

On Semi α -Regular Weakly Open Sets in Topological Spaces

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(Received on: October 30, Accepted: November 2, 2017)

ABSTRACT

In this research paper, a new class of open sets called semi α -regular weakly open sets (sarw open set) in topological space are introduced, studied and some of their basic properties have been investigated. We also introduce sarw-closure, sarw-interior, sarw neighbourhood, and sarw-limit points discuss some of their properties.

2010 Mathematics Classification: 54A05, 54A10.

Keywords: sarw-closed sets, sarw-open sets, sarw-closure, sarw-interior, sarw-neighbourhood and sarw-limit points.

1. INTRODUCTION

Regular open sets and rw-open sets have been introduced and investigated by Stone¹¹ and Benchalli and Wali² respectively. Levine⁶, Nagaveni⁹, Pushpalatha¹⁰ introduced and investigated semi open sets, generalized closed sets, weakly generalized closed sets respectively. Maki *et al.*⁷ introduced and studied generalized α -closed sets and α -generalized closed sets. S. S. Benchalli *et al.*³, R. S. Wali and P. S. Mandalgeri¹³ studied $w\alpha$ -closed, arw closed sets in topological spaces. Basavaraj M. Ittanagi and Mohan V¹⁴ introduced and studied sarw-closed sets. We introduce and study the sarw-open sets in topological space and obtain some of their properties. Also we introduce sarw-interior, sarw-neighbourhood, sarw-limits points in topological space.

2. PRELIMINARIES

Throughout this paper X or (X, τ) represents a topological space. For a subset A of a topological space (X, τ) , $cl(A)$, $int(A)$, $scl(A)$, $\alpha cl(A)$, $spcl(A)$ and $gcl(A)$ denotes the closure

of A , the interior of A , the semi-closure of A , the α -closure of A , the semi pre closure of A and the g -closure of A in X respectively.

Definition 2.1: Let (X, τ) be topological space and $A \subseteq X$.

The intersection of all semi closed (respectively pre-closed, α -closed and semi-pre closed) subsets of space X containing A is called the Semi closure (respectively pre-closure, α -closure and Semi-pre-closure) of A and is denoted by $scl(A)$ (respectively $pcl(A)$, $\alpha cl(A)$, $spcl(A)$).

Definition 2.2: A subset A of a topological space (X, τ) is called

- 1) Generalized semi-closed set (briefly gs closed)¹ if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 2) An α -generalized closed set (briefly αg -closed)⁷ if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 3) Generalized semi pre regular closed (briefly $gspr$ -closed) set¹⁰ if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
- 4) Generalized pre regular closed set (briefly gpr -closed)² if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
- 5) Weakly generalized closed set (briefly, wg -closed)⁹ if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 6) A regular weakly generalized closed set (briefly, rwg -closed)⁹ if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
- 7) α -generalized regular closed (briefly αgr -closed) set¹² if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
- 8) $W\alpha$ -closed set³ if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is w -open in (X, τ) .
- 9) Regular w - closed (briefly rw -closed) set² if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi- open in (X, τ) .
- 10) Generalized pre closed (briefly gp closed) set⁸ if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 11) α regular weakly closed(αrw closed)¹³ whenever $\alpha-cl(A) \subseteq U$ and U be rw -open in (X, τ) .

The compliment of the above mentioned closed sets are their open sets respectively.

3. SEMI α -REGULAR WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1¹⁴: Let A be subset of topological space (X, τ) , A defined semi α - regular weakly closed set ($sarw$ closed set) whenever $scl(A) \subseteq U$ and $A \subseteq U$ where U be αrw -open in X . The family of all semi α - regular weakly closed sets in (X, τ) , is denoted by $sarw C(X)$.

Example 3.2: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology then $Sarw$ closed sets are $X, \phi, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$

Results 3.3: From¹⁴

- 1) Every closed set is sarw -closed set in X
- 2) Every α -closed set in X is sarw -closed set
- 3) Every regular closed set is sarw -closed set in X .
- 4) Every sarw -closed set is sg -closed (also αgr -closed, gs -closed, gspr closed, rwg -closed, gp -closed, gpr -closed) set in X

4. SEMI α -REGULAR WEAKLY OPEN SETS IN TOPOLOGICAL SPACES

In this section, we define sarw -open sets in topological spaces and obtain some of their properties.

Definition 4.1: A subset A in (X, τ) is called Semi α -regular weakly open (briefly, sarw -open) in X if A^c is sarw -closed in (X, τ) . We denote the family of all sarw -open sets in (X, τ) by $\text{SaRWO}(X)$.

Example 4.2: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology then Sarw open sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{1, 4\}, \{2,3\}, \{1, 2, 3\}, \{2, 3, 4\}$

Theorem 4.3: For any topological spaces (X, τ) we have the following

- (i) Every open (respectively α -open, regular-open) set is sarw -open.
- (ii) Every sarw -open set is sg -open set.
- (iii) Every sarw -open set is αgr -open (respectively gs -open, gspr -open, wg -open, rwg open, gp -open and gpr -open) set.

Theorem 4.4: A subset of A of a topological space X is sarw -open iff $U \subseteq \alpha\text{int}(A)$, whenever U is αrw -closed and $U \subseteq A$.

Proof: Assume that A is sarw -open set in X and U is αrw -closed set of (X, τ) such that $U \subseteq A$.

Then $X-A$ is a αrw -closed set in (X, τ) . Also $X - A \subseteq X - U$ and $X - U$ is sarw -open set of (X, τ) . This implies that $\text{scl}(X - A) \subseteq X - U$. But $\text{scl}(X - A) = X - \alpha\text{int}(A)$. Thus $X - \alpha\text{int}(A) \subseteq X - U$, so $U \subseteq \alpha\text{int}(A)$.

Conversely:

Suppose $U \subseteq \alpha\text{int}(A)$ whenever U is αrw -closed and $U \subseteq A$. To prove that A is sarw -open set. Let F be αrw -open set of (X, τ) such that $X - A \subseteq F$. Then $X - F \subseteq A$. Now $X - F$ is αrw -closed set containing A , So $X - F \subseteq \alpha\text{int}(A)$, $X - \alpha\text{int}(A) \subseteq F$ but $\text{scl}(X - A) = X - \alpha\text{int}(A) \subseteq F$. Thus $\text{scl}(X - A) \subseteq F$ that is $X - A$ is sarw -closed set and hence A is sarw -open set.

Theorem 4.5: If $\alpha\text{int}(A) \subseteq B \subseteq A$ and A is sarw -open set, then B is sarw -open set.

Proof: Let $\alpha\text{int}(A) \subseteq B \subseteq A$, Thus $X - A \subseteq X - B \subseteq X - \alpha\text{int}(A)$, that is $X - A \subseteq X - B \subseteq \text{cl}(X - A)$. Since $X - A$ is sarw -closed set, $X - B$ is sarw -closed set. Therefore B is sarw -open set.

Theorem 4.6: If $A \subseteq X$ is sarw -closed then $\text{scl}(A) - A$ is sarw -open set.

Proof: Let A be sarw -closed. Let $F \subseteq \text{scl}(A) - A$, where F is αrw -closed. Therefore $F \subseteq \alpha\text{int}(\text{acl}(A) - A)$ and $\text{scl}(A) - A$ is sarw -open set.

Theorem 4.7: A set A is sarw -open set in (X, τ) if and only if $U = X$ whenever U is αrw -open and $\alpha\text{int}(A) \cup (X - A) \subseteq U$.

Proof: Suppose that A is sarw -open set in X . Let U be αrw -open and $\alpha\text{int}(A) \cup (X - A) \subseteq U$. $U^c \subseteq (\alpha\text{int}(A) \cup A^c)^c = (\alpha\text{int}(A))^c \cap A$ that is, $U^c \subseteq (\alpha\text{int}(A))^c \cap A^c$ (because $A - B = A \cap B^c$). Thus $U^c \subseteq \alpha\text{cl}(A^c) - A^c$ (because $(\alpha\text{int}(A))^c = \text{scl}(A^c)$). Now A^c is also sarw -closed and U^c is αrw -closed then it follows $U^c = \phi$ then $U = X$.

Conversely: Suppose F is sarw -closed and $F \subseteq A$. Then $\alpha\text{int}(A) \cup (X - A) \subseteq \alpha\text{int}(A) \cup (X - F)$. It follows that $\alpha\text{int}(A) \cup (X - F) = X$.

Definition 4.8: For a subset A of (X, τ) , sarw -closure of A is denoted by $\text{sarwcl}(A)$ and defined as $\text{sarwcl}(A) = \bigcap \{G : A \subseteq G, G \text{ is } \text{sarw}\text{-closed in } (X, \tau)\}$ or $\bigcap \{G : A \subseteq G, G \in \text{sarwC}(X)\}$.

Theorem 4.9: If A and B are subsets of space (X, τ) then

- (i) $\text{sarwcl}(X) = X, \text{sarwcl}(\phi) = \phi$.
- (ii) $A \subseteq \text{sarwcl}(A)$.
- (iii) If B is any sarw -closed set containing A , then $\text{sarwcl}(A) \subseteq B$.
- (iv) If $A \subseteq B$ then $\text{sarwcl}(A) \subseteq \text{sarwcl}(B)$.
- (v) $\text{sarwcl}(A) = \text{sarwcl}(\alpha\text{rwcl}(A))$.

Proof:

- i) By definition of sarw -closure, X is only sarw -closed set containing X . therefore $\text{sarwcl}(X) = \text{Intersection of all the } \text{sarw}\text{-closed set containing } X = \bigcap \{X\} = X$ therefore $\text{sarwcl}(X) = X$ and again by definition of sarw -closure $\text{sarwcl}(\phi) = \text{Intersection of all } \text{sarw}\text{-closed sets containing } \phi = \phi \cap \text{any } \alpha\text{rw}\text{-closed set containing } \phi = \phi$ Therefore $\text{sarwcl}(\phi) = \phi$.
- ii) By definition of sarw -closure of A , it is obvious that $A \subseteq \text{sarwcl}(A)$.
- iii) Let B be any sarw -closed set containing A . Since $\text{sarwcl}(A)$ is the intersection of all sarw -closed set containing A , $\text{sarwcl}(A)$ is contained in every sarw -closed set containing A . Hence in particular $\text{sarwcl}(A) \subseteq B$.
- iv) Let A and B be subsets of (X, τ) such that $A \subseteq B$ by definition sarw closure, $\text{sarwcl}(B) = \bigcap \{F : B \subseteq F \in \text{sarwC}(X)\}$. If $B \subseteq F \in \alpha\text{RwC}(X)$, then $\text{sarwcl}(B) \subseteq F$. since $A \subseteq B, A \subseteq B \subseteq F \in \alpha\text{RwC}(X)$, we have $\text{sarwcl}(A) \subseteq F, \text{sarwcl}(A) \subseteq \bigcap \{F : B \subseteq F \in \alpha\text{RwC}(X)\} = \text{sarwcl}(B)$. Therefore $\text{sarwcl}(A) \subseteq \text{sarwcl}(B)$.

- v) Let A be any subset of X by definition of sarw-closure , $\text{sarwcl}(A) = \bigcap \{F: A \subseteq F \in \alpha\text{RwC}(X)\}$. If $A \subseteq F \in \alpha\text{RwC}(X)$ then $\text{sarwcl}(A) \subseteq F$, since F is sarw-closed set containing $\text{sarwcl}(A)$ by (iii) $\text{sarwcl}(\text{arwcl}(A)) \subseteq F$. Hence $\text{sarwcl}(\text{arwcl}(A)) = \bigcap \{F: A \subseteq F \in \alpha\text{RwC}(X)\} = \text{sarwcl}(A)$. Therefore $\text{sarwcl}(\text{arwcl}(A)) = \text{sarwcl}(A)$.

Theorem 4.10: If $A \subseteq X$ is sarw-closed set then $\text{sarwcl}(A) = A$

Proof: Let A be sarw-closed subset of X . we know that $A \subseteq \text{sarwcl}(A)$. Also $A \subseteq A$ and A is sarw-closed , $\text{sarwcl}(A) \subseteq A$. Hence $\text{sarwcl}(A) = A$.

Theorem 4.11: For an $x \in X$, $x \in \text{sarwcl}(A)$ if and if $A \cap V \neq \emptyset$ for every sarw-open set V containing x .

Proof: Let $x \in \text{sarwcl}(A)$. To prove $A \cap V \neq \emptyset$ for every sarw-open set V containing x by contradiction. Suppose \exists sarw-open set V containing x such that $A \cap V = \emptyset$ then $A \subseteq X - V$, $X - V$ is sarw-closed set, $\text{sarwcl}(A) \subseteq X - V$. This shows that $x \notin \text{sarwcl}(A)$ which is contradiction. Hence $A \cap V \neq \emptyset$ for every sarw-open set V containing x .

Conversely:

Let $A \cap V \neq \emptyset$ for every sarw-open set V containing x . To prove $x \in \text{sarwcl}(A)$. We prove the result by contradiction. Suppose $x \notin \text{sarwcl}(A)$ then there exist a sarw-closed subset F containing A such that $x \notin F$. Then $x \in X - F$ is sarw-open . Also $(X - F) \cap A = \emptyset$ which is contradiction. Hence $x \in \text{sarwcl}(A)$.

Theorem 4.12: If A is subset of space X then

- (i) $\text{sarwcl}(A) \subseteq \text{cl}(A)$
- (ii) $\text{sarwcl}(A) \subseteq \text{scl}(A)$

Proof:

- (i) Let A be subset of space X by definition of Closure $\text{cl}(A) = \bigcap \{F: A \subseteq F \in C(X)\}$. If $A \subseteq F \in C(X)$ then $A \subseteq F \in \text{sarwC}(X)$ because every closed set is sarw-closed that is $\text{sarwcl}(A) \subseteq F$ therefore $\text{sarwcl}(A) \subseteq \bigcap \{F: A \subseteq F \in C(X)\} = \text{cl}(A)$. Hence $\text{sarwcl}(A) \subseteq \text{cl}(A)$
- (ii) Let A be subset of space X by definition of s -closure $\text{scl}(A) = \bigcap \{F: A \subseteq F \in C(X)\}$. If $A \subseteq F \in C(X)$ then $A \subseteq F \in \text{sarwC}(X)$ because every s -closed set is sarw-closed that is $\text{sarwcl}(A) \subseteq F$ therefore $\text{sarwcl}(A) \subseteq \bigcap \{F: A \subseteq F \in C(X)\} = \text{scl}(A)$ Hence $\text{sarwcl}(A) \subseteq \text{scl}(A)$.

Theorem 4.13: If A is subset of space X then $\text{gprcl}(A) \subseteq \text{sarwcl}(A)$ where $\text{gprcl}(A) = \bigcap \{F: A \subseteq F \in \text{gprC}(X)\}$.

Proof: Let A be a subset of X by definition of sarw-closure , $\text{sarwcl}(A) = \bigcap \{F: A \subseteq F \in \text{sarwC}(X)\}$. If $A \subseteq F \in \text{sarwC}(X)$ then $A \subseteq F \in \text{gprC}(X)$, because every sarw-closed is gpr-closed that is $\text{gprcl}(A) \subseteq F$ therefore $\text{gprcl}(A) \subseteq \bigcap \{F: A \subseteq F \in \text{sarwC}(X)\} = \text{sarwcl}(A)$. Hence $\text{gprcl}(A) \subseteq \text{sarwcl}(A)$

Definition 4.14: For a subset A of (X, τ) , sarw -interior of A is denoted by $\text{sarwint}(A)$ and defined as $\text{sarwint}(A) = \{G: G \subseteq A \text{ and } G \text{ is sarw-open in } X\}$ or $\cup\{G: G \subseteq A \text{ and } G \in \text{sarwO}(X)\}$ that is $\text{sarwint}(A)$ is the union of all sarw -open set contained in A .

Theorem 4.15: Let A and B be subset of space X then

- i) $\text{sarwint}(X) = X$, $\text{sarwint}(\phi) = \phi$
- ii) $\text{sarwint}(A) \subseteq A$
- iii) If B is any sarw -open set contained in A , then $B \subseteq \text{sarwint}(A)$
- iv) If $A \subseteq B$ then $\text{sarwint}(A) \subseteq \text{sarwint}(B)$
- v) $\text{sarwint}(A) = \text{sarwint}(\text{sarwint}(A))$

Proof: i) and ii) by definition of sarw -interior of A , it is obvious.

iii) Let B be any sarw -open set s.t $B \subseteq A$. Let $x \in B$, B is an sarw -open set contained in A , x is an sarw -interior of A i.e. $x \in \text{sarwint}(A)$. Hence $B \subseteq \text{sarwint}(A)$.

iv), v), similar proof as theorem 5.2 and definition of sarw -interior

Theorem 4.16: If a subset A of X is sarw -open then $\text{sarwint}(A) = A$

Proof: Let A be sarw -open subset of X . We know that $\text{sarwint}(A) \subseteq A$ -----eqn (1).

Also A is sarw -open set contained in A from Theorem 5.14 iii) $A \subseteq \text{sarwint}(A)$ – eqn (2).

Hence from eqn (1) and eqn (2) $\text{sarwint}(A) = A$.

Theorem 4.17: If A is a subset of X then

- i) $\text{int}(A) \subseteq \text{sarwint}(A)$.
- ii) $\alpha\text{int}(A) \subseteq \text{sarwint}(A)$.

Proof:

i) Let A be a subset of a space X . Let $x \in \text{int}(A) \Rightarrow x \in \cup\{G: G \text{ is open, } G \subseteq A\} \Rightarrow \exists$ an open set G such that $x \in G \subseteq A \Rightarrow$ an sarw -open set G s.t. $x \in G \subseteq A$, as every open set is an sarw -open set in $X \Rightarrow x \in \cup\{G: G \text{ is sarw-open set in } X\} \Rightarrow x \in \text{sarwint}(A)$.

Thus $x \in \text{int}(A) \Rightarrow x \in \text{sarwint}(A)$. Hence $\text{int}(A) \subseteq \text{sarwint}(A)$.

ii) Let A be a subset of a space X . Let $x \in \alpha\text{int}(A) \Rightarrow x \in \cup\{G: G \text{ is } \alpha\text{-open, } G \subseteq A\} \Rightarrow \exists$ an α -open set G such that $x \in G \subseteq A \Rightarrow \exists$ an sarw -open set G s.t. $x \in G \subseteq A$, as every α -open set is an sarw -open set in $X \Rightarrow x \in \cup\{G: G \text{ is sarw-open set in } X\} \Rightarrow x \in \text{sarwint}(A)$. Thus $x \in \alpha\text{-int}(A) \Rightarrow x \in \text{sarwint}(A)$. Hence $\alpha\text{int}(A) \subseteq \text{sarwint}(A)$.

Theorem 4.18: If A is subset of X , then $\text{sarwint}(A) \subseteq \text{gpr-int}(A)$, where $\text{gpr-int}(A)$ is given by $\text{gpr-int}(A) = \{G \subseteq X: G \text{ is gpr-open, } G \subseteq A\}$

Proof: Let A be a subset of a space X . Let $x \in \text{sarwint}(A) \Rightarrow x \in \cup\{G: G \text{ is sarw-open, } G \subseteq A\} \Rightarrow \exists$ an sarw -open set G such that $x \in G \subseteq A$, as every sarw -open set is an gpr -open set in $X \Rightarrow x \in \cup\{G: G \text{ is gpr-open, } G \subseteq A\} \Rightarrow x \in \text{gpr-int}(A)$. Thus $x \in \text{sarwint}(A) \Rightarrow x \in \text{gpr-int}(A)$ Hence $\text{sarwint}(A) \subseteq \text{gpr-int}(A)$.

Theorem 4.19: For any subset A of X

- (i) $X - \text{sarwint}(A) = \text{sarwcl}(X - A)$

(ii) $X - \text{sarwcl}(A) = \text{sarwint}(X - A)$

Proof: If $x \in X - \text{sarwint}(A)$, then x is not in $\text{sarwint}(A)$, i.e. every sarw -open set G containing x such that $G \subseteq A$. This implies every sarw -open set G containing x intersects $(X - A)$ i.e. $G \cap (X - A) \neq \emptyset$. Then $x \in \text{sarwcl}(X - A)$. Therefore $X - \text{sarwint}(A) \subseteq \text{sarwcl}(X - A)$ ---eqn (1). Let $x \in \text{sarwcl}(X - A)$, then every sarw -open set G containing x intersects $(X - A)$ i.e. $G \cap (X - A) \neq \emptyset$, i.e. every sarw -open set G containing x s.t. $G \subseteq A$. Then, x is not in $\text{sarwint}(A)$, i.e. $x \in X - \text{sarwint}(A)$ and so $\text{sarwcl}(X - A) \subseteq X - \text{sarwint}(A)$ ---eqn(2). Thus $X - \text{sarwint}(A) = \text{sarwcl}(X - A)$

Definition 4.20: Let (X, τ) be a topological space and Let $x \in X$, A subset of N of X is said to be sarw -neighbourhood of x if there exists an sarw -open set G s.t. $x \in G \subseteq N$.

Definition 4.21:

- (i) Let (X, τ) be a topological space and A be a subset of X , A subset of N of X is said to be sarw -neighbourhood of A if there exists an sarw -open set G s.t. $A \subseteq G \subseteq N$.
- (ii) The collection of all sarw -neighbourhood of $x \in X$ is called sarw -neighbourhood system at x and shall be denoted by $\text{sarw-N}(x)$

Definition 4.22: Let (X, τ) be a topological space and A be a subset of X , then a point $x \in X$ is called a sarw -limit point of A iff every sarw -neighbourhood of x contains a point of A distinct from x i.e. $(N - \{x\}) \cap A \neq \emptyset$ for each sarw -neighbourhood N of x . Also equivalently iff, every sarw -open set G containing x contains a point of A other than x . The set of all sarw limit points of the set A is called derived set A and is denoted by $\text{sarwd}(A)$.

Theorem 4.23: Every neighbourhood N of $x \in X$ is a sarw -neighbourhood of X .

Proof: Let N be neighbourhood of point $x \in X$. To prove that N is a sarw -neighbourhood of x by definition of neighbourhood, \exists an open set G such that. $x \in G \subset N$. Hence N is sarw neighbourhood of x .

Theorem 4.24: If a subset N of a space X is sarw -open, then N is a sarw -nbhd of each of its points.

Proof: Suppose N is sarw -open. Let $x \in N$. We claim that N is sarw -nbhd of x . For N is a sarw -open set such that $x \in N$. Since x is an arbitrary point of N , it follows that N is a sarw -nbhd of each of its points.

Theorem 4.25: Let X be a topological space. If F is a sarw -closed subset of X , and $x \in F^c$. Prove that there exists a sarw -nbhd N of x such that $N \cap F = \emptyset$.

Proof: Let F be sarw -closed subset of X and $x \in F^c$. Then F^c is sarw -open set of X . So F^c contains an sarw -nbhd of each of its points. Hence there exists a sarw -nbhd N of x such that $N \subseteq F^c$. That is $N \cap F = \emptyset$.

Theorem 4.26: Let X be a topological space and for each $x \in X$, Let $\text{sarw-N}(x)$ be the collection of all sarw -nbhd of x . Then we have the following results.

- (i) $\forall x \in X, \text{sarw-N}(x) \neq \emptyset$.
- (ii) $N \in \text{sarw-N}(x) \Rightarrow x \in N$.
- (iii) $N \in \text{sarw-N}(x), M \supset N \Rightarrow M \in \text{sarw-N}(x)$.
- (iv) $N \in \text{sarw-N}(x) \Rightarrow$ there exists $M \in \text{sarw-N}(x)$ such that $M \in N$ and $M \in \text{sarw-N}(y)$ for every $y \in M$.

Proof:

- (i) Since X is a sarw-open set, it is a sarw-nbhd of every $x \in X$. Hence there exists at least one sarw-nbhd for each $x \in X$. Hence $\text{sarw-N}(x) \neq \emptyset$ for every $x \in X$.
- (ii) If $N \in \text{sarw-N}(x)$, then N is a sarw-nbhd of x . So by definition of $\text{sarw-nbhd} \in N$.
- (iii) Let $N \in \text{sarw-N}(x)$ and $M \in N$. Then there is a sarw-open set G such that $x \in G \subseteq N$. Since $N \in M, x \in G \subseteq M$ and so M is sarw-nbhd of x . Hence $M \in \text{sarw-N}(x)$.
If $N \in \text{sarw-N}(x)$, then there exists a sarw-open set M such that $x \in M \subseteq N$. Since M is a sarw-open set, it is sarw-nbhd of each of its points. Therefore $M \in \text{sarw-N}(y)$ for every $y \in M$.

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