, N_1,the radiation parameter and s, the non–dimensional gap of the coaxial duct. The shear stress on the inner and outer cylinder, and the Nusselt number on the boundaries have been analytically evaluated.

2. FORMULATION OF THE PROBLEM

We consider a fully developed laminar convection flow in co-axial cylindrical duct filled with porous medium. The axis of cylindrical duct is taken to be z-axis in the vertical upwards direction and r-axis is the radial direction. The inner and outer cylinders of duct are mentioned at constant temperatures T_0 and T_1 respectively. The temperature gradient in the flow field is sufficient to cause natural convection in the flow region. A constant axial pressure gradient is also imposed so that this resultant flow is mixed convection flow. The porous medium is assumed to isotropic and homogenous with constant porosity and effective thermal diffusivity. The thermo physical properties of porous matrix are also assumed to be constant and boussineq approximation is invoked by confining the density variation to buoyancy term. In absence of any extraneous forces the flow is unidirectional along the axis of the duct assumed to be of infinite span.

The Brinkman–Forchheinner extended Darcy Equation which account for boundary inertial effects in the momentum equation is used to obtain the velocity field. Based on the above assumptions the governing equations the vector form are

$\nabla . \bar{q} = 0$ (Equation of continuity) (2.1)

$$\frac{\rho}{\delta}\frac{\partial \bar{q}}{\partial t} + \frac{\rho}{\delta^2}\left(\bar{q}.\nabla\right)\bar{q} = -\nabla p + \rho g - \frac{\mu}{k}\bar{q} - \frac{\rho F}{\sqrt{k}}\left|\bar{q}.\bar{q}\right| + \mu\nabla^2\bar{q}$$

(Equation of momentum) (2.2)

$$\rho c_p\left[\frac{\partial T}{\partial t} + \left(\bar{q}.\nabla\right)\mathrm{T}\right] = \lambda\nabla^2\mathrm{T} + Q - \frac{\partial q_r}{\partial r}$$

(Equation of Energy) (2.3)

$$\rho - \rho_0 = -\beta\rho_0\left(\mathrm{T} - \mathrm{T}_0\right)$$

(Equation of state) (2.4)

where $\bar{q}$ = (0, 0, u) is the velocity, T is the temperature, p is the pressure, ρ is the density of fluid, c_p is the specific heat at constant pressure, k is the permeability of porous medium, μ is the coefficient of viscosity of the fluid, δ is the porosity of the medium, β is the coefficient of thermal expansion, λ is co-efficient of thermal conductivity and F is a function that depends on the Reynolds number and the microstructure of porous medium. Here, the thermo physical properties of solid and fluid have been assured to be constant except for the density variation in the body force term (Boussinesq approximation), and the solid particles and fluid are considered to be in local thermal equilibrium. Since the flow is unidirectional the equation of continuity (2.1) reduce to

$$\frac{\partial u}{\partial z} = 0 .$$

where u is the axial velocity implies
u = u (r).

The momentum and energy equation in the scalar form reduce to

$$\frac{\partial p}{\partial z} + \frac{\mu}{\delta}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{\mu}{k}u - \frac{\rho\delta F}{\sqrt{k}}u^2 + \rho g\left(\mathrm{T} - \mathrm{T}_0\right)\beta_\lambda = 0 , \quad (2.5)$$

$$\rho c_p u \frac{\partial T}{\partial z} = \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + Q - \frac{\partial q_r}{\partial r} \quad (2.6)$$

The boundary conditions are

u(a) = u(b) = 0 (2.7)

T(a) = T_0, T(b) = T_1 (2.8)

The axial temperature gradient $\frac{\partial T}{\partial z}$ assumed to be a constant by A.

Invoking Rosseland Approximation for radiation

$$q_r = - \frac{4\sigma}{3\beta_r} \frac{\partial T^4}{\partial u_r}$$

Expanding T^4 about T^∞ in Taylor series

$T^4 \approx 4 T T_\infty^3 - 3 T_\infty^4$

We define the following non–dimensional variables

$$z^* = \frac{z}{a}, r^* = \frac{r}{a}, u^* = \frac{a}{\upsilon} u,$$

$$p* = \frac{p a^2 \delta}{\rho r^2}, \theta^* = \frac{T - T_0}{T_1 - T_0} \quad \delta^* = \frac{b\beta}{a}$$

Introducing these non–dimensional variables then the governing equation in the non–dimensional form reduce to (on removing the stars)

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} = P + \delta D^{-1} u + \delta^2 A u^2 - \delta G \theta \quad (2.9)$$

$$\frac{d^2 \theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} = P_1 N_1 u + \alpha_1 \quad (2.10)$$

where $A = F D^{-\frac{1}{2}}$ (inertia parameter of Forchhenner number)

$$G = \frac{g\beta \left(T_1 - T_0 \right) a^3}{\gamma^2} \text{ (Grashoff number),}$$

$$D^{-1} = \frac{a^2}{k} \text{ (inverse darcy parameter),}$$

$$N = \frac{Aa}{T_1 - T_0} \text{ (Non-dimensional temperature gradient)}$$

$$P = \frac{\rho c_p \gamma}{\lambda} \text{ (Prandtl number),}$$

$$\alpha = Q \, a^2 / \lambda \text{ (Heat source parameter),}$$

$$N_1 = \frac{4\sigma^*}{\beta_R \lambda} \text{ (radiation parameter),}$$

$$\alpha_1 = \frac{3N_1 \alpha}{3N_1 + P}, \quad P_1 = \frac{3N_1 P}{3N_1 + P}$$

The corresponding boundary conditions are

u (1) = u (s) = 0 (2.11)

θ (1) = 0, θ (s) = 0 (2.12)

3. SOLUTION OF THE PROBLEM

The governing equations of flow and temperature are coupled non–linear differential equations. Assuming the porosity δ to be small we write

$u = u_0 + \delta u_1 + \delta u_2 + \ldots\ldots$ (3.1)

$$\theta = \theta_0 + \delta\theta_1 + \delta^2 \theta_2 + \ldots\ldots\ldots\ldots \quad (3.2)$$

Substituting the above equations (2.8) and (2.9) and equating the like powers of δ, we obtain equations to the zeroth order as

$$\frac{d^2u_0}{dr^2} + \frac{1}{r}\frac{d.u_0}{dr} = \pi \qquad (3.3)$$

$$\frac{d^2\theta_0}{dr^2} + \frac{1}{r}\frac{d\theta_0}{dr} = P_1 N_1 u_0 + \alpha_1 \qquad (3.4)$$

The first order equations are

$$\frac{d^2u_1}{dr^2} + \frac{1}{r}\frac{d.u_1}{dr} = D^{-1}u_0 - G\theta_0 \qquad (3.5)$$

$$\frac{d^2\theta_1}{dr^2} + \frac{1}{r}\frac{d\theta_1}{dr} = P_1 N_1 u_1 \qquad (3.6)$$

The second order equations are

$$\frac{d^2u_2}{dr^2} + \frac{1}{r}\frac{d.u_2}{dr} = D^{-1}u_1 - G\theta_1 + Au_0 \qquad (3.7)$$

$$\frac{d^2\theta_2}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} = P_1 N_1 u_2 \qquad (3.8)$$

The corresponding boundary conditions are

$u_0(1) = u_0(s) = 0,$

$$\theta_0(1) = 0 \; \theta_0(s) = 1 \qquad (3.9)$$

$u_1(1) = u_1(s) = 0,$

$$\theta_1(1) = 0, \theta_1(s) = 0 \qquad (3.10)$$

$u_2(1) = u_2(s) = 0$

$$\theta_2(1) = 0, \qquad \theta_2(s) = 0 \qquad (3.11)$$

The differential equations (3.3) to (3.8) have been discussed numerically by reducing the differential equations into difference equations which are solved using Gauss–Seidel iteration method.

4. SHEAR STRESS AND NUSSELT NUMBER

The shear stresses on the inner and outer cylinders are evaluated using the formula.

$$\tau_{inner} = \left(\frac{du}{dr}\right)_{r=1} \quad \tau_{outer} = -\left(\frac{du}{dr}\right)_{r=s}$$

The rate of the heat transfer (Nusselt number) on the inner and outer cylinders are given by

$$Nu_{inner} = \left(\frac{d\theta u}{dr}\right)_{r=1} \quad Nu\tau_{outer} = -\left(\frac{du}{dr}\right)_{r=s}$$

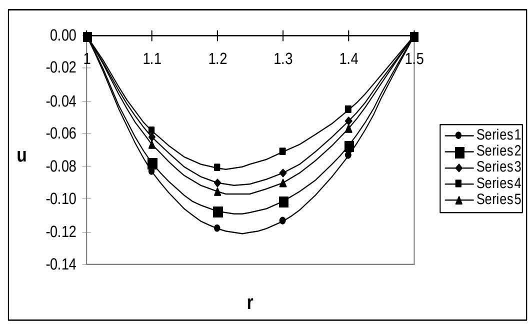

Fig.1 Variation of velocity u with G , s=1.5
G 10^2 $5x10^2$ $8x10^2$ -10^2 $-5x10^2$

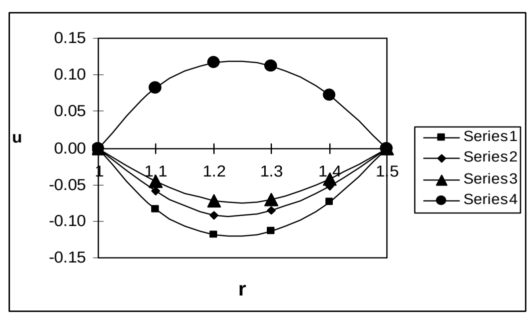

Fig.2 Variation of velocity u with D^{-1}, s=1.5
D^{-1} 10^3 $3x10^3$ $5x10^3$ $7x10^3$

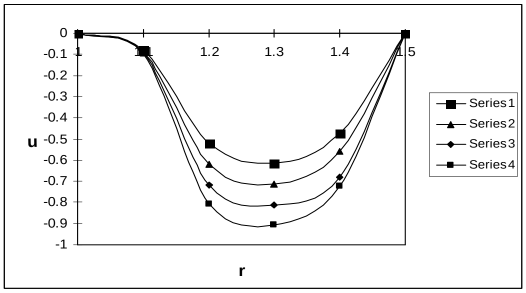

Fig. 3 Variation of velocity u with N_1, s=1.5
N_1 0.5 2.5 4 10

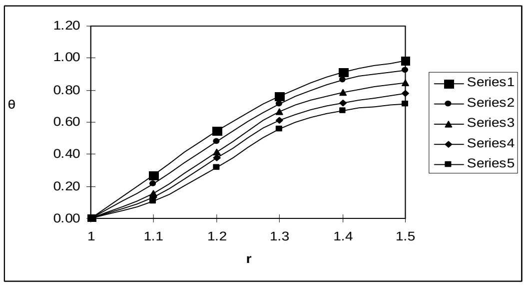

Fig. 4 Variation of temperature θ with G, s=1.5
G 10^2 $5x10^2$ $8x10^2$ -10^2 $-5x10^2$

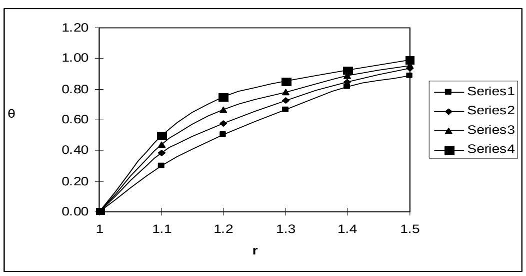

Fig. 5 Variation of temperature θ with D^{-1}, s=1.5
D^{-1} 10^3 $3x10^3$ $5x10^3$ $7x10^3$

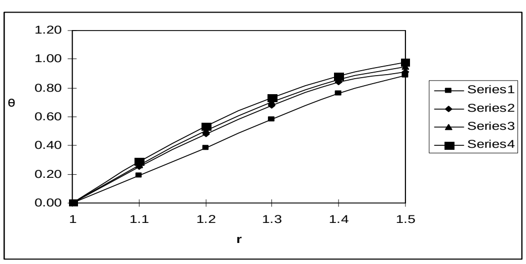

Fig.6 Variation of temperature θ with N_1, s=1.5
N_1 0.5 2.5 4 10

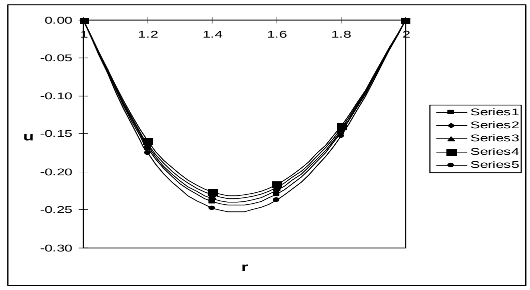

Fig.7 Variation of velocity u with G, s=2
G 10^2 $5x10^2$ $8x10^2$ -10^2 $-5x10^2$

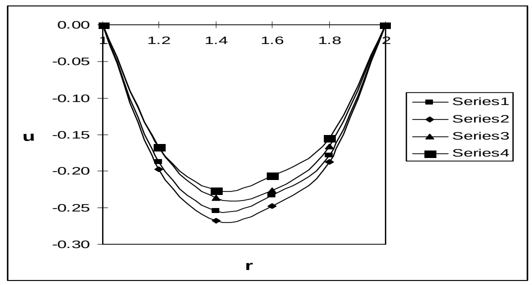

Fig.8 Variation of velocity u with D^{-1}, s=2
D^{-1} 10^3 $3x10^3$ $5x10^3$ $7x10^3$

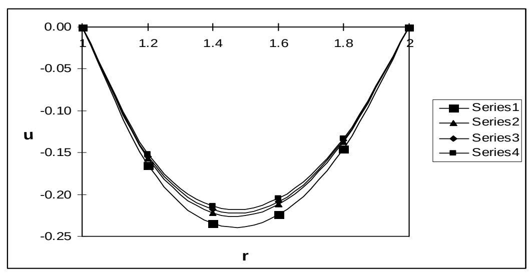

Fig.9 Variation of velocity u with N_1, s=2
N_1 0.5 2.5 4 10

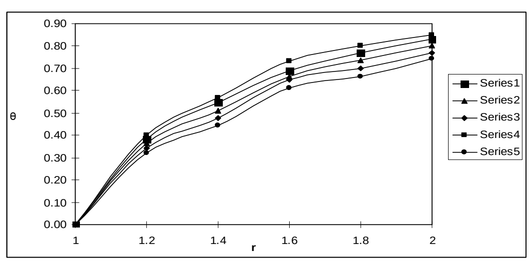

Fig. 10 Variation of temperature θ with G, s=2
G 10^2 $5x10^2$ $8x10^2$ -10^2 $-5x10^2$

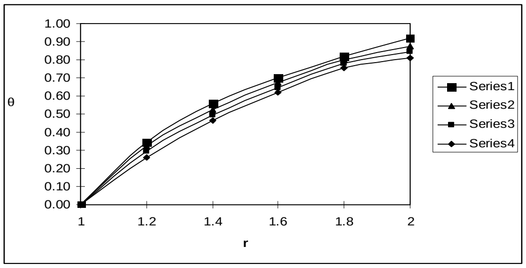

Fig.11 Variation of temperature θ with D^{-1}, s=2

D^{-1} 10^3 $3x10^3$ $5x10^3$ $7x10^3$

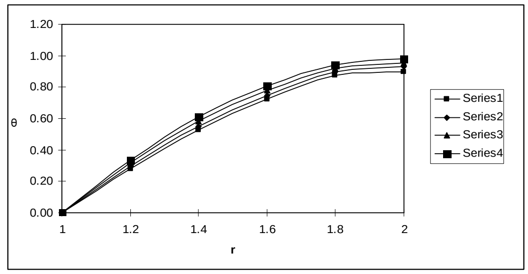

Fig.12 Variation of temperature θ with N_1, s=2

N_1 0.5 2.5 4 10

Table-1
Shear Stress at the inner Cylinder r=1,

	I	II	III	IV
$D^{-1}= 10^3$	- 0.52782	- 0.25996	- 0.21633	- 0.14021
$D^{-1}= 3x10^3$	- 0.50586	- 0.28956	- 0.25595	- 0.19522
$D^{-1}= 5x10^3$	- 0.53853	-0 .37378	- 0.35017	-0.30477
$D^{-1}= 7x10^3$	- 0.62579	- 0.51256	-0.49898	- 0.46891

N_1	0.5	2.5	4	10

Table-2
Shear Stress at the inner Cylinder r=1

	I	II	III	IV
$D^{-1}= 10^3$	- 0.60674	- 0.12584	- 0.51912	0.08448
$D^{-1}= 3x10^3$	- 0.59658	- 0.20829	- 0.15151	- 0.4277
$D^{-1}= 5x10^3$	- 0.65960	- 0.36390	- 0.32446	- 0.24321
$D^{-1}= 7x10^3$	- 0.79577	- 0 .59 273	- 0.57049	- 0.51681

N_1	0.5	2.5	4	10

Table-3
Shear Stress at the outer Cylinder r=s

	I	II	III	IV
$D^{-1}= 10^3$	0.41455	0.17856	0.11884	0.04123
$D^{-1}= 3x10^3$	0.39691	0.20855	0.15898	0.09601
$D^{-1}= 5x10^3$	0.43025	0.28953	0.25009	0.20173
$D^{-1}= 7x10^3$	0.51456	0.42147	0.39216	0.35845

N_1	0.5	2.5	4	10

Table -4
Shear Stress at the outer Cylinder r=s

	I	II	III	IV
$D^{-1}= 10^3$	0.46873	0.04399	- 0.06119	- 0.20092
$D^{-1}= 3x10^3$	0.46282	0.12372	0.03610	- 0.07734
$D^{-1}= 5x10^3$	0.52522	0.27175	0.20172	0.11455
$D^{-1}= 7x10^3$	0.65590	0.48809	0.43562	0.37474

N_1	0.5	2.5	4	10

Table – 5
Nusselt Number at the inner Cylinder r=1

	I	II	III	IV
$D^{-1}= 10^3$	- 1.00882	-0.50059	-0.08760	0.03784
$D^{-1}= 3x10^3$	-1.02895	-0.56109	-0.11870	-0.01992
$D^{-1}= 5x10^3$	-1.05881	-0.64171	-0.20582	-0.09433
$D^{-1}= 7x10^3$	-1.09842	-0.74249	-0.28894	-0.18536

N_1	0.5	2.5	4	10

Table – 6
Nusselt Number at the inner Cylinder r=1
P=1, P_r = 7 , δ =0.01, α =2 ,N=1, s=2

	I	II	III	IV
$D^{-1}= 10^3$	-0.91018	-0.28413	0.11333	0.25895
$D^{-1}= 3x10^3$	-0.94056	-0.38088	0.03122	0.16525
$D^{-1}= 5x10^3$	-0.98399	-0.50464	-0.7233	0.04927
$D^{-1}= 7x10^3$	-.1.04047	-0.65535	-0.19735	-0.02901

N_1	0.5	2.5	4	10

5. DISCUSSION

The analysis has been carried out when the outer cylinder is at a higher temperature than that of the inner cylinder. The actual flow is along the gravitational field and hence the axial velocity (u) in the vertical direction represents the actual flow. u>0 indicates the reversal flow. For computational purpose we have chosen the non-dimentional temperature(θ) on the inner cylinder to be 1, on the outer cylinder it continues to be 0. We find from figs.1 and 7 that no reversal flow appears in the region for any value of $|G|(\gtrless 0)$ .The velocity depreciates in the heating of the cylindrical wall and enhances in the cooling case with maximum attained at r = 1.25 in the narrow gap case and r = 1.4 in the wide gap case. The variation of u with D^{-1} shows that in the narrow gap, we notice a reversal flow in the region for the higher $D^{-1} \geq 7 \times 10^3$ and no such reversal flow appears in the wide gap case. Also lesser the porous permeability of the medium, smaller the magnitude of u in the entire flow region while in a wide gap case |u| enhances with $D^{-1} \leq 3x10^3$ and depreciates for further increase in $D^{-1} \geq 7x10^3$ (Figs. 2 and 8). The effect of radiation parameter is to enhance |u| in the narrow gap and depreciates in the wide gap case (Figs.3&9). The temperature gradually increases from inner cylinder to the outer cylinder. We find that in a given porous medium the temperature depreciates with increase in $|G|(\gtrless 0)$ in a narrow gap case(fig.4) while in a wide gap case it depreciates with increase in G and enhances with |G|(<0)(fig.10). Figs.5&11 shows that lesser the permeability of the porous medium larger the temperature in a narrow gap and smaller θ in a wide gap case. Also an increase in the radiation parameter N_1 enhances θ in the flow region in both narrow and wide gap cases(figs.6 and 12).The variation of τ with radiation parameter, shows that at $D^{-1} \leq 3 \times 10^3$, $|\tau|$ depreciates with increase in smaller values of N_1 and at higher $D^{-1} \geq 5x10^3$,$|\tau|$ experiences an enhancement while it enhances with higher N_1 at both the cylinders(tables.1-4) Also the effect of the radiation parameter N_1,is to enhance |Nu| at r= 1 (tables.5-6).

REFERENCES

1. A.Y.Bakier, R.S.R.Gorla: “Thermal radiation effects on mixed convection from horizontal surfaces in porous media, *Transport in Media, Vol*. 23, pp. 357–362 (1996).
2. C.L. Tien and J.T. Hong : “Natural Convection in porous media under non–Darcian and non–uniform permeability conditions”, in natural convection (Edited by S. Kakac W. Aung and R.Viskanta),Hemisphere, Washington DC (1985).
3. D. Poulikakas : “A departure from the Darcy model in boundary layer natural convection in a vertical porous layer with uniform heat flux”, *J. Heat Transfer* 107, 716-720 (1985).
4. D. Poulikakos and A. Bejan: “The departure from Darcy flow in natural convection in a vertical porous layer”, *Physics Fluids* 28,3477-3484 (1985).
5. N. Kalidas and V. Prasad: “Benard Convection in porous media effects of Darcy and Prandtl numbers”. Int symp convection in porous media Non– Darcy Effects. Proc 25th Nat. *Heat Transfer Conf., Vol.* I. pp 593-604 (1988).
6. P. Cheng: “Heat Transfer in geothermal system”, *Adv. Heat Transfer* 14, 1-105 (1978).
7. P. Cheng: “Wall effects on fluid flow and heat transfer in porous media”. *Proc. ASME/ TSME Heat transfer Conf.*, pp. 297-303 (1983).
8. Raptis A: “Radiation and free convection flow through a porous medium”. *Int. Commun. Heat Mass Transfer, Vol*. 25, pp. 289-295 (1998).
9. V. Prasad and A. Tuntomo: “Inertia effects on natural convection in a vertical porous cavity”, *Numerical Heat Transfer* 11, 295-320 (1987).
10. V. Prasad, G. Lauriat and N. Kalidas: “Reexamination of Darcy Brinkman

solutions for free convection in porous media", Int. Symp. Convection in Porous Media. Non-Darcy effects, *Proc 25th Nat.Heat Transfer Conf., Vol*.1, pp569-580 (1988).

11. V. Prasad, F.A. Kulacki and M. Keyhani: "Natural convection in a porous media", *J. Fluid Mech*. 150, 89-119 (1985).