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Solving System of Nonlinear Equations using Genetic Algorithm

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ABSTRACT

Finding solution of nonlinear system of equations is a peculiar task for engineering and science fraternities. In this paper, we propose solution of nonlinear systems through a stochastic method namely, Genetic algorithm. Comprehensive experimentation is performed on standard benchmark problems for validating the present scheme. The original optimization problem is first assigned a fitness function and then, the minimization task is carried out by selection of the parameters and tuning of the algorithm. Further, the sensitivity analysis is studied to analyze the effect of variations of parameters. The results are duly compared and ensure the effectiveness of the work in handling complex nonlinear system of equations.

Keywords: Genetic Algorithm, Nonlinear equations, Optimization, Soft Computing Techniques.

1. INTRODUCTION

Nonlinear system of equations arise in all engineering, science, and medical domains. These complex problems comprises of functions which are nonlinear and transcendental in nature. Many problems arising in discrete systems like travelling salesman problem, continuous systems like designing airfoil in aerospace engineering, elasticity theory, kinetic theory of gases, molecular potential energy and other domains are reduced to system of nonlinear equations in order to find their potential solution. Many optimization techniques are used to solve such problems by deterministic approach such as Newton's method, gradient method, steepest descent etc[1,2]. These methods demand the differentiability property of the objective function. In practice, the objective functions might not be necessarily differentiable, or may be nonsmooth. This gave motivation to many researchers to work with stochastic global optimization approaches such as Particle swarm optimization, Genetic algorithm, Cuckoo search, Bat algorithm, Firefly algorithm, Gravitational search etc.

With the advent of technology, many innovative ideas have been developed to solve nonlinear system of equations. This includes parameters estimation by multi-crossover real coded genetic algorithm[3], improving the conventional Genetic algorithm by well conditioning of nonlinear system of equations using calculus based variant gradient descent method[4], line search sequential quadratic programming[5], arranging the equations in Cycle Gas Turbine simulation by fixed point method[6], multi-objective optimization using Genetic algorithm for standard benchmark problems[7], analytical existence of solution based on Matrix inversion principle with Schur complement for road safety measure[8]. Also, many attempts have been made to enhance the exploration and exploitation efficiency of GA, numerous researchers solicited to enhance the basic architect of GA by assimilating particle swarm optimization[9], augmented langrangian function[10], symmetric and harmonious individuals[11], global and Tabu search[12], stochastic design solver based on variants of GA[13]. All these attempts provided motivation to develop a reliable and effective methodology for solving nonlinear systems.

In the present work, we are applying Genetic Algorithm (GA), a form of Evolutionary Algorithm (EA) to solve system nonlinear of equations. In our earlier work, we tried solving nonlinear systems with single equation and further worked on two dimensional nonlinear system of equations[14,15]. Here, the proposed technique is applied on benchmark problem adopted from Grosan[7]. A comparative analysis is made to substantiate the effectiveness and reliability of the proposed scheme in handling nonlinear systems involving transcendental functions. The data is obtained by independent execution with the help of GA Solver. Sensitivity analysis is also made to validate the selection of parameters of GA.

The paper has been organized in 6 sections as follows. In section 2, brief review of Genetic algorithm has been narrated. Section 3 comprises methodology, figure depicting procedure and parameters setting of GA. In section 4, experimentation results of the simulation performed along with comparison with other evolutionay technique are presented by numerical and graphical illustrations. Also, to validate the efficiency of the work, the sensitivity analysis has been performed and results are presented with section 5. The last section presents the findings and conclusion.

2. GENETIC ALGORITHM

Genetic algorithm (GA) was developed by John Holland and his collaborators in 1960 [16]. GA is a model or abstraction of biological evolution based on theory of natural selection[17]. To solve an optimization problem, GA uses the value of objective (fitness or merit) function assigned to the problem[10]. It can deal with any type of optimization problem where the objective function may be stationary, non-stationary, differentiable, non-differentiable, linear, nonlinear, continuous, discontinuous, or having random noise. The algorithm starts with encoding the solutions as arrays of bits or strings to represent chromosomes and forming an initial poputaion matrix[17]. These matrix includes individuals that are part of the solution space of the problem. A set of genetic operators viz. reproduction, crossover and mutation are applied to generate a new generation from the initial population matrix based on the best fitness value. This process is iterated until the stopping criteria is met. Once the algorithm terminates, the final chromosome represent the optimal solution to the problem.

3. MATERIAL AND METHODS

In this section, we present the methodology adapted for finding effective global solution of nonlinear system of equations using optimization technique based on genetic algorithm.

3.1 Methodology: The work scheme involves encoding the original set of equation which is phenotype in nature, into genotype structure. This is further followed by setting the parameters of GA. Then, the population size is varied, and experiments are repeated for various sizes to decide the number of chromosomes in the population. Rank scaling function has been used and experiments are carried out for both tournament and Roulette wheel. The empirical analysis showed that both could generate the optimal results in given time. To carry out meaningful empirical analysis, the number of generations are varied. The results are obtained by varying the number of generations.

3.2 Parameters of GA: The parameters of Genetic Algorithm have been stated in Table 1. The procedure has been depicted in Figure 1. The process has been peformed, and the results are given in the subsequent section.

Table 1 Parameters of GA

Parameters	Values
Population size	200/ 250/ 500
Mutation function	Gaussian
Scaling function	Rank
Crossover function	Single point
Selection function	Tournament/ Roulette wheel
Generations	50/51/100/150/200/300
Ratios/ Fractions	Default

It may be noted here that variations in generations have been considered. The reason is, more the number of generations more would be the computational time. It is, therefore, necessary to first see if optimization can be achieved by slightly varying the number of generations but, if it is not possible, then larger leaps are required. Therefore, we have taken various generations as 50, 51, 100, 150, 200, 300.

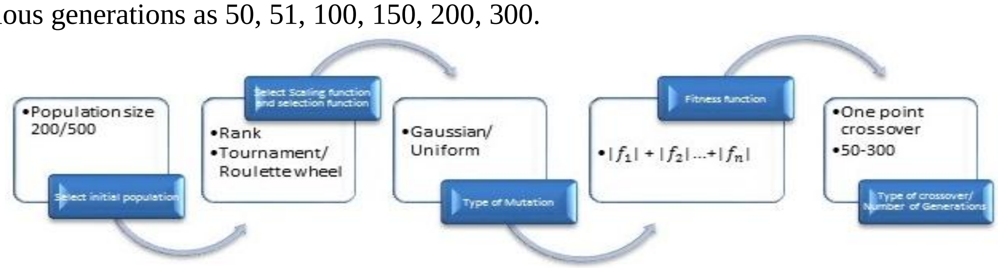

Figure 1 Adapted Procedure of GA

4. RESULTS AND DISCUSSIONS

For evaluating the proposed approach, the algorithm has been employed on standard benchmark problems which appear in engineering and science domains[7,10,18-21]. For each function, the experiment is carried out by varying the number of generations (50, 51, 100, 150, 200 and 300). For each generation, the experiment is repeated by varying the number of chromosomes in the initial population. Moreover, both Tournament ad Roulette wheel selection are employed. This is because the empirical study carried out earlier did not indicate to which of the two is better. The experiment is repeated 100 times, and the minimum values are taken, and are shown in Table 2 and Table 3. Moreover, the comparison of the results for the two methods of selection has also been shown diagrammatically (Figure 2 & Figure 3).

4.1 Arithmetic Application Problem

This test problem is proposed from arithmetic[7, 9,19, 20]. This problem consists of following set of complex system of nonlinear equations having ten variables:

$$\begin{aligned}
f_1(x) &= x_1 - 0.25428722 - 0.18324757\, x_4 x_3 x_9 \\
f_2(x) &= x_2 - 0.37842197 - 0.16275449\, x_1 x_{10} x_6 \\
f_3(x) &= x_3 - 0.27162577 - 0.16955071\, x_1 x_2 x_{10} \\
f_4(x) &= x_4 - 0.19807914 - 0.15585316\, x_7 x_1 x_6 \\
f_5(x) &= x_5 - 0.44166728 - 0.19950920\, x_7 x_6 x_3 \\
f_6(x) &= x_6 - 0.14654113 - 0.18922793\, x_8 x_5 x_{10} \\
f_7(x) &= x_7 - 0.42937161 - 0.21180486\, x_2 x_5 x_8 \\
f_8(x) &= x_8 - 0.07056438 - 0.17081208\, x_1 x_7 x_6 \\
f_9(x) &= x_9 - 0.34504906 - 0.19612740\, x_{10} x_6 x_8 \\
f_{10}(x) &= x_{10} - 0.42651102 - 0.21466544\, x_4 x_8 x_1
\end{aligned}$$

The problem has been considered as

$$\min(f_1(x), f_2(x), \ldots, f_n(x))$$

where $n = 10$ and $x = (x_1, x_2, x_3, \cdots, x_{10})$

Using the proposed methodology, the system was solved. Table 2 lists the minimum values of the fitness function taken as

$$z = |f_1| + |f_2| + |f_3| + |f_4| + |f_5| + |f_6| + |f_7| + |f_8| + |f_9| + |f_{10}|$$

The obtained results have been depicted in Figure 2.

The fitness function attains its optimal value as 0.040217 at roulette wheel selection function with highest population 500 and 150 generation.

Table 2 Minimum values obtained for Arithmetic Application Problem

Selection \ No. of generations	300	200	150	100	51	50
Tournament (Population size 200)	*	0.175594	0.193638	0.199685	0.132563	0.102964
Roulette (Population size 200)	*	0.184894	0.121656	0.06607	0.058034	0.104628
Tournament (Population size 500)	0.085699	0.055994	0.06434	0.062285	0.065633	0.059031
Roulette (Population size 500)	0.089772	0.061663	0.040217	0.062998	0.042824	0.058634

*average change in the fitness value less than options. TolFun.

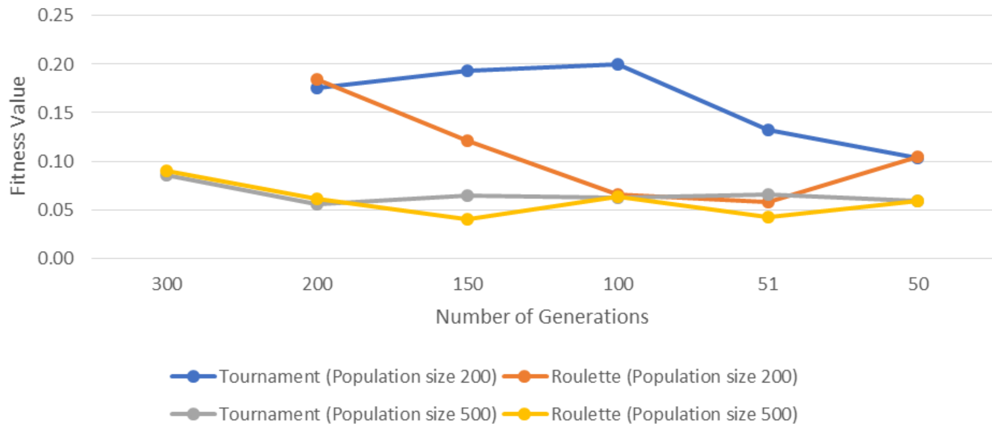

Figure 2 Minimum values obtained for Arithmetic Application Problem

4.2 Combustion Problem

In this test problem, a complex nonlinear algebraic system governing combustion problem[7,9,10] has taken under consideration.

The equations are as follows:

$$f_1(x) = x_2 + 2x_6 + x_9 + 2x_{10} - 10^{-5}$$
$$f_2(x) = x_3 + x_8 - 3.10^{-5}$$
$$f_3(x) = x_1 + x_3 + 2x_5 + 2x_8 + x_9 + x_{10} - 5.10^{-5}$$
$$f_4(x) = x_4 + 2x_7 - 10^{-5}$$
$$f_5(x) = x_1x_2 - (0.5140437)10^{-7}x_5$$
$$f_6(x) = 2x_2^2 - (0.1006932)10^{-6}x_6$$
$$f_7(x) = x_4^2 - (0.7816278)10^{-15}x_7$$
$$f_8(x) = x_1x_3 - (0.1496236)10^{-6}x_8$$
$$f_9(x) = x_1x_2 - (0.6194411)10^{-7}x_9$$
$$f_{10}(x) = x_1x_2^2 - (0.2089296)10^{-14}x_{10}$$

Changing the above system as multi-objective optimization problem with fitness functions as

$$z = |f_1| + |f_2| + |f_3| + |f_4| + |f_5| + |f_6| + |f_7| + |f_8| + |f_9| + |f_{10}|$$

with $\min(f_1(x), f_2(x), \dots, f_n(x))$

where n = 10 and $x = (x_1, x_2, x_3, \cdots, x_{10})$

The minimum values of z obtained by the present approach have been listed in Table 3. From the table, it can be assessed that the present results are much closer to zero. Also, the comparison of values at different population size has been shown in Figure 3.

Table 3 Minimum values obtained for Combustion Problem

Selection \ No. of generations	300	200	150	100	51	50
Tournament (Population size 200)	*	0.188594	0.224595	0.299335	0.061145	0.078137
Roulette wheel (Population size 200)	*	0.085871	0.145396	0.16594	0.108382	0.030947
Tournament (Population size 500)	0.117213624	0.153625	0.090191	0.108775	0.032489	0.038349
Roulette wheel (Population size 500)	0.024030224	0.045464	0.043204	0.051433	0.040961	0.052011

*average change in the fitness value less than options. TolFun.

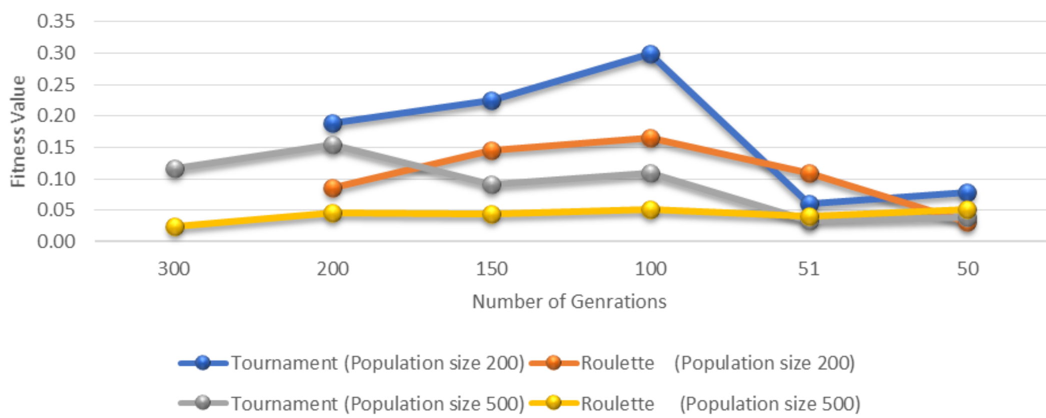

Figure 3 Minimum values obtained for Combustion Problem

The minimum value of fitness function is 0.024030224. The optimal value is obtained with population size 500 and number of generations 300 and with roulette wheel selection function.

5. SENSITIVITY ANALYSIS

In the previous subsection, the results of experiments in which single point crossover was applied, has been presented. Along with this, roulette wheel and tournament selection were used to carry out the experiments. Genetic algorithm convergence depends on its parameters. To substantiate the selected parameters, sensitivity analysis experiment has been performed by taking multiple crossover rates and varying the selection function. Data pertaining to results carried out through change in the parameters of GA, with MATLAB 2015 solver has been shown. The variations include change in following parameters as shown in Table 4.

Table 4 Variations in Parameters in further experiments

Parameters	Values
Population size	Taken as minimum obtained in earlier experiment
Scaling function	Proportional/ Rank/ Top/ Custom/ Shift linear
Selection function	Roulette wheel
Crossover function	Single point/ Double Point / Scattered
Mutation function	Gaussian
Generations	As per previous experiment
Ratios/ Fractions	Default

The following table shows the findings obtained by using the above mention variations of the parameters in the benchmark problems under consideration. In Table 4, further experimentation results for Arithmetic Application Problem are presented. The minimum value obtained is 0.007596631 for single point crossover, double vector type, population size 500, rank scaling function and tolerance level 1e-8 and 150 number of generations. Lastly, Table 5 shows the results for Combustion Problem, in which minimum value obtained is 0.026835676 for scattered crossover, double vector type, population size 500, rank scaling function and tolerance level 1e-8 and 300 generations. All minimum values obtained have been marked yellow in the corresponding tables.

Table 4 Results of Sensitivity Analysis for Arithmetic Application Problem

	RW/500/SPC/ DV/150(G)					
	Rank [TL 1e-8]	Rank [TL 1e-6]	Propotional	Top	Shift linear	Custom
	0.013354149	0.08422097	0.705596721	#	0.6517111	##
	0.015403967	0.062797845	0.60044319		0.53104126	
	0.074409318	0.099241388	0.516143405		0.66105715	
	0.084282827	0.076579396	0.592388655		0.69688449	
	0.081231483	0.09810198	0.861020728		0.7158219	
	0.007596631	0.091755229	0.541809058		0.61812188	
	0.11263849	0.087140607	0.517943514		0.67103865	
	0.036420603	0.148493508	0.583721582		0.61372521	
	0.084192518	0.152815155	0.61065068		0.68837643	
	0.057664145	0.130961994	0.435998124		0.18293108	
	0.044120873	0.137390085	0.342497291		0.60157629	
	0.059846618	0.082227738	0.776162245		0.64896883	
	0.017051364	0.087426477	0.704470932		0.79148642	
	0.072524701	0.249927666	0.741866759		0.80352079	
	0.064654268	0.16808303	0.687584129		0.93102365	
	0.151609346	0.120882056	0.568282861		0.4861942	
	0.094820881	0.089681196	0.573867272		0.65326981	
	0.05051856	0.104478368	0.850247044		0.90973173	
	0.067104061	0.125351759	0.745022268		0.4877586	
	0.086485542	0.061109545	0.704871743		0.75599281	
	0.108294159	0.114574396	0.433763305		0.43392565	
	0.108495509	0.0785906	0.528699284		0.71195455	
	0.034747318	0.096449426	0.726972865		0.44486311	
	0.033370206	0.155456539	0.381315148		0.40148775	
	0.065001542	0.163933516	0.933760477		0.74160241	
Average	0.065033563	0.114706819	0.626603971		0.63336263	
Minmum	0.007596631	0.061109545	0.342497291		0.18293108	

	RW/500/DPC/DV				
	RANK	Propotional	Top	Shift Linear	Custom
	0.102295867	0.69133343	#	0.51728408	##
	0.084790729	0.93178341		0.59832035	
	0.094949835	0.66580937		0.70259633	
	0.135203571	0.85514026		0.70007709	
	0.067963907	0.67081497		0.60994363	
	0.162413174	0.41613796		0.62562502	
	0.150097809	0.45196346		0.66718818	
	0.083069968	0.36215769		0.54243607	
	0.082767977	0.77475148		0.67820349	
	0.076071914	0.65679959		0.63983338	
	0.073609272	0.80815516		0.60515668	
	0.103520041	0.49564792		0.93176226	
	0.09550819	0.53989139		0.69601713	
	0.092692594	0.60800364		0.66270997	
	0.057198385	0.86567168		0.60986461	
	0.103405442	0.54320703		0.47565485	
	0.088294288	0.66075843		0.4330305	
	0.124101973	0.65638908		0.58291998	
	0.030496088	0.35165757		0.90770877	
	0.14774329	0.70933094		0.64974459	
	0.05577817	0.72914432		1.11897755	
	0.155307832	0.46888785		0.85287061	
	0.085376226	0.6479829		0.81939387	
	0.061821161	0.48859376		0.5440738	
	0.13234564	0.50312889		0.7640215	
Average	0.097872934	0.62212569		0.67741657	
Minmum	0.030496088	0.35165757		0.4330305	

	RW/500/SCATTERED/DV				
	RANK	Propotional	Top	Shift Linear	Custom
	0.064359522	0.58538951	#	0.57829735	##
	0.068171904	0.86315959		0.90557845	
	0.044155933	0.92331461		0.91762554	
	0.039435003	0.85439881		0.98606787	
	0.022843878	0.81140381		0.67254626	
	0.030042072	0.9564957		0.84698214	
	0.033480298	1.12120546		0.51579937	
	0.024144143	0.75510828		0.51981751	
	0.050804198	0.5082461		0.79057229	
	0.049194758	0.67994192		0.60472894	
	0.042922474	0.4026456		1.07065128	
	0.041066304	0.45001356		0.50948146	
	0.04273859	0.88409335		0.4578281	
	0.098823203	0.54227772		1.02155955	
	0.076300436	0.8658514		1.10995608	
	0.062329977	0.68106432		0.72875511	
	0.046754796	0.8137694		0.8364383	
	0.039758492	0.9469617		0.72970968	
	0.059915257	0.79779158		0.8176442	
	0.023438321	1.15890068		0.64114184	
	0.026890388	0.73954364		0.83247384	
	0.061025423	0.95886569		0.62180105	
	0.032137724	1.0468335		1.02555675	
	0.054700001	0.81842046		0.59222174	
	0.052596183	0.63662325		1.0174675	
Average	0.047521171	0.79209279		0.77402809	
Minmum	0.022843878	0.4026456		0.4578281	

Attempted to access thisPopulation(0,:); index must be a positive integer or logical.
Too many input arguments
SPC - Single point crossover
DPC- Double Point Crossover
(G)- No. of Generations
TL- Tolerence Level
RW- Roulette wheel
DV- Double Vector

Table 5 Results of Sensitivity Analysis for Combustion Problem

	RW/500/SPC/DV/300(G)					
	Rank [TL 1e-8]	Rank [TL 1e-6]	Propotional	Top	Shift linear	Custom
	0.132263194	0.09153738	0.898655804	#	0.721221901	##
	0.079243158	0.06977203	1.287323136		0.842438707	
	0.266711567	0.139109831	1.321767656		0.768630828	
	0.14558825	0.112700291	0.940998346		1.153753412	
	0.196132981	0.221337675	1.042682856		0.774090973	
	0.154659076	0.148691506	1.058564035		1.107558198	
	0.114865326	0.148621263	1.234665883		1.239527542	
	0.125160618	0.083831091	0.458390699		0.96450181	
	0.082860477	0.159885481	1.283442496		0.786187236	
	0.134437129	0.145752008	0.575686068		0.823824756	
	0.188300122	0.119608627	0.493449836		0.694855447	
	0.091346646	0.134516954	0.752415387		1.271358832	
	0.031256708	0.120756798	1.427341105		1.305962068	
	0.148623505	0.105640553	0.995488358		1.274142428	
	0.062461131	0.100213051	0.852008641		1.061850022	
	0.070147005	0.177842066	0.52790703		0.71593132	
	0.119128402	0.100087751	1.587259645		1.170337981	
	0.076649428	0.148178866	1.30213048		0.644662689	
	0.089202374	0.14167771	0.373333696		0.390745191	
	0.092622686	0.240965229	1.166008243		0.735893263	
	0.115073276	0.336883976	0.803569043		0.974205868	
	0.086361538	0.157860027	0.99787494		0.944999071	
	0.057520814	0.393176625	1.630434272		0.481876985	
	0.046794274	0.166098378	0.265204319		1.14242614	
	0.076395772	0.109549489	0.794305333		1.334905076	
Average	0.111352218	0.154971786	0.962836292		0.93303551	
Minimum	0.031256708	0.06977203	0.265204319		0.390745191	

	RW/500/DPC/DV/300 (G)					
	Rank [TL 1e-8]	Rank [TL 1e-6]	Propotional	Top	Shift Linear	Custom
	0.119601689	0.075412785	0.97199553	#	1.47219614	##
	0.073863027	0.062814174	1.21140836		0.97404737	
	0.070586783	0.150917323	1.20058534		1.50090547	
	0.036081489	0.230512266	1.18874521		1.76817068	
	0.046924615	0.206920069	0.48006885		0.771374	
	0.090907396	0.13624692	1.21300367		0.86554674	
	0.128690787	0.115050181	0.77023792		1.14678315	
	0.122642778	0.12188685	0.93507897		0.67797867	
	0.07993613	0.088740305	0.44010117		1.26174695	
	0.040341418	0.196397249	0.35304939		0.69840224	
	0.060647997	0.202380324	0.66720872		0.98600753	
	0.041312452	0.068988134	1.67632968		0.96313258	
	0.045713495	0.140169648	0.67463696		1.43956471	
	0.126852121	0.195545583	1.71993689		1.1382813	
	0.182048287	0.066061083	0.85381698		0.78316673	
	0.03233399	0.067141057	1.19777881		1.55018719	
	0.150689977	0.04620377	1.15034611		0.92746265	
	0.27954007	0.090640413	1.14937497		0.95652583	
	0.222116907	0.112393715	1.03278366		0.75030494	
	0.10099377	0.144061972	1.79413303		0.91490531	
	0.07351391	0.093661053	0.84156006		0.7965818	
	0.077321189	0.135874932	1.37202531		0.93534164	
	0.089766213	0.034838351	1.30850614		0.53862206	
	0.04200132	0.104408815	1.35063554		1.03726012	
	0.07158146	0.076264581	0.86119675		0.65023993	
Average	0.096240371	0.118541262	1.05658176		1.02018943	
Minimum	0.03233399	0.034838351	0.35304939		0.53862206	

	RW/500/SCATTERED/DV/300(G)					
	Rank [TL 1e-8]	Rank [TL 1e-6]	Propotional	Top	Shift Linear	Custom
	0.060113787	0.052375536	0.69855398	#	0.86356177	##
	0.091834675	0.270767831	1.78289884		1.02749815	
	0.058827243	0.097856396	1.13069524		1.43511558	
	0.191928769	0.054553318	1.41295864		1.25218714	
	0.03675482	0.102745138	0.95316899		0.90559419	
	0.091835347	0.06083516	1.23438391		1.58718346	
	0.080115316	0.060354313	1.45391422		1.55076617	
	0.065007472	0.066110253	0.93148126		0.95767736	
	0.049702156	0.158981258	1.49666515		1.18952835	
	0.03873713	0.035122176	1.34137635		1.30520252	
	0.035110258	0.116733419	1.03502286		1.97357	
	0.036367369	0.064678947	1.49180234		0.84340724	
	0.096307072	0.06729036	1.07014159		1.26286396	
	0.056702925	0.07668117	1.63368974		1.31028532	
	0.111308634	0.033008047	1.39739206		0.95356757	
	0.026835676	0.044271514	1.14751134		1.60141321	
	0.066693061	0.134254393	1.41347576		1.77440565	
	0.171512396	0.027734199	1.18505477		1.32397063	
	0.088988779	0.044250823	1.35673972		1.10644822	
	0.073742703	0.091690543	1.4542545		1.17547276	
	0.058412169	0.055408529	1.60408457		1.28629105	
	0.16274599	0.129966958	1.78000846		1.80557193	
	0.086354703	0.090511234	1.27824008		1.67582146	
	0.083770984	0.046078047	1.27736266		1.46082662	
	0.034244136	0.083770105	1.56110505		1.19374994	
Average	0.078158143	0.082641187	1.32487928		1.31287921	
Minimum	0.026835676	0.027734199	0.69855398		0.84340724	

Attempted to access thisPopulation(0,:); index must be a positive integer or logical.
Too many input arguments
SPC - Single point crossover
DPC- Double Point Crossover
(G)- No. of Generations

TL- Tolerence Level
RW- Roulette wheel
DV- Double Vector

The optimal value obtained by the present approach for each function mentioned are presented in Table 6. With the help of this table, the results are compared with the work[7].

Table 6 Comparison Table

Functions	Previous Work [7]	Present Technique (Minimum based approach)	Sensitivity Analysis
Arithmetic Application	2.231845643	0.040217	**0.007596631**
Combustion Problem	0.359195584	**0.024030224**	0.026835676

6. CONCLUSION

In this work, a method has been proposed to enhance the exploration and exploitation scheme of GA. The parameters of GA have been set by empirical studies. Multiple experiments have been carried out. Population size in the experiment are varied to make empirical analysis as effective as possible. The efficiency of the proposed work is encountered by standard benchmark problems. By careful and considerate analysis of the data obtained in the experiments, we concluded, in most cases, Roulette wheel selection method give better results as compared to Tournament selection method. The results by the present approach are much better than the, Evolutionary algorithm used by Grosan[7]. Further, sensitivity analysis has been performed for evaluating the effect of variations in parameters. We see that improving the tolerance level to 1e-8 in the sensitivity analysis resulted in better outcome. We observe that for Arithmetic application problem, the optimal value obtained by Grosan[7] is 2.231845643 whereas the values obtained by improving the tolerance level is 0.007596631 which is better in comparison with the later one. One major observation by the present work is that the convergence of GA or simply, attainment of better optimal results, majorly depends on the selection of parameters which help in tuning the algorithm. Thus, one should always try to tune the algorithm by suitable selection of the parameters which is commonly, problem dependent.

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