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A Study on Non-Split Domination Number of A Graph and Its Applications

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ABSTRACT

In this paper we analyze the domination parameters corresponding to non-split domination number of a graph and obtained several results on this parameters. A Dominating set *D* of a graph *G= (V, E)* is a nonsplit dominating set if the induced subgraph ⟨V-D⟩ is connected. The nonsplit domination number γ_{ns}(G) of G is the minimum cardinality of a nonsplit dominating set. In this paper, many bounds on γ_{ns} (G) are obtained and its exact values for some standard graphs are found. Finally we conclude that this paper will be useful in the field of medicine and biochemistry.

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Keywords: Non-split domination Number; Graph.

1. INTRODUCTION

Graphs considered here are finite, undirected nontrivial and connected without loops and multiple edges. Let *G=(V,E)* be a graph. A set $S \subseteq V$ is a dominating set of G if every vertex in V – S is adjacent to some vertex in S. The domination number γ(G) of G is the minimum cardinality of a dominating set. A dominating set S of G is a connected dominating set if the induced subgraph ⟨*S*⟩ is connected. The connected domination number γ(G) of G is the minimum cardinality of a connected dominating set. Recently Kulli and Janakiram introduced the concept of split domination number[5]. A dominating set *S* of a graph G=*(V,E)* is a split dominating set if the induced subgraph ⟨*V* – *S*⟩ is disconnected. The split domination number

$\gamma_s(G)$ of G is the minimum cardinality of a split dominating set. The reader is refer to[1,2,3] for survey of results on domination. Any undefined term in this paper may be found in Harary[4]. Unless stated, the graph has p vertices and q edges. The purpose of this paper is to introduce the concept of Nonsplit Domination.A dominating set S of a graph $G=(V,E)$ is a nonsplit dominating set if the induced subgraph $\langle V-S\rangle$ is connected. The nonsplit domination number $\gamma_{ns}(G)$ of G is the minimum cardinality of a nonsplit dominating set.We call a set of vertices γ- set if it is a dominating set with cardinality $\gamma(G)$. Similarly a γ_c- set, γ_s- set and γ_{ns}- set are defined.

2. PRELIMINARIE RESULTS

Theorem 2.1

For any graph G, $\gamma(G) \leq \gamma_{ns}(G)$

Theorem 2.2

For any graph G, $\gamma(G) = \min\{\gamma_s(G), \gamma_{ns}(G)\}$

In[3], Cockayne and Hedetneimi gave necessary and sufficient condition for a minimal dominating set.

Theorem 2.3

A dominating set S of a graph G is minimal if and only if for each vertex $v \in S$, one of the following is satisfied

(i) There exists a vertex $u \in \langle V-S\rangle$ such that $N(u) \cap S=\{v\}$

(ii) V is an isolated vertex in $\langle S\rangle$

Theorem 2.4

A non splitting dominating set S of G is minimal if and only if for each vertex $v \in S$, one of the following is satisfied

(i) There exists a vertex $u \in \langle V-S\rangle$ such that $N(u) \cap S=\{v\}$

(ii) V is an isolated vertex in $\langle S\rangle$

(iii) $N(v) \cap (V-S)= \emptyset$

Proof

Suppose S is minimal. On the contradiction if there exists a vertex $v \in S$ such that v doesnot satisfy any of the given conditions, then by theorem 2.3, $S`=S-\{v\}$ is a dominating set of G by (iii), $\langle V-S`\rangle$ is connected. This implies that S` is a nonsplit dominating set of G, which is a contradiction. This proves the necessity.Sufficiently directly we can prove.

Note

Next we obtain a relations between $\gamma_{ns}(G)$ and $\gamma_{ns}(H)$ where H is any spanning subgraph of G.

Theorem 2.5

For any spanning subgraph H of G,$\gamma_{ns}(G) \leq \gamma_{ns}(H)$In the following two results, we obtain lower and upper bounds on $\gamma_{ns}(G)$ respectively.

Theorem 2.6

For any graph G, $\gamma_{ns}(G) \geq (2p - q - 1)/2$

Proof

Let S be a γ_{ns}-set of G. Since $\langle V - S \rangle$ is connected, $q \geq |V - S| + |V - S| - 1$. Hence the proof.

Theorem 2.7

For any graph G, $\gamma_{ns}(G) \leq p - \omega(G) + 1$ where $\omega(G)$ is the clique number of G.

Proof:

Let D be a set of all vertices of G such that $\langle D \rangle$ is complete with $|D| = \omega(G)$. Then for any $u \in D$, $(V - D) \cup \{u\}$ is a nonsplit dominating set of G. Hence the proof.

PROPOSITION: 3

Now we list the exact values of $\gamma_{ns}(G)$ for some standard graphs.

(i) For any complete graph K_p with $p \geq 2$ vertices, $\gamma_{ns}(K_p) = 1$

(ii) For any complete bipartite graph $K_{m,n}$ with $2 \leq m \leq n$, $\gamma_{ns}(K_{m,n}) = 2$

(iii) For any cycle C_p, $\gamma_{ns}(C_p) = p - 2$

(iv) For any wheel W_p, $\gamma_{ns}(W_p) = 1$

(v) For any path P_p with $p \geq 3$ vertices, $\gamma_{ns}(P_p) = p - 2$

Theorem 3.1

If T is a tree which is not a star, then, $\gamma_{ns}(T) \leq p - 2$

Proof

Since T is not a star, there exist two adjacent cut vertices u and v with deg u, deg $v \geq 2$. This implies that $V - \{u, v\}$ is a non split dominating set of T.

Theorem 3.2

If $K(G) > \beta_0(G)$, then $\gamma_{ns}(G) = \gamma(G)$ where K(G) is the connectively of G and $\beta_0(G)$ is the independence number of G.

Proof

Let S be a γ-set of G. Since $K(G) > \beta_0(G) \geq \gamma(G)$, it implies that $\langle V - S \rangle$ is connected. This proves that S is a γ_{ns} set of G.

Theorem 3.3

Let S be a γ_{ns} set of a connected graph of G. If no two vertices in V – S are adjacent to a common vertex in S, then $\gamma_{ns}(G) + \mathcal{E}(T) \geq p$ where $\mathcal{E}(T)$ is the maximum number of end vertices in any spanning tree T of G.

Proof

Let S be a γ_{ns}- set of G, given in the hypothesis. Since for two vertices $u, v \in V - S$, there exist two vertices $u_1, v_1 \in S$ such that u_1 is adjacent to u but not to v and v_1 is adjacent to v but not to u_1. This implies that there exists a spanning tree T of $\langle V - S \rangle$ in which each vertex of $V - S$ is adjacent to a vertex of S. This proves that $\mathcal{E}(T) \geq |V - S|$.

Theorem 3.4
If $\delta(G) + \omega(G) \geq p+1$, then $\gamma_c(G) + \gamma_{ns}(G) \leq p$ where $\delta(G)$ is the minimum degree of G.
Proof
By theorem 2.7 $\gamma_{ns}(G) \leq p - \omega(G) + 1 \leq \delta(G)$. Let S be a γ_{ns}- set of G. Then every vertex in S is adjacent to some vertex in V – S. Thus $\langle V - S \rangle$ is a connected dominating set of G, since $\langle V - S \rangle$ is connected.

Note
In the next result we obtain another upper bound on $\gamma_{ns}(G)$.

Theorem 3.5
For any graph G, $\gamma_{ns}(G) \leq p - \mathrm{diam}(G) + h + 1$where diam(G) is the diameter of G and h is the minimum number of vertices in a γ_{ns} – set of G which lie in between shortest u-v path and d(u,v)=diam(G).
Proof
Let diam (G) =k. we consider the following cases.

Case 1
Suppose u, v$\in$ V –S. Then V – S has atleast k+1 vertices.

Case 2

Suppose u$\in$S and v$\in$ V – S. If there exists a vertex $u_1 \in V - S$ such that u_1 is connected to u through the vertices of S then, $d(u_1,v) \geq k-(h+1)$ and hence V – S has at least k-h vertices. For otherwise, for every vertex $u_1 \in$ V – S there exists a vertex w adjacent to u_1 such that $d(u,w)=d(u,v)+d(v,u_1)+d(u_1,w) \geq k+1$, which is a contradiction. This implies that V – S ={v} and hence G=K_2 or $K_{1,2}$

Case 3

Suppose u,v$\in$ S. If there exists two vertices $u_1,v_1 \in V - S$ such that u is connected to u_1 and v is connected to v_1 through the vertices of S, then $d(u_1,v_1) \geq k-(h+2)$ and hence V – S has at least k-h-1 vertices. For otherwise, there exists exactly one vertex $u_1 \in V - S$ which is adjacent to both u and v and $\{u_1\}=(V - S)$. This implies that G is a star with at least three vertices. Thus from the above all three cases, it follows that V – S has at least k-h-1 vertices.

4. CONCLUSION

In this paper we analyze the domination parameters corresponding to non-split domination number of a graph and obtained several results on this parameters. Finally we conclude that this paper will be useful explicitly in the field of medicine and biochemistry.

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