Journal of Computer and Mathematical Sciences, *Vol.8(9), 430-441 September 2017* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Soft Minimal Open and Soft Maximal Open Maps in Soft Topological Spaces

Chetana C and K. Naganagouda

Research Scholar,
Sri Siddhartha Academy of Higher Education, Tumakuru-57210, Karnataka, INDIA.
Department of Mathematics,
Sri Siddhartha institute of Technology, Tumakuru-572107, Karnataka, INDIA.
email: chetanachandran@gmail.com, kngoud15@gmail.com.

(Received on: August 14, Accepted: September 12, 2017)

ABSTRACT

In this paper we introduce a new class of soft maps called soft minimal open maps and soft maximal open maps in soft topological space. A map f:$(X, \tau, E) \rightarrow (Y, \mu, E)$ is called soft minimal open (briefly soft min-open) if f(F,E) is soft open set in Y for every soft minimal open set (F, E) in X. A map f:$(X, \tau, E) \rightarrow (Y, \mu, E)$ is called soft maximal open (briefly soft max-open) if f(F,E) is soft open set in Y for every soft maximal open set (F, E) in X. Also some of their properties have been investigated.

2010 Mathematics classification: 54C05.

Keywords: soft minimal open maps and soft maximal open maps.

1. INTRODUCTION AND PRELIMINARIES

In year 2001 and 2003, F. Nakaoka and N. Oda[1,2,3] presented and contemplated minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets, which are subclasses of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are known as maximal closed sets and minimal closed sets respectively.In year 1999, Russian specialist Molodtsov[4], started the idea on soft sets as another scientific instrument to manage vulnerabilities while demonstrating issues in building material science, software engineering, financial aspects, sociologies and restorative sciences. In Molodtsov[5], connected effectively in bearings, for example, smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and hypothesis to estimation. The soft set is an accumulation of inexact portrayals of an article. Likewise he also indicated why soft set hypothesis is excluded from the parametrization deficiency disorder of fuzzy set hypothesis, rough set hypothesis, probability theory and game theory.

In year 2002 and 2003, Maji, Biswas and Roy[6], introduce few new statements on soft sets and displayed first practical use of soft sets in decision making problems that depends on the lessening of parameters to keep the ideal decision objects. In 2003, Maji, Biswas and Roy[7], examined the hypothesis of the soft sets started from Molodtsov. They characterized equity of two soft sets, subset and super set of the soft set, complement of the soft set, null soft set and absolute soft set along cases. The Soft binary operations like AND, OR furthermore the operations of union and the intersection were also characterized. In year 2005, D. Chen[8], introduced another meaning of the soft set parametrization lessening and correlation with property decrease on soft set hypothesis. In 2005 year, D. Pie, D. Miao[9], examined the difference between soft sets and data frameworks. They demonstrated soft sets are a class of unique data frameworks. In 2008, Z. Kong, L. Gao, L. Wong, S. Li[10], presented the thought of ordinary parameter decrease of soft sets and its utilization to explore the issue of imperfect decision and included a parameter set in soft sets.

As of late, specialists have contributed a lot towards fuzzification of Soft Set Theory. In 2001, Maji P. K., Biswas R and Roy A.R.[11], presented the idea of Fuzzy Soft Set and a few properties with respect to fuzzy soft union, intersection, supplement of a fuzzy soft set, De Morgan Law and so forth. In 2007, X. Yang, D. Yu, J. Yang, C. Wu[12], consolidated the interim esteemed fuzzy set and soft set models and presented the idea of interim esteemed fuzzy soft set.

Topological of soft set and fuzzy soft set of topological structures are studied by a few creators as of late. In 2011, Muhammad Shabir and Munazza Naz and Naim Cagman *et al*. started the investigation of soft topology and soft topological spaces independtly. Muhammad Shabir and Munazza Naz[13], presented the thought of soft topological spaces which are characterized over an underlying universe with a settled arrangement of parameters and demonstrated that a soft topological space gives a parameterized group of topological spaces. They presented the meanings of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. Additionally they got few fascinating results for soft separation axioms, which are truly profitable for exploration in this field. N. Cagman, S. Karatas and S. Enginoglu[14], characterized the soft topology on a soft set, and displayed its related properties and establishments of the hypothesis of soft topological spaces. The thought of soft topology by Naim Cagman *et al*. is broad than that by Shabir and Naz.

In the meantime, Abdulkadir Aygunoglu and Halis Aygun[15], presented soft topological spaces and soft continuity of soft mappings. They additionally explored starting soft topologies and soft compactness. In 2011, Sabir Hussain and Bashir Ahmad[16], examined the properties of soft open (closed), soft neighborhood and soft closure. Likewise characterized and examined the properties of soft interior, soft exterior and soft boundary which are essential for further research on soft topology and establishments of the hypothesis of soft topological spaces. In 2012, Bashir Ahmad and Sabir Hussain[17], characterized soft exterior and examined its essential properties and set up a few critical results relating soft interior, soft exterior, soft closure, and soft boundary in soft topological spaces. In addition, they described soft open sets, soft closed sets and soft clopen defines by means of soft boundary. In 2007, H. Hazra, P. Majumdar and S. K. Samanta[18], presented the thoughts of topology on soft subsets and soft topology. Some essential properties of these topologies are studied. In 2004, Metin Akdag and Alkan Ozkan[19], presented and examined the idea of soft α-Open sets and soft α-constant functions. In 2015, A. Selvi and I. Arockiarani[20,21], presented and contemplated the idea of soft almost g-continuous functions. In 2014, Metin Akdag and Alkan Ozkan[22], presented and considered the idea of soft β-Open Sets and soft β-Continuous functions. In 2010, Pinaki Majumdar and S. K. Samanta,[23] presented and examined the idea of soft mappings in soft topological spaces. In 2011, S. S. Benchalli, Basavaraj M.I and R. S. Wali[24] presented the idea On Minimal Open Sets and Maps in Topological Spaces. In 2015, Hai-Long Yang, Xiuwu Liao and Sheng-Gang Li[25] presented the idea On soft continuous mappings and soft connectedness of soft topological spaces. In 2017, Chetana C. and Naganagouda K.[27], presented the idea on Soft Minimal Continuous and Soft Maximal Continuous Maps in soft topological spaces.

We review the accompanying statements, which are requirements for present study.

1.1 Definition [4]: Let U be an initial universe and E be the set of parameters. Let P(U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

1.2. Definition [5]: For two soft sets (F, A) and (G,B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

i) $A \subseteq B$ and

ii) for all $e \in A$, F(e) and G(e) are identical approximations.

We write (F, A) $\tilde{\subseteq}$ (G, B). (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by (F, A) $\tilde{\supseteq}$ (G, B).

1.3. Definition [5]: Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

1.4. Definition [5]: Let $E = \{e_1, e_2, e_3 . . . , e_n\}$ be a set of parameters. The NOT set of $\neg P$ denoted by $\neg E$ is defined by $\neg E=\{e_1, e_2, e_3 . . . , e_n\}$, where $\neg e_i$ = not e_i for all i.

1.5. Definition [5]: The complement of a soft set (F, A) is denoted by $(F,A)^c=(F^c, \neg A)$ where, $F^c: \neg A \to P(U)$ is a mapping given by $F^c(\alpha) = U \setminus F(\neg\alpha)$, for all $\alpha \in \neg A$

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F , A)$.

1.6. Definition [5]: A soft set (F, A) over U is said to be a NULL soft set denoted by "ϕ" if $\forall\ \varepsilon \in A$, $F(\varepsilon) = \phi$, (null-set).

1.7. Definition [5]: If (F, A) and (G, B) are two soft sets then (F, A) AND (G, B) denoted by (F, A)Λ(N, B) is defined by (F, A) Λ (G, B) = (H, A × B), where $H((\alpha, \beta)) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \cap A \times B$.

1.8. Definition [5]: If (F, A) and (G, B) are two soft sets then (F, A) OR (G, B) denoted by (F, A) V (G, B) is defined by (F, A) V (G, B) = (O, A × B) where, $O((\alpha, \beta)) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \cap A \times B$.

1.9. Definition [5]: The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where C = A∪B and for all e ε C,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$ We write (F, A) ∪ (G, B) = (H, C).

1.10. Definition [8]: The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted (F, A) ∩ (G, B), is defined as C=A∩B, and H(e) = M (e) ∩ N(e) for all e ∈ C

1.11. Definition [13]: Let τ be the collection of soft sets over X; then τ is called a soft topology on X if τ satisfies the following axioms:

i) Φ, X belong to τ.

ii) The union of any number of soft sets in τ belongs to τ.

iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be soft open in X. A soft set (F, E) over X is said to be soft closed in X if its relative complement $(F, E)^c$ belongs to τ .

1.12. Definition [13]: Let (F, E) be a soft set over X and x ∈ X. We say that x ∈ (F, E) read as 'x' belongs to the soft set (F, E), whenever x∈ F(α) for all α ∈ E.

Note that for x ∈ X, x ∉ (F, E) if x ∉ F(α) for some α ∈ E.

1.13. Definition [13]: Let x ∈ X; then (x, E) denotes the soft set over X for which x(α) = {a}, for all α ∈ E.

1.14. Definition [2]: Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X and x∈ X. Then (G, E) is said to be a soft neighborhood of a if there exists a soft open set (F, E) such that x ∈ (F, E) $\tilde{\subseteq}$ (G, E).

1.15. Definition [2]: Let (X, τ, E) be a soft topological space and (A, E) be a soft set over X.

i) The soft interior of (A, E) is the soft set sint(A, E) = ∪{(O, E): (O, E) is soft open and (O, E) $\tilde{\subseteq}$ (A, E)}.

ii) The soft closure of (A, E) is the soft set scl(A, E) = ∩{(F, E) : (F, E) is soft closed and (A, E) $\tilde{\subseteq}$ (F, E)}.

1.16 Definition [1]: A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is ϕ or U.

1.17 Definition [2]: A proper nonempty open subset U of a topological space X is said to be maximal open set if any open set which contains U is X or U.

1.18 Definition [3]: A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ϕ or F.

1.19 Definition [3]: A proper nonempty closed subset F of a topological space X is said to be maximal closed set if any closed set which contains F is X or F.

1.20 Definition [26]: A proper nonempty soft open subset (F,E) of (X,τ,E) is said to be a soft minimal open set if and only if any soft open set which is contained in (F,E) is either ϕ or (F,E) itself.

1.21 Definition [26]: A proper nonempty soft open subset (F,E) of (X,τ,E) is said to be a soft maximal open set if and only if any soft open set which is contains in (F,E) is either X or (F,E) itself.

1.22 Definition [22]: A soft set (F, E) of a soft topological space (X, τ, E) is called soft α-open set if (F, E)$\tilde{\subset}$ int(cl(int(F,E))). The complement of soft α-open set is called soft α-closed set.

1.23 Definition [22]: A soft set (F, E) is called soft preopen set (resp., soft semiopen) in a soft topological space *X* if (F, E)$\tilde{\subset}$ int(cl(F, E)) (resp., (F, E)$\tilde{\subset}$ cl(int(F, E))).

1.24 Definition [22]: A soft mapping f:X → Y is said to be soft α- continuous if the inverse image of each soft open subset of Y is a soft α-open set in X.

1.25 Definition[22]:A soft mapping f:X → Y is called soft precontinuous(resp., soft semicontinuous) if the inverse image of each soft open set in Y is soft preopen (resp., soft semiopen) in X.

1.26 Definition [20]: A function f:X → Y is called soft almost open(soft almost closed), if the image of every soft regular open subset of X is soft open(soft regular closed) subset of Y.

1.27 Definition [22]: A subset (F, E) of a topological space X is called soft generalized-closed (soft g -closed), if cl(F,E) ⊂ (G,E) whenever (F,E) ⊂ (G,E) and (G,E) is soft open in X.

1.28 Definition [19]: A subset (F, E) of a soft topological space X is called soft regular closed, if cl(int(F,E)) = (F,E). The complement of soft regular closed set is soft regular open set.

1.29 Definition [19]: A soft set (*F*, *E*) of a soft topological space (*X*, τ, *E*) is said to be soft β-open if (F,E)$\tilde{\subset}$cl(int(cl(F,E))).

1.30 Definition [19]: Let X and Y are two nonempty sets and E' be a parameter set. Then the mapping f:X^1→U(Y^1) is called a soft mapping from X to Y under E',where Y' is the collection of all mappings from X to Y.

1.31 Definition[17]: A soft mapping f : X → Y is called soft β-continuous (resp., soft α-continuous, soft precontinuous , and soft semicontinuous) if the inverse image of each soft open set in *Y* is soft β-open (resp., soft α-open, soft preopen, and soft semiopen) set in *X*.

1.32 Definition[24]: Consider X and Y are topological spaces. The mapping f:X → Y is known as

i) minimal continuous (min-continuous) if f^{-1}(F,E) is an open set in X for each minimal open set (F,E) in Y.

ii) maximal continuous (max-continuous) if f^{-1}(F,E) is an open set in X for each maximal open set (F,E) in Y.

1.33 Definition[25]: Let (X, τ_1, E) and (Y, τ_2 , E) be two soft topological spaces over X and Y respectively, and f be a mapping from X to Y. If $\forall (G,E) \in \tau_2$ we have mapping $\overleftarrow{f}(G,E) \in \tau_1$ then f is called a soft continuous mapping from(X, τ_1, E) to (Y, τ_2 , E).

1.34 Definition [27]:]: Consider (X, τ, E) and (Y, μ, E) are soft topological spaces. The mapping f: (X, τ, E) →(Y, μ, E) is said to be
i) Soft minimal continuous (soft min-continuous) if f^{-1}((F,E)) is a soft open set in X for each soft minimal open set (F,E) in Y.
ii)Soft maximal continuous (soft max-continuous) if f^{-1}((F,E)) is a soft open set in X for each soft maximal open set (F,E) in Y.

1.35 Definition [28]: Consider (X, τ, E) and (Y, μ, E) are soft topological spaces. The mapping f: (X, τ, E) →(Y, μ, E) is said to be
i) Soft minimal irresolute(briefly soft min-irresolute) if f^{-1}(F,E) is soft minimal open set in X for every soft minimal open set (G,E) in Y.
ii) Soft maximal irresolute (briefly soft max-irresolute) if f^{-1}(F,E) is soft maximal open set in X for every soft maximal open set (G,E) in Y.
iii) Soft minimal-maximal continuous (briefly soft min-max continuous) if f^{-1} ((F,E)) is soft maximal open set in X for every soft minimal open set (G,E) in Y.
iv) Soft maximal-minimal continuous (briefly soft max-min continuous) if f^{-1} ((F,E)) is soft minimal open set in X for every soft maximal open set (G,E) in Y.

2. SOFT MINIMAL OPEN MAPS AND MAXIMAL OPEN MAPS

2.1 Definition: Let X and Y be soft topological spaces. A map f: X→Y is called
i) soft minimal open (soft min-open) if f(F,E) is soft open set in Y for every soft minimal open set (F,E) in X.
ii) soft maximal open (soft max-open) if f(F,E) is soft open set in Y for every soft maximal open set (F,E) in X.

2.2 Theorem: Every soft open map is soft minimal open map but not conversely.
Proof: Let f:X→Y be soft open map and let (F,E) be any soft minimal open set in X. Since every soft minimal open set is soft open set, (F,E) is soft open set in X. Since f is soft open map, f(F,E) is soft open set in Y. Hence f is a soft minimal open map.

2.3 Example: Let $X = Y = \{a,b,c\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E), (F_2, E)\}$ *where*
$F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$ and
$\mu = \{Y, \phi, (F_1, E), (F_2, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$;
$F_2(e_1) = \{a,c\}, F_2(e_2) = \{b,c\}$.

Let f: X→Y be soft identity map. Then f is a soft minimal open map but it is not soft open map, since for the soft open set (F_2, E) in X, $f((F_2, E))=(F_2, E)$ which is not soft open set in Y.

2.4 Theorem: Every soft open map is soft maximal open map but not conversely.
Proof: Similar to that of Theorem 2.2.

2.5 Example: Let $X = Y = \{a,b,c\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E), (F_2, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$ and $\mu = \{Y, \phi, (F_1, E), (F_2, E)\}$ *where* $F_1(e_1) = \{b\}, F_1(e_2) = \{c\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$.

Let f: X→Y be soft identity map. Then f is a soft maximal open map but it is not soft open map, since for the soft open set (F_1, E) in X, $f((F_1, E)) = (F_1, E)$ which is not soft open set in Y.

2.6 Remark: soft Minimal open and soft maximal open maps are independent of each other.

2.7 Example: In Example 2.3, f is a soft minimal open map but it is not a soft maximal open map. In Example 2.5, f is a soft maximal open map but it is not a soft minimal open map.

2.8 Remark: soft Minimal (resp. soft maximal) open and almost soft open maps are independent of each other.

2.9 Example: Let $X = Y = \{a,b,c\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{b\}, F_2(e_2) = \{c\}$; $F_3(e_1) = \{a,b\}, F_3(e_2) = \{b,c\}$ and $F_4(e_1) = \{a,c\}, F_4(e_2) = \{a,b\}$; $\mu_1 = \{Y, \phi, (F_1, E), (F_2, E), (F_3, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{b\}, F_2(e_2) = \{c\}$; $F_3(e_1) = \{a,b\}, F_3(e_2) = \{b,c\}$ and $\mu_2 = \{Y, \phi, (F_1, E), (F_2, E), (F_3, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$; $F_3(e_1) = \{a,c\}, F_3(e_2) = \{a,b\}$ and $\mu_3 = \{Y, \phi, (F_1, E), (F_2, E)\}$ *where* $F_1(e_1) = \{b\}, F_1(e_2) = \{a\}$; $F_2(e_1) = \{b,c\}, F_2(e_2) = \{b,c\}$. Let f: (X, τ) → (Y, μ_1) be soft identity map. Then f is a soft minimal open map but it is not soft almost open map, since for the regular soft open set (F_2, E) in A, f((F_2, E))= (F_2, E) which is not soft open set in Y. Let g: (X, τ) → (Y, μ_2) be soft identity map. Then g is a soft maximal open map but it is not soft almost open map, since for the regular soft open set (F_2, E) in A, $g((F_2, E))=(F_2, E)$ which is not soft open set in Y. Let h: (X, τ) → (Y,μ_3) be soft identity map. Then h is soft almost open map but it is not a soft minimal (resp. soft maximal) open map, since for the soft minimal (resp. soft maximal) open set (F_1, E) (resp. (F_1, E)) in , $h((F_1, E))=(F_1, E)$ (resp. h((F_1, E))= (F_1, E)) which is not soft open set in Y.

2.10 Remark: soft Minimal (resp. soft maximal) open and soft preopen (resp. soft α-open, soft β-open) maps are independent of each other.

2.11 Example: Let $X = Y = \{a,b,c\}, E = \{e_1, e_2\}, \tau_1 = \{X, \phi, (F_1, E), (F_2, E)\}$*where*

$F_1(e_1)=\{b\}, F_1(e_2)=\{a\}$; $F_2(e_1)=\{b,c\}, F_2(e_2)=\{a,c\}$ and $\tau_2=\{X,\phi,(F_1,E),(F_2,E)\}$ *where* $F_1(e_1)=\{c\}, F_1(e_2)=\{b\}$; $F_2(e_1)=\{a,c\}, F_2(e_2)=\{a,b\}$ and $\tau_3=\{X,\phi,(F_1,E)\}$ *where* $F_1(e_1)=\{a,b\}, F_1(e_2)=\{b,c\}$; and $\mu_1=\{Y,\phi,(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$ *where* $F_1(e_1)=\{a\}, F_1(e_2)=\{b\}$; $F_2(e_1)=\{b\}, F_2(e_2)=\{c\}$; $F_3(e_1)=\{a,b\}, F_3(e_2)=\{b,c\}$; $F_4(e_1)=\{a,c\}, F_4(e_2)=\{a,b\}$; $\mu_2=\{Y,\phi,(F_1,E)\}$ *where* $F_1(e_1)=\{a\}, F_1(e_2)=\{b\}$.Let *f*: (X, τ_1) $\to$ (Y, μ_1) be soft identity map. Then *f* is a soft minimal open map but it is not a soft preopen (resp. soft α-open, soft β-open) map, since for the soft open set (F_2,E) in X, *f*((F_2,E))= (F_2,E) which is not a soft preopen (resp. soft α-open, soft β-open) set in Y. Let *g*: (Y, τ_2) $\to$ (Y, μ_1) be soft identity map. Then *g* is a soft maximal open map but it is not a soft preopen (resp. soft α-open, soft β-open) map, since for the soft open set (F_1,E) in X, *g*((F_1,E))= (F_1,E) which is not a soft preopen (resp. soft α-open, soft β-open) set in Y. Let *h*: (X, τ_3) $\to$ (Y, μ_2) be soft identity map. Then *h* is a soft preopen (resp. soft α-open, soft β-open) map but it is not a soft minimal (resp. soft maximal) open map, since for the soft minimal (resp. soft maximal) open set (F_1,E) in X, *h*((F_1,E))= (F_1,E) which is not soft open set in Y.

2.12 Remark: soft Minimal (resp. soft maximal) open and soft g-open maps are independent of each other.

2.13 Example: Let $X=Y=\{a,b,c\}, E=\{e_1,e_2\}, \tau=\{x,\phi,(F_1,E)\}$ *where* $F_1(e_1)=\{a,b\}, F_1(e_2)=\{b,c\}$; and $\mu=\{Y,\phi,(F_1,E)\}$ *where* $F_1(e_1)=\{a,b\}, F_1(e_2)=\{b,c\}$; .Let *f*: X→Y be soft identity map. Then *f* is a soft g-open map but it is not a soft minimal (resp. soft maximal) open map, since for the soft minimal open (resp soft. maximal open) set (F_1,E) in X, $f(F_1,E)$)= (F_1,E) which is not soft open set in Y.

2.14 Example: Let $X=Y=\{a,b,c\}, E=\{e_1,e_2\}, \tau_1=\{X,\phi,(F_1,E),(F_2,E)\}$ *where* $F_1(e_1)=\{b\}, F_1(e_2)=\{c\}$; $F_2(e_1)=\{b,c\}, F_2(e_2)=\{a,c\}$ and $\tau_2=\{Y,\phi,(F_1,E),(F_2,E)\}$ *where* $F_1(e_1)=\{b\}, F_1(e_2)=\{c\}$; $F_2(e_1)=\{a,c\}, F_2(e_2)=\{a,b\}$ and $\mu=\{Y,\phi,(F_1,E),(F_2,E),(F_3,E)\}$ *where* $F_1(e_1)=\{a\}, F_1(e_2)=\{c\}$; $F_2(e_1)=\{b\}, F_2(e_2)=\{a\}$; $F_3(e_1)=\{a,b\}, F_3(e_2)=\{a,c\}$ .Let *f*: (X, τ_1)→(Y, μ) be soft identity map. Then *f* is a soft minimal open map but it is not a soft g-open map, since for the soft open set (F_2,E) in X, *f*((F_2,E))= (F_2,E) which is not a soft g-open set in Y. Let *g*: (X, τ_2) $\to$ (Y, μ) be a soft map defined by *g*(a)=*g*(c)=b and *g*(b)=c. Then *g* is a soft maximal open map but it is not a soft g-open map, since for the soft open set (F_1,E) in X, *g*({b})={c} which is not a soft g-open set in X.

2.15 Remark: From the above discussion and known results we have the following implications.

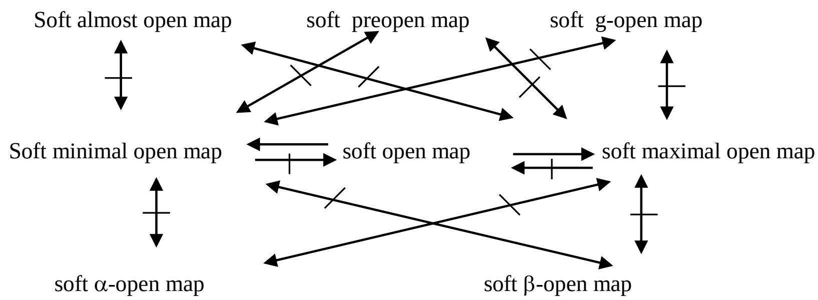

Diagram 2.4

2.16 Theorem: A map *f*: X→Y is soft minimal open if and only if for any soft subset (G,E) in Y and each soft maximal closed set (F,E) in X containing f^{-1}(G,E), there is a soft closed set (H,E) in Y such that (G,E) ⊂(H,E) and f^{-1}(H,E) ⊂ (F,E).

Proof: Suppose *f* is a soft minimal open map. Let (G,E) be any soft subset in Y and (F,E) is a soft maximal closed set in X such that f^{-1}(G,E) ⊂(F,E). Then (H,E) =Y−*f*(A-(F,E)) is a soft closed set in Y containing (G,E) such that f^{-1}(H,E) ⊂(F,E).
Conversely, suppose that (I,E) is a soft minimal open set in X. Then f^{-1}(Y−*f*(I,E))⊂X-(I,E)and X-(I,E)is a soft maximal closed set in X. By hypothesis, there is a soft closed set (H,E) in Y such that Y−*f*(I,E)⊂ (H,E) and f^{-1}(H,E) ⊂X−(I,E). Therefore (I,E)⊂X−f^{-1}(H,E). Hence Y-(H,E) ⊂*f*(I,E)⊂*f*(X−f^{-1}(H,E))⊂Y−(H,E) which implies *f*(I,E)=Y−(H,E). Since Y−(H,E) is soft open set in Y, *f*(I,E) is soft open set in Y. Hence *f* is a soft minimal open map.

2.17 Theorem: A map *f*: X→Y is a soft maximal open if and only if for each soft subset (G,E) in Y and each soft minimal closed set (I,E) in X containing f^{-1}(G,E), there is a soft closed set (H,E) in Y such that (G,E) ⊂(H,E) and f^{-1}(H,E) ⊂(I,E).
Proof: Similar to that of Theorem 2.16

1.4.18 Remark: If *f*: X→Y is soft minimal open map and (D,E)⊂X then f_D: (D,E)→Y need not be a soft minimal open map.

2.19 Example: Let $X = Y = \{a,b,c\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E), (F_2, E)\}$ *where*
$F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$ and
$\mu = \{Y, \phi, (F_1, E), (F_2, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,c\}, F_2(e_2) = \{b,c\}$.
Let *f*: X→Y be soft identity map. Then clearly *f* is a soft minimal open map. Let (D,E)= $F(e_1) = \{b,c\}, F(e_2) = \{a,c\}$ be with relative topology τ_D={ϕ, (F_1, E),(D,E)} where $F_1(e_1) = \{b\}, F_1(e_2) = \{a\}$. Then f_D: (D,E)→Y is not a soft minimal open map, since for the soft minimal open set (F_1, E) in (D,E), f_D((F_1, E))= (F_1, E) which is not soft open set in Y.

2.20 Remark: If *f*: X→Y is soft maximal open map and (D,E)⊂X then f_D: (D,E)→Y need not be a soft maximal open map.

2.21 Example: In Example 2.19, f is a soft maximal open map but f_X is not a soft maximal open map, since for the soft maximal open set (F_1, E) in (D,E), $f_D((F_1, E))= (F_1, E)$ which is not soft open set in Y.

2.22 Remark: The composition of soft minimal open maps is need not be a soft minimal open map.

2.23 Example: Let $X = Y = Z = \{a,b,c\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E)\}$ *where* $F_1(e_1) = \{a,b\}, F_1(e_2) = \{b,c\}$; $\mu = \{Y, \phi, (F_1, E), (F_2, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$ and $\eta = \{Z, \phi, (F_1, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$. Let f: X→Y and g: Y→Z be the soft identity maps. Then clearly f and g are soft minimal open maps but gof: X→Z is not a soft minimal open map, since for the soft minimal open set (F_2, E) in X, $(g\text{o}f)((F_2, E))=(F_2, E)$ which is not soft open set in Z.

2.24 Theorem: Let f: X→Y be a soft minimal open and g: Y→Z be soft open maps. Then gof: X→Z is a soft minimal open map.

Proof: Let (F, E) be any soft minimal open set in X. Since f is soft minimal open, $f(F, E)$ is soft open set in Y. Again since g is soft open, $g(f(F, E))=(g\text{o}f)(F, E)$ is soft open set in Z. Hence gof is a soft minimal open map.

2.25 Remark: The composition of soft maximal open maps is need not be a soft maximal open map.

2.26 Example: : Let $X = Y = Z = \{a,b,c\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $\mu = \{Y, \phi, (F_1, E), (F_2, E)\}$ *where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$ and $\eta = \{Z, \phi, (F_1, E)\}$ *where* $F_1(e_1) = \{a,b\}, F_1(e_2) = \{b,c\}$.Let f: X→Y and g: Y→Z be the soft identity maps. Then clearly f and g are soft maximal open maps but gof: X→Z is not a soft maximal open map, since for the soft maximal open set (F_1, E) in X, $(g\text{o}f)((F_1, E))= (F_1, E)$ which is not soft open set in Z.

2.27 Theorem: Let f: X→Y be a soft maximal open and g: Y→Z be soft open maps. Then gof: X→Z is a soft maximal open map.

Proof: Similar to that of Theorem 2.24.

2.28 Theorem: Let f: X→Y and g: Y→Z be the soft maps and let gof: X→Z be a soft minimal open map. Then

i) g is soft minimal open if f is soft minimal irresolute and surjective.

ii) f is soft minimal open if g is soft continuous and injective.

Proof: i) Let (F, E) be any soft minimal open set in Y. Since f is soft minimal irresolute and surjective, $f^{-1}(F, E)$ is a soft minimal open set in X. Again since gof is soft minimal open, $(g\text{o}f)(f^{-1}(F, E))$ is soft open set in Z. But $(g\text{o}f)(f^{-1}(F, E)) = g(f(f^{-1}(F, E)))=g(F, E)$ is soft open set in Z. Hence g is a soft minimal open map.

ii) Let (F, E) be any soft minimal open set in X. Since $g \circ f$ is soft minimal open, $(g \circ f)(F, E)$ is soft open set in Z. Again since g is soft continuous and injective, $g^{-1}((g \circ f)(F, E)$ is soft open set in Y. But $g^{-1}((g \circ f)(F, E))= g^{-1}(g(f(F, E))=f(F, E)$ is soft open set in Y. Hence f is a soft minimal open map.

2.29 Theorem: Let f: X→Y and g: Y→Z be soft maps and let $g \circ f$: X→Z be a soft maximal open map. Then

i) g is soft maximal open if f is soft maximal irresolute and surjective.

ii) f is soft maximal open if g is soft continuous and injective.

Proof: Similar to that of Theorem 2.28.

REFERENCES

1. Nakaoka,F.,Oda,N., "Some properties of maximal open sets," *Int. J. Math. Sci*., 21, pp. 1331- 1340 (2003).
2. Nakaoka,F.,Oda,N., "Some applications of maximal open sets," *Int. J. Math. Sci*., 27 (8), pp.471- 476 (2001).
3. Nakaoka,F.,Oda,N., "Minimal closed sets and maximal closed sets," *Int. J. Math. Sci*., pp.1-8 (2006).
4. Molodtsov,D.A., "Soft set theory-first results," *Comp. and Math. with Appls*.,37,pp-19-31 (1999).
5. Molodtsov, D.A., Leonov,V.Y., Kovkov, D.V., "Soft sets technique and its application," Nechetkie sistemy myagkie vychisleniya,1(1),pp.8-39 (2006).
6. Maji,P.K.,Biswas,R.,Roy,R., "Soft set theory," *Comp. and Math*.,45,pp.555-562 (2003).
7. Maji,P.K.,Biswas,R.,Roy,R., "An application of soft sets in a decision making problem," *Comp. and Math*.,44,pp.1077-1083 (2002).
8. Chen,D., "The parametrization reduction of soft sets and its applications," *Comp. and Math. with Appls*.,49,pp-757-763 (2005).
9. Pie,D.,Miao,D., "From soft sets to information systems, granular computing," *IEEE Int. Conf*.,2,pp-617-621 (2005).
10. Kong,Z.,Gao,L.,Wong,L.,Li,S., "The normal parameter reduction of soft sets and its algorithm," *J. Comp. Appl. Math*.,56,pp.3029-3037 (2008).
11. Maji,P.K.,Biswas,R.,Roy.A.R., "Fuzzy soft sets," *J. of Fuzzy Math*.,9 (3),pp.589-602 (2001).
12. Yang,X.,Yu,D.,Yang,J.,Wu,C., "Generalization of soft sets theory: from crisp to fuzzy case," *Fuzzy Inform. Engin*.,40,pp.345-354 (2007).
13. Muhammad Shabir, Munazza Naz, "On soft topological spaces," *Comp. Math. Appl*., 61,pp.1786-1799 (2011).
14. Cagman,N., Karatas, S., Enginoglu, S., "Soft topology," *Comp. and Math. with Appl*., 62, pp.351- 358 (2011).
15. Aygunoglu, A., Aygun, H., "Some note on soft topological spaces," *Neural Comp. Appl*.,21,pp.113-119 (2012).

16. Hussain,S.,Ahmad,B., “Some properties of soft topological spaces,” *Comp. Math. Appl*., 62, pp.4058-4067 (2011).
17. Bashir Ahmad, Sabir Hussain, “On some structures of soft topology,” *J. of Math. Sciences*, 6(64),pp.1-7 (2012).
18. Hazra,H., Majumdar,P.,Samanta, S.K., “Soft topology,” *Fuzzy Inf. Eng*.,1,pp.105-115 (2012).
19. Metin Akdag, Alkan Ozkan, 2014,”On soft β-open sets and soft β-continuous function”. *The Scientific world J*., Article ID 843456 (2014).
20. Selvi,A., Arockiarani,I., “On soft almost g-continuous function,” *Int. J. of Math. Research*, 7(2),pp.109-123 (2015).
21. Selvi,A.,Arockiarani, I., “Soft πg-operators in soft topological spaces, *Int. J. of Math. Archive*, 5 (4),pp.37-43 (2014).
22. Metin Akdag, Alkan Ozkan, “On soft α-open sets and soft α-continuous functions, Abstract and applied analysis (2014).
23. Pinaki M., Samanta,S.K., “On soft mappings,” *J. of Comp. and Math. with Appl*., 60, pp.2666-22 (2010).
24. Benchali, S. S., Basavaraj, M. I., WALI, R. S., “On Minimal Open Sets and Maps in Topological Spaces,” *J. of Comp. and Math. Sci*, 2 (2), pp.170-398 (2011).
25. Hai-Long Yang, Xiuwu Liao, Sheng-Gang Li, “On soft continuous mappings and soft connectedness of soft topological spaces,” *J. of Math and Statistics*, 44 (2),pp. 385 – 398 (2015).
26. Naganagouda, K., Chetana, C., “Soft minimal open sets and soft maximal open sets in soft topological spaces,” *J. of Comp. and Math. Sci*, 4(2),pp.149-155 (2015).
27. Chetana, C., Naganagouda, K., Soft minimal continuous maps and soft maximal continuous maps in soft topological spaces, *Global Journal of Pure and Applied Mathematics*, Vol. 13, No 7, pp. 3033-3047 (2017).
28. Chetana, C., Naganagouda, K., On soft minimal irresolute maps in soft topological spaces, *Journal of Engineering and Applied Sciences*, Accepted.