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Graph Theoretic Approach to Characterize Food Web

Bhagya Jyoti Nath

Department of Mathematics,
Kaliabor College, Nagaon, Assam, INDIA.
email: nathbhagyajyoti@gmail.com

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ABSTRACT

A food web is a natural interconnection between different species in an ecological community. From a food web we can have the informations of what-eats-what in an ecological community. These food webs can be considered as graph and they can be studied in graph theoretic approach. The main goal of this article is to observe how the graph theoretic concepts can be used to characterize a food web. We can use digraphs to model complex trophic relations and use the concepts of graph to calculate the relative importance of each species in the food web. In this article we have considered some examples of ecological community and try to apply the graph theory concepts to describe it. We have also included some observations and results of food webs using different graph theory concepts.

Keywords: Food web, Digraph, Trophic level and Trophic status.

1. INTRODUCTION

Food webs are the networks formed by the trophic interactions between species in ecological communities. Each organism depends for food on one or many other organisms in an ecosystem. The exceptions are primary producers. They produce their energy from sunlight through photosynthesis or from chemicals through chemosynthesis. The other species in the ecosystem depends on the primary producers directly or indirectly. Their dependence on each other for food in an ecosystem form a complex food web. Descriptions of food web relationships first appeared more than a century ago. Generally the ecologists were working in this field but a few mathematicians became interested in the graph-theoretic properties of food webs and their corresponding competition graphs. When we want to discuss the history of the study of food webs we have to go back at least to the late 1800s. By the 1910, the researchers began to produce images not like food webs we seen today, such a network of insect predators and parasites on cotton feeding weevils. By the 1920, the first relatively detailed emperical descriptions of terrestrial and marine food webs appeared. Elton (1927) introduced the term food chain and all the food chains in a community as a food cycle. Now this food cycle is known as food web. Different mathematicians studied these food webs and try to characterize the food webs whose competition graphs are interval graphs. It was first observed by Joel Cohen in 1968. Cohen[2] took a statistical approach to this problem of food web. He generated food webs randomly and determined whether or not their competition graph were interval graphs. Lundgren and Maybee[8] found a characterization of acyclic digraphs whose competition graph is an interval graph. Steif[10] approached this problem from such a structural point of view. But he obtained negative result. Sugihara[11] showed statistically that the frequency with which food webs have interval graph competition graphs could be accounted for by requiring that the competition graph have no cycles of length greater than 3 as generated sub graphs. Some other mathematician like Hefner, Jones, Kim, Lundgren and Roberts[5] approached the problem by studying digraphs with limited in degrees and out degrees. They characterized these food webs with digraphs with in degree and out degree at most 2 at each vertex and which have interval competition graph. But this problem remains open for higher in degrees and out degrees. In this article we try to study the food webs in some ecological system and try to characterize them using the concepts of graph theory. In the first part of this article we introduced some definitions and backgrounds which are required to observe the food webs from graph theory point of view.

2. PRELIMINARIES

There are many terms of graph theory which are frequently used in the study of food webs in graph theory approach. In this section we have mentioned some of such important terms and their definitions.

Definition 2.1: A graph G is defined by (V,E,γ) where the elements of the set V are called vertices of graph G and the elements of the set E are called edges of graph G. Here γ is a function which assigns each edge of the graph G with two vertices.

Definition 2.2: A Digraph or Directed graph consists of a vertex set V and an edge set E such that each edge e in E is associated with an ordered pair of vertices. In digraph every edge has a direction.

In food web the set of vertices is the collection of all species in an ecosystem and we connect the vertices by edges depending on the energy transfer that is from a prey species to a predator of that prey.

Definition 2.3: In a digraph the number of head ends adjacent to a vertex is called the indegree of the vertex and the number of tail ends adjacent to a vertex is its outdegree.

Definition 2.4: A walk on a graph is a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident to its preceding and following vertices. Length of a walk is given by the number of edges in the walk.

Definition 2.5: A path or simple path is an open walk (walk whose beginning and ending vertices are not same) in which no vertex appears twice or more. The number of edges in the path is called the length of path. Using length of path we can define the trophic level in a food web.

3. MAIN SECTION

Food web matrix and its characteristics

The basic information that is required to construct a food web is contained in food web matrix[11]. Food web matrix is a binary matrix whose columns are corresponding to the set of consumer species or the predators and rows are corresponding to the set of resources or prey in the system. The food web matrix for a pond ecosystem is shown in the figure 1. The 1's in the matrix signify that a given consumer uses a given prey item which in turn generates the ordered pair (C_i, R_i). The given coloumn C_i in food web matrix identifies the subset of resources used by that consumer and each row R_i identifies the subset of consumers that share a given resources. These ordered pairs in a food web matrix can be represented in graphical form by constructing digraph of trophic flows (Figure 2). In the digraph *G(V,E)* the set *V* contains the union of consumers *C* and resources *R* that is all the species in the ecosystem and the set *E* contains the ordered pairs (*C*,*R*) . The set *E* indicates who eats whom. Thus digraph is used for representing food webs.

Example 3.1: To describe the concept of food web matrix and food web digarph we consider a pond ecological system where we consider some species as given below:

1. Large fish. 2. Medium fish. 3. Small fish. 4. Benthic invertebrates.
5. Filter feeder. 6. Zooplankton. 7. Phytoplankton.

Now depending on the food structure we have the following food web matrix and digraph:

Consumers→ Producers ↓	Large fish	Medium fish	Small fish	Benthic invertibrates	Filter feeder	Zoo plankton	Phyto plankton
Large fish	0	0	0	0	0	0	0
Medium fish	1	0	0	0	0	0	0
Small fish	1	1	0	0	0	0	0
Benthic invertibrates	0	1	1	0	0	0	0
Filter feeder	0	0	0	0	0	0	0
Zooplankton	0	0	1	1	1	0	0
Phytoplankton	0	0	0	1	1	1	0

Figure 1: Community Food web matrix for pond ecosystem.

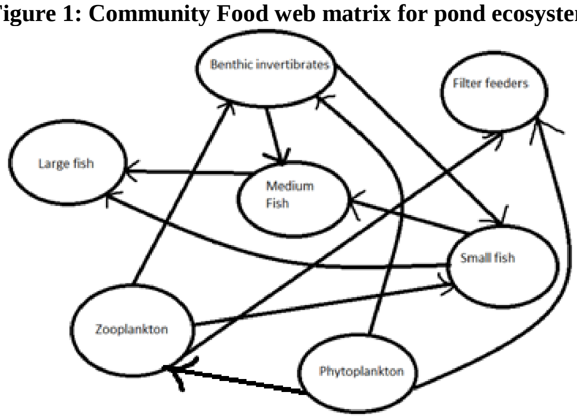

Figure 2: Digraph for pond ecosystem.

We can have some observations from the food web matrix as well as digraph as follows:

Observation1: We observed that in food web matrix, the column corresponding to phytoplankton contains only 0's and in the digraph there is no inward edges to it. That means phytoplankton is only producers not consumers. So the in degree of the vertices corresponding to the only producers = 0.

Observation 2: Similarly, We observed that the rows corresponding to large fish and filter feeder contains only 0's and in the digraph there is no outward edges from it. That means they are only consumers in this food web not prey to any other species. So the out degree of the vertices corresponding to the only consumers = 0.

Observation 3: In case of some species like small fish, medium fish, benthic invertebrates they have both inward and outward edges. It means they are consumers and also food to some other species.

Thus using food web matrix and digraph concept we can characterize different food webs.

Trophic levels and trophic status:

Trophic levels in food webs provide a way of organizing species in a community food web into feeding groups. Generally the food webs give the direction of energy flow in a ecological system and by determining the trophic level of a species we can find the amount of energy that the species receives in the food web. Classification of species in food web into various feeding groups can be made by using different methods. Usually we divide them into primary producers, secondary producers and consumers etc. We can also define trophic levels using the path length in a digraph. Trophic levels can be defined in two ways as mentioned below.

Definition 3.1: (Shortest path definition)

The trophic level of a species *X* is:

(i) 0, if *X* is a primary producer in food web.

(ii) *k*, if the shortest path from a level 0 species to *X* is of length *k*.

The trophic level of different species using shortest path definition in the Example 3.1 of pond ecological system is given in the Table 1.

Table 1: Trophic level of different species in Example 3.1 using shortest path definition.

Species	Trophic level	Shortest path
Large fish	3	Phytoplankton - Zooplankton – Small fish - Large fish
Medium fish	2	Phytoplankton –Benthic invertebrates - Medium fish
Small fish	2	Phytoplankton -Zooplankton - Small fish
Benthic invertebrates	1	Phytoplankton - Benthic invertebrates
Filter feeder	1	Phytoplankton - Filter feeder.
Zooplankton	1	Phytoplankton - Zooplankton
Phytoplankton	0	

But there are some difficulties in finding the trophic level using this shortest path definition like two species which have prey-predator relationship can have same trophic level by this definition. So, we use another alternate definition of trophic level as given below.

Definition 3.2: (Longest path definition)
The trophic level of a species X is:
(i) 0, if X is a primary producer in food web.
(ii) k, if the longest path from a level 0 species to X is of length k.

The trophic level of different species using longest path definition in the Example 3.1 of pond ecological system is given in the Table 2.

Table 2: Trophic level of different species in Example 3.1 using longest path definition.

Species	Trophic level	Longest path
Large fish	5	Phytoplankton - Zooplankton - Benthic invertebrates - Small fish - Medium fish - Large fish
Medium fish	4	Phytoplankton - Zooplankton - Benthic invertebrates - Small fish - Medium fish
Small fish	3	Phytoplankton - Zooplankton – Benthic invertebrates - Small fish
Benthic invertebrates	2	Phytoplankton - Zooplankton - Benthic invertebrates
Filter feeder	2	Phytoplankton - Zooplankton - Filter feeder
Zooplankton	1	Phytoplankton – Zooplankton.
Phytoplankton	0	

Therefore we observed that when we use longest path definition then the highest trophic level is five but when we use shortest path definition highest trophic level is three. From the above two tables we have the following assumption.

Assumption: If a species X is predator of the species Y then the trophic level of species X is greater than the trophic level of Y.
But the above assumption may not always applicable in case of shortest path definition. For example: In the Table 1 , medium fish and small fish have same trophic level 2, but medium fish is a predator for small fish. But we observed that this assumption holds in case of longest path definition. So we have the following theorem.

Theorem 3.1: In longest path definition, if a species X is predator of the species Y then the trophic level of species X is at least 1 greater than the trophic level of Y.

Proof: Consider the trophic level of Y is k (≥ 0). Then by longest path definition there exists a longest path say $(v_1, e_1, v_2, e_2, \ldots, v_k, e_k, Y)$ from a level 0 species v_1 to Y. Again, X is a predator of the species Y, so there is an edge say e from Y to X. So $(v_1, e_1, v_2, e_2, \ldots, v_k, e_k, Y, e, X)$ is a path from level 0 species v_1 to *X* having path length *k+1*. So there exists a path from v_1 to *X* having length *k+1*. So trophic level of *X* is greater than or equal to *k+1*.

Corollary: If two species in a food web have same trophic level (using longest path definition) there cannot be a prey-predator relationship in between them.

Thus we observed that the longest path definition is stronger and more applicable in characterization of food web than the shortest path definition. Using this longest path definition we can define the trophic status of a species.

Definition 3.3: The trophic status of a species v is defined as $T(v)=\sum kn_k$, where n_k is the number of species whose longest path to v has length k and this sum is taken over all k.

Now we can consider the Example 3.1 of the pond ecosystem. We calculate the trophic status of large fish.

The longest path to large fish from phytoplankton is k=5 and here n_k=1.
The longest path to large fish from zooplankton is k=4 and here n_k=1.
The longest path to large fish from Benthic invertebrates is k=3 and here n_k=1.
The longest path to large fish from small fish is k=2 and here n_k=1.
The longest path to large fish from medium fish is k=1 and here n_k=1.
Therefore the trophic status of large fish= $1 \times 1 + 2 \times 1 + 3 \times 1 + 4 \times 1 + 5 \times 1 = 15$.

Similarly, the trophic status of the other species in the Example 3.1 is given in the following table.

Table 3: Trophic status of the species in Example 3.1.

Species	Trophic status
Medium fish	10
Small fish	6
Benthic invertebrates	3
Filter feeder	3
Zooplankton	1
Phytoplankton	0

Thus we observed that the trophic statuses of different species in the ecosystem are different. The trophic status of a lower level species in a ecosystem is always less than the upper level species.

In the above discussions of trophic level and trophic status, we have considered that if a species has more than one food resource then it prefers each of the food equally. But in general, it is not true. The species may not prefer all foods equally. In digraph representation we represent the relations between different predators and prey. But to show the food preferences of a species we need to give some weights to the edges depending upon the preference. For example – if we give weight 0.7 to the edge from zooplankton to small fish and 0.3 to the edge from benthic invertebrates to small fish then we understand that 70% of small fish’s diet comes from zooplankton and 30% comes from Benthic invertebrates.

Therefore we have a new definition for trophic level in case of weighted graph representation of food web. It is called flow-based trophic level.

Definition 3.3:

Flow based trophic level (TL) is defined as:
TL of a species i= 1 + $\sum(weight\ of\ each\ food\ source\ for\ i) \times TL\ (food\ source\ j)$
Thus TL(i) = $1 + \sum(w_{ij}) \times TL(\ food\ source\ j)$.
Where w_{ij} is the weight of the arc between vertex i and j.
Observation: In case of primary producer, all w_{ij} are 0, so TL = 1.
Using flow based trophic level we can characterize the food web of an ecological community considering their food preferences. Therefore among all the definitions of trophic level, the flow based trophic level is more powerful and best.

4. CONCLUSION

The main objective of this article was to describe the applications of graph theory concepts in describing and characterizing a food web. So we consider some example of food web and try to describe them using graph theory terms like digraph, weighted graph etc. and also use different definitions of graph theory to find the trophic levels of different species in the food web. But the construction of real food web is not easy. So in our example we consider some selected species and try to find their relations. In an ecological system there are lots of species including parasites etc. But when in general we construct a food web we never included these parasites in a food web, but they have enormous impact on the survivability of other species. Therefore we observed that though there are some limitations, different characteristics of a food web like importance of different species in a food web, their trophic level etc. can be explained using graph theory concepts.

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