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R-Harmonic Mean Labeling of Graphs

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ABSTRACT

In this paper, I introduce new labeling techniques called R-Harmonic mean labeling. A graph G with p vertices and q edges is called a R-Harmonic mean graph if there is a function τ from the vertex set of G to {0, 1, 2, 3, …, q} with τ is one-one and τ induces a bijection τ^* from the edge set of G to {1, 2, 3, …, q}, where $\tau^*(e = uv) = \begin{cases} \left\lceil \frac{2\tau(u)\tau(v)}{\tau(u)+\tau(v)} \right\rceil & if\ \tau(u) \neq 0\ and\ \tau(v) \neq 0 \\ 1 & \text{otherwise} \end{cases}$ and the function τ is called as R-Harmonic mean labeling of G. Further I investigate some of R-Harmonic mean graphs.

Keywords: Graph, Harmonic mean labeling, Harmonic mean graph, R-Harmonic mean labeling and R-Harmonic mean graph.

1. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. For a detailed survey of graph labeling I refer to Gallian[1]. For standard terminology and notations I follow Harary[2]. Simple, finite, connected and undirected graphs are considered here.

S. Somasundaram and R. Ponraj introduced the concept of mean labeling of graphs in[8]. S. Somasundaram and S.S Sandhya introduced the concept of harmonic mean labeling and studied their behavior in[6] and[7]. C. David Raj, S. S Sandhya and C. Jayesekaran investigated several Harmonic mean graphs in[4] and[5]. The concept of one modulo three harmonic mean labeling was introduced by C.David Raj, S.S Sandhya and C.Jayesekaran[3] and showed several graphs are harmonic mean graphs. S.S Sandhya and C. David Raj introduce the concept of super Harmonic mean labeling in[9]. In this paper I introduce new type of labeling called R-Harmonic mean labeling. The following definitions are useful for the present investigation.

Definition 1.1. A graph G = (V, E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices x∈V with distinct labels f(x) from 1, 2, …, q + 1 in such a way that when each edge e = xy is labeled with f(e = xy) = $\left\lceil\frac{2f(x)f(y)}{f(x)+f(y)}\right\rceil$ or $\left\lfloor\frac{2f(x)f(y)}{f(x)+f(y)}\right\rfloor$, then the edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition 1.2. The corona of two graphs G_1 and G_2 is the graph G = $G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.3. $\lceil x \rceil$ (ie ceil(x)) returns the smallest integer which is greater than or equal to x. $\lfloor x \rfloor$ (ie floor(x)) returns the largest integer which is less than or equal to x.

2. R-HARMONIC MEAN GRAPH

Definition 2.1. A graph G with p vertices and q edges is called a R-Harmonic mean graph if there is a function τ from the vertex set of G to {0, 1, 2, 3, …, q} with τ is one-one and τ induces a bijection τ^* from the edge set of G to {1, 2, 3, …, q}, where τ^*(e = uv) $=\begin{cases} \left\lceil\frac{2\tau(u)\tau(v)}{\tau(u)+\tau(v)}\right\rceil & if\ \tau(u) \neq 0\ and\ \tau(v) \neq 0 \\ 1 & \text{othewise} \end{cases}$ and the function τ is called as R-Harmonic mean labeling of G.

Remark 2.2. In a R-Harmonic mean graph vertices get labels from {0, 1, 2, …, q} and edges get labels from {1, 2, 3, …, q}.

Remark 2.3. If p > q+1, then the graph G is not a harmonic mean graph, since we don't have sufficient labels from {0, 1, 2, 3, …, q} for the vertices of G.

Remark 2.4. In a R-Harmonic mean graph one of the vertices must get the label 0 since an edge must get the label 1.

Theorem 2.5. If G is a R harmonic mean graph, then G has at least one vertex of degree one.
Proof.

Suppose G is a R harmonic mean graph. Then by remark 2.3 one of the vertices must get the label 0. If a vertex u gets the label 0, then any edge incident with u gets the label 1. Since τ^* is a bijection from an edge set of G to {1, 2, 3, …, q}, only one edge incident with u. Hence the degree of u is one.

Theorem 2.6. Any K-regular graph, K>1, is not a R-Harmonic mean graph.

Proof.

Suppose G is a R-Harmonic mean graph. Then by theorem 2.5 there exists atleast one vertex of degree one. Which is a contraction to K>1. Hence the theorem.

Corollary 2.7. Any cycle C_n is not a R-Harmonic mean graph.
Proof.
C_n is a 2-regular graph and hence the corollary follows from theorem 2.6.

Corollary 2.8. If n>2, K_n is not a R-Harmonic mean graph.
Proof.
K_n, n>2 is a K-regular graph and hence the corollary follows from theorem 2.6.

Theorem 2.9. Any Path P_n is a R-Harmonic mean graph.
Proof.
Let P_n be the path $s_1s_2\ldots s_n$. Define a function τ: $V(P_n) \rightarrow \{0, 1, 2, \ldots, q\}$ by $\tau(s_i) = i - 1$, $1 \leq i \leq n$.
Then τ induces a bijection τ^*: $E(P_n) \rightarrow \{1, 2, \ldots, q\}$, where
$\tau^*(s_is_{i+1}) = i$, $1 \leq i \leq n - 1$.
Hence P_n is a R-Harmonic mean Graph.
Example 2.10. R-Harmonic mean labeling of P_8 is given in the following figure.2.1

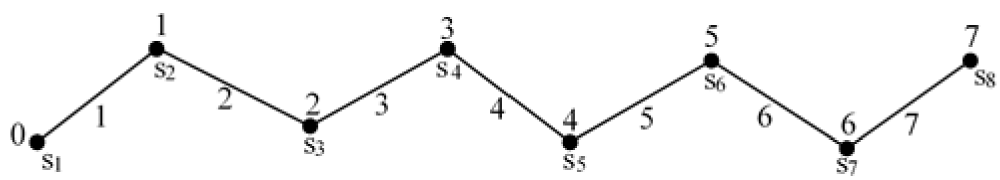

Figure 2.1.

Theorem 2.11. A comp $P_n \odot K_1$ is a R-Harmonic mean graph.

Proof.

Let $s_1s_2\ldots s_n$ be the path P_n. Join each vertex t_i with s_i, $1 \leq i \leq n$. Then the resultant graph is $P_n \odot K_1$ whose edge set is $E = \{s_is_{i+1}/1 \leq i \leq n - 1\} \cup \{s_it_i/1 \leq i \leq n\}$. Define a function

τ: $V(P_n \odot K_1) \rightarrow \{0, 1, 2, \ldots, q\}$ by

$\tau(s_i) = 2i - 1$, $1 \leq i \leq n$;
$\tau(t_i) = 2(i - 1)$, $1 \leq i \leq n$;

Then τ induces a bijection τ^*: $E(P_n \odot K_1) \rightarrow \{1, 2, \ldots, q\}$, where

$\tau^*(s_is_{i+1}) = 2i$, $1 \leq i \leq n - 1$;
$\tau^*(s_it_i) = 2i - 1$, $1 \leq i \leq n$

In the view of the above labeling pattern, τ provides R-Harmonic mean labeling for $P_n \odot K_1$.

Hence $P_n \odot K_1$ is a R-Harmonic Mean graph.

Example 2.12. R-Harmonic mean labeling of $P_7 \odot K_1$ is given in the following figure 2.2.

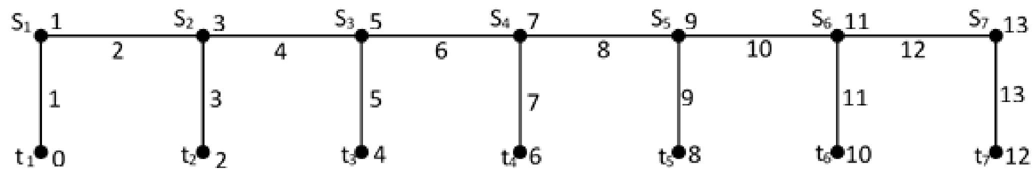

Figure 2.2.

Theorem 2.13. $C_n \odot K_1$, $n > 3$, is a R-Harmonic Mean Graph.

Proof. Let $s_1s_2\ldots s_ns_1$ be the cycle C_n. For $1 \le i \le n$, let t_i be the vertex of i^{th} copy of K_1 which is adjacent to s_i. The resultant graph is $C_n \odot K_1$ whose edge set is $E = \{s_ns_1, s_is_{i+1}/1 \le i \le n - 1\} U$

$\{s_it_i/ 1 \le i \le n\}$. Define a function τ: $V(P_n \odot K_1) \rightarrow \{0, 1, 2, \ldots, q\}$ by

$\tau(s_i) = 2i$, $1 \le i \le n$;

$\tau(t_1) = 1$; $\tau(t_2) = 0$; $\tau(t_i) = 2i - 1$; $3 \le i \le n$.

Then τ induces a bijection τ^*: $E(C_n \odot K_1) \rightarrow \{1, 2, \ldots, q\}$, where

$\tau^*(s_is_{i+1}) = 2i + 1$, $1 \le i \le n - 1$; $\tau^*(s_ns_1) = 4$;

$\tau^*(s_1t_1) = 2$; $\tau^*(s_2t_2) = 1$; $\tau^*(s_it_i) = 2i$; $3 \le i \le n$.

Hence $C_n \odot K_1$ is a R-Harmonic mean graph.

Example 2.14. R-Harmonic mean labeling of $C_8 \odot K_1$ is given in the following figure 2.3.

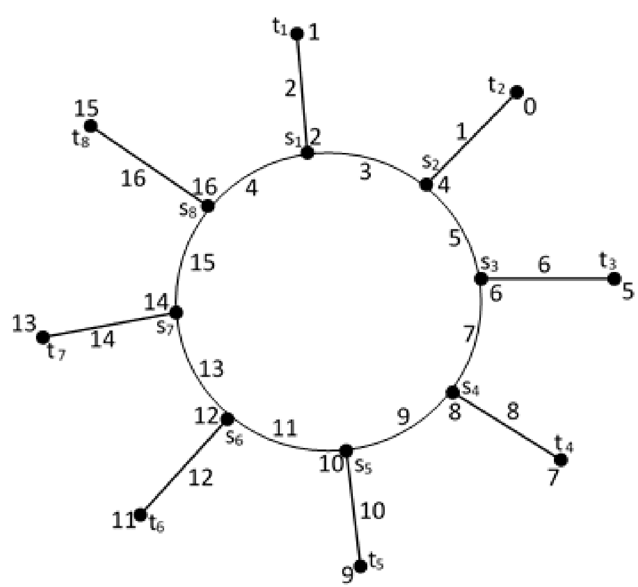

Figure 2.3.

3. CONCLUSION

In harmonic mean labeling we use either ceil function or floor function for labeling the edges. In R-Harmonic mean labeling we use only the roof function. As all graphs are not R-Harmonic mean labeling, it is very interesting to find the graphs which provides R-Harmonic mean labeling.

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