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Solvable Potentials for Transform Form of Confluent Hypergeometric Differential Equation

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ABSTRACT

The aim of this paper is to evaluate Solvable Potential for transform form of Confluent Hypergeometric Differential Equation.

Keywords: Solvable Potential, Confluent Hypergeometric differential equation.

1. INTRODUCTION

The object of this paper is to derive solvable potential for transform form of confluent hypergeometric differential equation. The method employed is due to Bhattacharjie and Sudarshan[1].

2. POTENTIAL FOR TRANSFORM FORM OF CONFLUENT HYPERGEOMETRIC DIFFERENTIAL EQUATION

The transform form of confluent hyper geometric differential equation[2] is:

$$z\frac{d^2y}{dz^2}+\left(a+\frac{1}{2}\right)\frac{dy}{dz}-y=0 \qquad (2.1)$$

where |z|<1. The general solution of (2.1) is given by

$$y(z)=A\ {}_0F_1\left[-;a+\frac{1}{2};\frac{1}{4}z^2\right]+B(z^2)^{\frac{1}{2}-a}\ {}_0F_1\left[-;\frac{3}{2}-a;\frac{1}{4}z^2\right] \qquad (2.2)$$

where A and B are arbitrary constants.

Following Bhattacharjie and Sudharshan[1], (2.1) can be transformed to the following s-wave radial Schrodinger equation (adopting units such that $E = k^2$):

$$\phi''(r)+[k^2-v(r)]\phi(r)=0. \qquad (2.3)$$

By making the following substitutions

$$z = f(r),\ y(z) = g(r)\phi(r),\ h(r) = \frac{d}{dr}\log g(r) \quad (2.4)$$

(2.1) takes the form

$$\phi''(r) + A(r)\phi'(r) + B(r)\phi(r) = 0 \quad (2.5)$$

where

$$A(r) = 2\frac{g'(r)}{g(r)} - \frac{f''(r)}{f'(r)} + \left(a + \frac{1}{2}\right)\frac{f'(r)}{f(r)} \quad (2.6)$$

$$B(r) = \frac{g''(r)}{g(r)} - \frac{f''(r)}{f'(r)}\frac{g'(r)}{g(r)} + \left(a + \frac{1}{2}\right)\frac{g'(r)}{g(r)}\frac{f'(r)}{f(r)} - \frac{\{f'(r)\}^2}{f(r)} \quad (2.7)$$

Now for (2.5) to be form (2.3), the following conditions should be satisfied:

$$A(r) = 0,\ B(r) = k^2 - v(r),\ \frac{\partial\{v(r)\}}{\partial k} = 0 \quad (2.8)$$

The third condition of (2.8) ensures the independence of v(r) from k completely.

Substituting the first condition of (2.8) into (2.6) and integrating, we get

$$f'(r) = M\{g(r)\}^2\{f(r)\}^{(a + 1/2)} \quad (2.9)$$

where M is the constant of integration.

Similarly, from the second condition of (2.8), (2.7) and (2.4), we arrive at

$$h'(r) - h^2(r) - \frac{\{f'(r)\}^2}{f(r)} = k^2 - v(r) \quad (2.10)$$

Considering as a particular choice

$$z = f(r) = 1 + e^{\alpha r} \quad (2.11)$$

we obtain from (2.10)

$$k^2 - v(r) = \frac{\alpha^2 e^{2\alpha r}[\left(a+\frac{1}{2}\right)\left(\frac{3}{2}-a\right)+4(1+e^{\alpha r})}{4(1+e^{\alpha r})^2} - \frac{\alpha^2}{4} \quad (2.12)$$

which suggests

$$k^2 = -\frac{\alpha^2}{4} \quad (2.13)$$

$$v(r) = -\frac{\alpha^2 e^{2\alpha r}[\left(a+\frac{1}{2}\right)\left(\frac{3}{2}-a\right)+4(1+e^{\alpha r})}{4(1+e^{\alpha r})^2} \quad (2.14)$$

3. SOLUTION AND S-MATRIX

For the potential derived in (2.14), using (2.2) the general solution $\phi(r)$ of the shodinger equation (2.3) can be written as,

$$\begin{aligned}\phi(r) = C_1\{1 - e^{-2ikr}\}^{\frac{1}{2}(a+\frac{1}{2})} e^{ikr}\ {}_0F_1\left[-; a + \frac{1}{2}; \frac{1}{4}\left(1 - e^{-2ikr}\right)^2\right] \\ +\{1 - e^{2ikr}\}^{\frac{1}{2}(a+\frac{1}{2})} e^{-ikr}\ C_2\{1 - e^{2ikr}\}^{1-2a} \\ \cdot\ {}_0F_1\left[-; \frac{3}{2} - a; \frac{1}{4}\left(1 - e^{2ikr}\right)^2\right]\end{aligned} \quad (3.1)$$

where the constant $\sqrt{M}$ being include in new constant C_1 and C_2. Constant C_1 and C_2 are to be determined so that $\phi(0) = 0$ and $\phi(0)$ is continuous normalized.

Now $\left(1 - e^{\mp 2ikr}\right) = 1 - \cos 2kr\ \pm\ i\sin 2kr$

Let $1-\cos 2kr \sim \beta (r \to \infty)$

and $\sin 2kr \sim \delta (r \to \infty)$

then, asymptotically,

$$\phi(r) \sim N_1\, e^{ikr}\ {}_0F_1\left[-; a+\frac{1}{2}; \frac{1}{4}(\beta+i\delta)^2\right] + N_2\, e^{-ikr}\ {}_0F_1\left[-; \frac{3}{2}-a; \frac{1}{4}(\beta-i\delta)^2\right], r \to \infty \quad (3.2)$$

where

$$N_1 = C_1(\beta+i\delta)^{\frac{1}{2}(a+\frac{1}{2})}, N_2 = C_2\,(\beta-i\delta)^{\frac{1}{2}(a+\frac{1}{2})}\,(\beta-i\delta)^{1-2a}$$

It is to be noted that the values of β oscillates between 0 and 2 when r tends to ∞ and it is likely to become 0 at some infinite points. Only such points enable us to evaluate the S-matrix.

Thus, when β is zero, (3.2) takes the form

$$\phi(r) \sim N_1\, e^{ikr}\ {}_0F_1\left[-; a+\frac{1}{2}; -\frac{1}{4}\delta^2\right] + N_2\, e^{-ikr}\ {}_0F_1\left[-; \frac{3}{2}-a; -\frac{1}{4}\delta^2\right], r \to \infty \quad (3.3)$$

(3.3) together with the requirement ϕ(r) = 0 at r = 0, yields the S-matrix

$$s(k) = -\frac{N_2}{N_1} = \frac{{}_0F_1\left[-; a+\frac{1}{2}; -\frac{1}{4}\delta^2\right]}{{}_0F_1\left[-; \frac{3}{2}-a; -\frac{1}{4}\delta^2\right]} \quad (3.4)$$

It should be noted, however, that the above expression for the S-matrix is valid only for such values of β which tend to zero when $r \to \infty$.

REFERENCES

1. Bhattacharjie, A and Sudarshan, E. C. G.: A class of solvable potentials, *Nuovo Cimento* (10) 25, p.864-879 (1962).
2. Rainville, E. D.: Special Functions, MacMillan, New York (1960).