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On gg-Open Sets in Topological Space

Basavaraj M. Ittanagi and Govardhana Reddy H G

Department of Mathematics,
Siddaganga Institute of Technology, Tumakuru-03,
Affiliated to VTU, Belagavi, Karnataka State, INDIA.
Department of Mathematics,
Alliance College of Engineering and Design,
Alliance University, Bangalore, Karnataka State, INDIA.
email: dr.basavarajsit@gmail.com, govardhanareddyhg@gmail.com.

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ABSTRACT

In this research paper, a new class of open sets called gg-open sets in topological space are introduced and studied. Also some of their properties have been investigated. We also introduce gg-closure, gg-interior, gg-neighbourhood, gg-limit points and discuss some properties.

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1. INTRODUCTION

The regular semi open sets are introduced and investigated by Cameron[2]. Levin[7] and Savithri[11] imtroduced and studied generalized closed sets and r^g closed sets respectively. Basavaraj M ittanagi and Govardhana Reddy[1] introduced and studied gg closed sets. We introduce and study gg-open sets, gg-interior, gg-neighbourhood, gg-closure and gg-limit points in topological space. Throughout this paper X or (X, τ) represents a topological space. For a subset A of a topological space (X, τ), cl(A), int(A), scl(A), αcl(A), spcl(A) and gcl(A) denote the closure of A, the interior of A, the semi-closure of A, the α-closure of A, the semi pre closure of A and the g-closure of A in X respectively.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X, τ) is called a

1. Regular open set[14] if A=int(cl(A)) and regular closed if A=cl(int(A))
2. Regular semi open set [2] if there exists a regular open set U such that U $\subseteq$ A $\subseteq$ cl(U)
3. θ Closed set[18] if $A = cl_{\theta}(A)$, where $cl_{\theta}(A) = \{x \in X : cl(U) \cap A \neq A, U \in \tau \ \& \ x \in U\}$
4. δ Closed set[18] if $A = cl_{\delta}(A)$, where $cl_{\delta}(A) = \{x \in X : \text{int}(U) \cap A \neq A, U \in \tau \ \& \ x \in U\}$
5. Generalized closed set (g-closed)[7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
6. θ-generalized closed set (θg- closed)[4] if clθ(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ)
7. δ-generalized closed set (δg- closed)[5] if clδ(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ)
8. ω-closed set[13] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi -open in (X, τ).
9. Strongly generalized closed set (g*-closed) [16] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g open in (X, τ).
10. $*$ gα –closed set[19] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gα - open in (X, τ).
11. $**$ gα -closed) set[20] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is *gα - open in (X, τ).
12. rb-closed set[9] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is b-open in (X, τ).
13. g# -closed set[17] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is αg- open in (X, τ).
14. gr-closed set[12] if rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
15. swg*-closed set[8] if gcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi -open in (X, τ).
16. βwg*-closed set[3] if gcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is β-open in (X, τ).
17. rwg-closed set[10] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular -open in (X, τ).
18. βwg**-closed set[15] if βwg* cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular-open in (X, τ).
19. r^g-closed set[11] if gcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular -open in (X, τ).

The complement of the closed sets mentioned above are their open sets respectively and vice versa.

3. gg-CLOSED SETS IN TOPOLOGICAL SPACE

Definition 3.1 [1]: A subset *A* of a topological space (X,τ) is called generalization of generalized closed (gg-closed) set if gcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi open in (X, τ).

Results 3.2: [1]

i) Every closed (respectively regular-closed , θ-closed, δ- closed, л- closed, g- closed, θg- closed, δg- closed, g*- closed, ω- closed, $g^{\#}$- closed, βwg*- closed,*gα- closed, $**$ gα- closed, rb- closed, swg*- closed, gr- closed) set is gg- closed set in X.

ii) Every gg- closed set is r^g- closed (respectively rwg- closed, βwg**- closed) set in X.

iii) The union of two gg-closed sets is gg-closed set in X.

4. gg-OPEN SETS IN TOPOLOGICAL SPACE

Definition 4.1: A subset A of a topological space (X, τ) is called a gg-open set in X if A^c is a gg-closed set in X.
The following Theorem is the analogue of Results 3.2

Theorem 4.2: For any topological space (X, τ) we have the following

i) Every open (respectively regular-open, θ-open, δ-open, л-open, g-open, θg- open, δg-open, g*-open, ω-open, $g^{\#}$-open, βwg*-open,*gα-open, ∗∗ gα-open, rb-open, swg*-open, gr-open) set is gg-open set in X.
ii) Every gg-open set is r^g-open (respectively rwg-open, βwg**-open) set in X.

Theorem 4.3 If A and B are gg-open sets in space X then (A∩B) is also a gg-open set in X.

Proof: Let A and B be gg-open sets in X. Then A^c and B^c are gg-closed sets in X. By Results 3.2 ($A^c \cup B^c$) is also gg-closed set in X. That is ($A^c \cup B^{c)}$ = $(A\cap B)^c$ is gg-closed set in X. Therefore (A∩B) is gg-closed set in X.

Remark 4.4 The union of gg-open sets in X is generally not a gg-open set in X.

Example 4.5 Let X = {p, q, r}, τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}}. If A= {q} and B={r} then A and B are gg-open sets but (A∪B) = {q, r} is not a gg-open set in X.

Theorem 4.6 A subset A of a topological space X is gg-open iff U⊆gint(A) whenever U⊆A and U is regular semiopen in X.

Proof: Suppose A is gg-open , U⊆A and U is regular semiopen. Then $A^c \subseteq U^c$ and U^c is also regular semiopen. By the definition of gg-closed gcl(A^c)⊆U^c . But gcl(A^c)=$(\text{gint}(A))^c$.=X-gint(A) This implies that U⊆gint(A)
Conversely: Suppose that U⊆gint(A) whenever U⊆A and U is regular semiopen in X. Let A^c⊆F where F is regular semiopen in X. F^c⊆A and F^c is regular semiopen in X. we know that F^c⊆gint(A) and $(\text{gint}(A))^c$⊆F. Since gcl(A^c)= $(\text{gint}(A))^c$ we have gcl(A^c)⊆U. Thus A^c is gg-closed set. That is A is gg-open set.

Theorem 4.7 If gint(A)⊆B⊆A and A is gg-open set then B is gg-open set.

Proof: Let gint(A)⊆B⊆A and A is gg open set in X. Let F be any regular semi open set in X such that F⊆B⊆A. Now by theorem 4.6, F⊆gint(A). We have gint(A)⊆B then A⊆gint(B) That is gint(A)⊆gint(B). This implies that F⊆int(B). Now by theorem 4.6, B is gg-open set.

Theorem 4.8 If A⊆X is gg-closed set then gcl(A)-A is gg-open set.

Proof: Ler A be gg-closed set. Let F⊆gcl(A)-A, where F is regular semiopen in X and hence F=ϕ. Then F⊆gint(gcl(A)-A) and by Theorem 4.6, gcl(A)-A is gg-open set.

Remark 4.9 If gcl(A)-A is gg-open set then A is need not be a gg-closed set in X.

Example 4.10 Let X = {p, q, r, s}, τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}}. A= {q, r}, gcl(A) – A={q, r, s}-{q, r}={s} is gg-open set in X but A is not gg-closed set.

Theorem 4.11 Let A and B be subsets of a topological space (X, τ). If A is gg-open and gint(B)⊆A then (A∩B) is gg-open.

Proof: Let A and B be gg-open in X and gint(B)⊆A. Now gint(B)⊆(A∩B)⊆B and B is gg-open set. By Theorem 4.7, (A∩B) is gg-open set in X.

gg-CLOSURE AND GG-INTERIOR IN TOPOLOGICAL SPACE

Definition 4.12 For a subset A of (X, τ), gg-closure of A is denoted by ggcl(A) and it is defined as ggcl(A)=∩{G: A⊆G, G⊆GGC(X)} or intersection of all gg-closed sets containing A.

Theorem 4.13 If A and B are subsets of space (X, τ) then

i) ggcl(X)=X, ggcl(ϕ)=ϕ
ii) A⊆ggcl(A)
iii) if B is any gg-closed set containing A then ggcl(A)⊆B
iv) If A⊆B then ggcl(A)⊆ggcl(B)
v) ggcl(A)=ggcl(ggcl(A))
vi) ggcl(A∪B)=(ggcl(A)∪ggcl(B))

Proof:

i) By the definition of gg-closure, ggcl(X)=Intersection of all gg-closed sets containing X =X ∩gg-closed set containing X=X∩X =X. Therefore ggcl(X)=X. By the definition of gg-closure, ggcl(ϕ)=intersection of all gg-closed sets containing ϕ =ϕ∩ any gg-closed set containing ϕ = ϕ∩ϕ=ϕ. Therefore ggcl(ϕ)=ϕ.

ii) By the definition of gg-closure of A, it is obvious that A⊆ggcl(A).

iii) Let B be any gg-closed set containing A. Since ggcl(A) is the intersection of all gg-closed set containing A. ggcl(A) is contained in every gg-closed set containing A. Hence in particular ggcl(A)⊆B.

iv) Let A and B be subsets of (X, τ) such that A⊆B. By the definition of gg-closure, ggcl(B)=∩{F: B⊆F∈ggcl(X) }. If B⊆F∈ggcl(X) then ggcl(B)⊆F. Since A⊆B ⊆F∈ggcl(X), we have ggcl(A)⊆F, ggcl(A)⊆∩{ F: B⊆F∈ggcl(X)}=ggcl(B). Therefore ggcl(A)⊆ggcl(B).

v) Let A be any subset of X. By the definition of gg-closure, ggcl(A)=∩{F: A⊆F∈ggcl(X)}. If A⊆F∈GGC(X) then ggcl(A)⊆F. Since F is a gg-closed set containing ggcl(A), by iii) ggcl(ggcl(A))⊆F. Hence ggcl(ggcl(A))⊆∩{F: A⊆F∈ggcl(X)}=ggC(A). That is ggcl(ggcl(A))=ggcl(A).

vi) Let A and B be subsets of (X, τ). Clearly A⊆(A∪B) and B⊆(A∪B). From iv) (ggcl(A)∪ggcl(B))⊆ggcl(A∪B)→(1) Now we have to prove that ggcl(A∪B) ⊆ggcl(A)∪ggcl(B). Suppose x∉**(**ggcl(A)∪ggcl(B)) then their exists gg-closed sets $A_1$and B_1 such that A⊆A_1, B⊆B_1 and x∉**(**A_1∪B_1) Thus we have (A∪B)⊆(A_1∪B_1) and (A_1∪B_1) is a gg-closed set by Results 3.2, such that x∉(A_1∪B_1) . Thus x∉ggcl(A∪B). Hence ggcl(A∪B)⊆(ggcl(A)∪ggcl(B))→(2).
From (1) and (2) we have ggcl(A∪B)=(ggcl(A)∪ggcl(B)).

Theorem 4.14 If A⊆X is gg-closed set then ggcl(A)=A.
Proof: Let A be any gg-closed set in X. we have A ⊆ggcl(A) →(1)**.** By Theorem 4.13, ggcl(A)⊆A → (2)**.** From (1) and (2), ggcl(A)=A.

Remark 4.15 If ggcl(A)=A then A is need not be a gg-closed set in X.

Example 4.16 Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, r}} be a topology on X. Then GGC(X) = {ϕ, X, {q}, {r}, {p, q}, {p, r}, {q, r}}. Now ggcl({p})={p}. But {p} is not a gg-closed set in X.

Theorem 4.17 If A and B are subsets of a topological space (X,τ) then ggcl(A∩B) ⊆ggcl(A)∩ggcl(B).

Proof: Let A⊆X and B⊆X**.** We know that (A∩B) ⊆A and (A∩B) ⊆B. Then by Theorem 4.13 we have ggcl(A∩B) ⊆ggcl(A) and ggcl(A∩B) ⊆ggcl(B).
Therefore ggcl(A∩B) ⊆ggcl(A) ∩ggcl(B).

Remark 4.18 In general ggcl(A)∩ggcl(B) is need not be a subset of ggcl(A∩B).

Example 4.19 Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, r}} be a topology on X. Let A = {p} and B = {r} are subsets of X. Now A∩B=ϕ, ggcl(A)={p}, ggcl(B)={p, r}and ggcl(A∩B)= ϕ but (ggcl(A)∩ggcl(B)) is not a subset of ggcl(A∩B).

Theorem 4.20 If A⊆X then

i) ggcl(A) ⊆cl(A)
ii) ggcl(A) ⊆gcl(A)
iii) r^g-cl(A) ⊆ggcl(A)

Proof:
i) Let A⊆X**.** We have $cl(A) = \cap \{F: A \subseteq F \in C(X)\}$. Since every closed set is gg-closed, if $A \subseteq F \in C(X)$ then $A \subseteq F \in GGC(X)$. That is $ggcl(A) \subseteq F$. Therefore $gg(cl(A)) = \cap$ $\{F: A \subseteq F \in C(X)\}$= $cl(A)$. Hence ggcl(A) ⊆cl(A).

ii) The proof is straight forward

iii) The proof is straight forward.

Theorem 4.21 Let A be any subset of a topological space X. Then x∈ggcl(A) if and only if U∩A≠ φ for every gg-open set U containing x.

Proof: We prove this theorem by contradiction. Let x∈ggcl(A) and suppose there is a gg-open set U in X such that x∈U and U∩A= φ. Therefore A⊆U^c which is gg-closed set in X. Also ggcl(A) ⊆ggcl(U^c)= U^c. Then x∉U ⇒ x∉ggcl(A). This is a contradiction.
Conversely, suppose that U∩A≠ φ for any gg-open set V containing x. To prove x∈ggcl(A). Suppose x∉ggcl(A), then there exists a gg-closed set F in X such that x∉F and A⊆F. That is x∈F^c which is gg-open set in X. Since A⊆F, A∩F= φ. This is a contradiction. Therefore x∈ggcl(A).

Definition 4.22 For a subset A of (X, τ), gg-interior of A is denoted by ggint(A) and it is defined as ggint(A)= ∪{G: G⊆A and G is gg-open in X } or ∪{G: G⊆A and G ∈GGO(X)} or ggint(A) is the union of all gg-open sets contained in A.

Theorem 4.23 Let A and B be subsets of space X then
i) ggint(X)=X, ggint(ϕ)=ϕ
ii) ggint(A)⊆A
iii) if B is any gg-open set contained in A then B⊆ggint(A)
iv) If A⊆B then ggint(A)⊆ggint(B)
v) ggint(A)=ggint(ggint(A))
vi) ggint(A∩B)=(ggint(A)∩ggint(B)).

Proof:
i) and ii) follows by the definition of gg-interior of A.

iii) Let B be any gg-open set such that B⊆A. Let x ∈B, B be gg-open set contained in A, x is an interior point of A that is x∈ggint(A). Hence B⊆ggint(A).

Proof of iv), v) and vi) is similar as Theorem 4.12 and definition of gg-interior.

Theorem 4.24 If a subset A of X is gg-open then ggint(A)=A

Proof: Let A be gg-open set of X. we know that ggint(A)⊆A → (1) Also A is gg-open set contained in A from Theorem 4.23 iii), A⊆ggint(A) → (2). Hence from (1) and (2) ggint(A) =A.

Theorem 4.25 If A and B are subsets of space X then (ggint(A)∪ggint(B))⊆ggint(A∪B)

Proof: We know that A⊆(A∪B) and B⊆(A∪B). We have Theorem 4.22 iv) ggint(A)⊆ggint(A∪B) and ggint(B)⊆ggint(A∪B). This implies that ggint(A)∪ggint(B) ⊆ggint(A∪B).

Remark 4.26 In general ggint(A∪B) is not a subset of (ggint(A) ∪ ggint(B)).

Example 4.27 Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, r}} be a topology on X. Let A = {q, r} and B = {p, q} are subsets of X. Now A∪B=X, ggint(A)={q, r}, ggint(B)={q}, ggint(A∪B)= X and (ggint(A) ∪ ggint(B))= {q, r}but ggint(A∪B) is not a subset of (ggint(A) ∪ ggint(B))

Theorem 4.28 If A⊆X then

i) int(A) ⊆ggint(A)
ii) gint(A) ⊆ggint(A)
iii) ggint(A) ⊆r^g-int(A)

Proof:

i) Let A⊆X. Let x∈int(A) ⇒ x ∈∪ $\{G: G\ is\ open, G \subseteq A\}$. Then there exists an open set G such that x ∈ G⊆A. Since every open set is gg-open, we have gg-open set G such that x ∈ G⊆A. This implies that x ∈∪ $\{G: G\ is\ gg - open, G \subseteq A\}$ ⇒ x ∈ ggint(A). Hence int(A) ⊆ggint(A).
ii) The proof is straight forward
iii) The proof is straight forward.

gg-NEIGHBOURHOOD AND GG-LIMIT POINTS IN TOPOLOGICAL SPACE

Definition 4.29 Let (X, τ) be a topological space and x∈X. A subset N of X is said to be gg-neighbourhood of x if their exists gg-open set G such that x∈G⊆N

Definition 4.30 Let (X, τ) be a topological space and A be a subset of X, a subset N of X is said to be gg-neighbourhood of A if their exists gg-open set G such that A⊆G⊆N.

Definition 4.31 The collection of all gg-neighbourhood of x∈X is called gg-neighbourhood system at x and it is denoted by gg-N(X)

Theorem 4.32 Every neighbourhood N of x belongs to X is a gg-neighbourhood of X.

Proof: Let N be a neighbourhood of x∈X. To prove that N is a gg-neighbourhood of x, by the definition of neighbourhood there exists an open set G such that x∈G⊆N. Hence N is a gg-neighbourhood of x.

Remark 4.33 In general, a gg-neighbourhood N of x belongs to X need not be a neighbourhood of x in X as seen from the following example.

Example 4.34 Let X = {p, q, r, s}, τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}}. Then GGO(X)={ ϕ, X, {p}, {q}, {r}, {s},{p, q},{q, r}, {r, s}, {p, q, r}}. The set {q, r, s} is a gg- neighbourhood of the point s. Since the gg-open set {r, s} is such that s⊆{r, s} ⊆{q, r, s}. However the set {q, r, s} is not a neighbourhood of the point d, since no open set G exists such that r∈G⊆{r, s}.

Theorem 4.35 If a subset N of a space X is gg-open then N is a gg-nbd of each of its points.

Proof: Suppose N is gg-open. Let x∈N. we claim that N is gg-nbd of X. For N is a gg-open set such that x∈N∈N. Since x is arbitrary point of N, it follows that N is a gg-nbd of each of its points.

Remark 4.36 The converse of the above theorem is not true in general as seen from the following example.

Example 4.37 Let X = {p, q, r} with the topology τ = {ϕ, X, {p}, {q}, {p, q}}. Then ggO(X)={ ϕ, X, {p}, {q}, {r},{p, q}}. The set {p, r} is a gg- neighbourhood of each of its points but not a gg-open set in X.

Theorem 4.38 Let X be a topological space. If F is a gg-closed subset of X and x∈F^c then prove that there exists a gg-nbd N of x such that N∩F=ϕ

Proof: Let F be gg-closed subset of X and x∈F^c. Then by Theorem 6.7, F^c contains a gg-nbd of each of its points. Hence there exists a gg-nbd of x such that N⊆ F^c. That is N∩F=ϕ.

Theorem 4.39 Let X be a topological space and for each x∈X. Let gg-N(x) be the collection of all gg-nbds of x. Then we have the following results.

i) for all x∈X, gg-N(x)≠ ϕ
ii) iN∈gg-N(x) ⟹ x∈N,
iii) N∈gg-N(x), M contains N ⟹ M∈gg-N(x)
iv) N∈gg-N(x), M∈gg-N(x) ⟹ (N∩M) ∈gg-N(x)
v) N∈gg-N(x) ⟹ There exists M∈gg-N(x) such that M⊆N and M∈gg-N(y) for every y∈M.

Proof:

i) Since X is a gg-open set, it is a gg-nbd of every x∈X. Hence there exists at least one gg-nbd (namely X) for each x∈X. Let gg-N(x) be the collection of all gg-nbds of x. Hence gg-N(x) ≠ ϕ.
ii) If N∈gg-N(x) then N is a gg-nbd of x. so by the definition of gg-nbd, x∈N.
iii) Let N∈gg-N(x), M contains N. Then there is a gg-open set G such that x∈G⊆N. Since N⊆M, x∈G⊆M and so M is a gg-nbd of x. Hence M∈gg-N(x).
iv) Let N∈gg-N(x) and M∈gg-N(x). Then by the definition of gg-nbd there exist gg-open sets $G_1$and G_2 such that x∈G_1⊆N and x∈G_2⊆N. Hence x∈(G_1∩G_2)⊆(N∩M). this implies that N∩M is a gg-nbd of x. Hence (N∩M)∈gg-N(x).
v) If N∈gg-N(x) then there exists a gg-open set M such that x∈M⊆N. Since M is a gg-open set it is a gg-nbd of each of its points. Therefore M∈gg-N(y), for every y∈M.

Definition 4.40 Let (X, τ) be a topological space and A be a subset of X, then a point x∈X is called a gg-limit point of A iff every gg-neighbourhood of x contains a point of A distinct from x that is ((N-{x})∩A)≠ϕ for each gg-neighbourhood N of x.

Also equivalently, every gg-open set G containing x contains a point of A which is other than x.

Definition 4.41 The set of all gg-limit points of the set A is called a derived set A and is denoted by ggd(A)

Theorem 4.42 Let (X,τ) be a topological space. Let A and B are any two subsets of X. If A⊂B then ggd(A)⊂ggd(B).

Proof: Let A and B be any two subsets of x such that A⊂B. Let x∈ggd(A) that is x is a gg -limit point of A. Let G be any gg-open set containing x. since x is gg-limit point of A,by the definition, G∩(A-{x})≠ ϕ → (1) since A⊂B we have A-{x}⊂B-{x} and G∩(A-{x})⊂ G∩(B-{x})⟹ G∩(A-{x})≠ ϕ..Therefore G∩(A-{x})≠ ϕ for every gg-open set containing x.that is x is a gg-limit point of B and x∈d(B).then x∈ggd(A)⇒ x∈ggd(B)
∴ggd(A)⊂ggd(B).

Theorem 4.43 Let (X,τ) be a topological space and A be any subset of X then A is gg-closed set iff ggd(A)⊂A.

Proof: Suppose A is gg-closed. To prove that ggd(A)⊂A. Let x∉A⇒x∈X-A and X-A is gg-open set containing x and (X-A)∩A=ϕ that is (X-A) ∩(A-{x})= ϕ. Therefore there exist gg-open set (X-A) containing x such that (X-A) ∩(A-{x})= ϕ. Therefore x is not a gg-limit point of A and x∉ggd(A) that is ggd(A)⊂A

Conversely: Suppose ggd(A)⊂A.To prove A is a gg-closed set that is to prove X-A is a gg-open. Let x∈(X-A)⇒x∉A and x∉ggd(A). Then there exist an gg open set G containing x such that G∩(A-{x})=ϕ ⇒G∩A= ϕ ⇒G⊂X-A. Therefore ∀x∈(X-A) there exist a gg-open set G such that x∈G⊂X-A. ∴ X-A is gg-open set and hence A is gg-closed set.

5. REFERENCES

1. Basavaraj M Ittanagi and Govardhana Reddy H G, On gg-Closed Sets in Topological Spaces, *International Journal of Mathematical Archive*-8(8), 126-133 (2017).
2. D. E. Cameron, Properties of s-closed spaces *Proc. Amer. Math. Soc.*, 72, 581-586 (1978).
3. C. Dhanapakyam J. Subashini and K. Indirani, ON - βwg* set and continuity in topological spaces, *IJMA*-5(5), 222-233 (2014).
4. J. Dontchev and H.Maki, On θ-generalized closed sets, *International J. Math and Math. Sci.* Vol 22, No 2, 239-249.
5. J.Dontchev and M.Ganster, On δ-generalized set $T_{3/4}$ spaces, *Mem. Fac. Sci. Kochi Uni. Ser. A. Math.*, 17, 15-31 (1996).
6. J. Dontchev and T. Nori, Quasi-normal spaces and πg-closed sets, *Acta Math. Hungar* 89(3), 211-219 (2000).
7. N. Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo*, 19(2), 89-96 (1970).

8. C. Mukundhan and N. Nagaveni, A Weaker Form of a Generalized Closed Set, *Int. J. Contemp. Math. Sciences*, Vol. 6, no. 20, 949 – 961 (2011).
9. A. Narmadha, N. Nagaveni and T. Noiri, On Regular b-Open Sets in Topological Spaces *Int. Journal of Math. Analysis*, Vol. 7, no. 19, 937 – 948 (2013).
10. Sanjay Mishra, Nitin Bhardwaj and Varun Joshi, On Regular Generalized weakly (rgw)-Closed Sets in Topological Spaces, *Int. Journal of Math. Analysis*, Vol. 6, no. 39, 1939 – 1952 (2012).
11. Savithiri D and Janaki C, .On Regular ^Generalized closed sets in topological spaces *International Journal of Mathematical Archive*-4(4), 162-169 (2013).
12. Sharmistha Bhattacharya, On Generalized Regular Closed Sets, *Int. J. Contemp. Math. Sciences*, Vol. 6, no. 3, 145 – 152 (2011).
13. M. Sheik John, On w-closed sets in Topology, *Acta Ciencia Indica*, 4, 389-392 (2000).
14. M. Stone, Application of the theory of Boolean rings to general topology, *Trans. Amer. Math Soci*, 41, 374-481 (1937).
15. Subashini Jesu Rajan, On βwg** set and Continuity in Topological Spaces, *International Journal of Computing*, Vol 4 Issue 3, July (2014).
16. M.K.R.S. Veera Kumar, Between closed sets and g-closed sets, *Mem. Fac. Sci. Kochi Univ., Math.* 21, 1-19 (2000).
17. M.K.R.S. Veera Kumar, g#-closed sets in topological spaces, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math* 24, 1-13 (2003).
18. N.V.Velicko, H-closed topological spaces. *Amer. Math. Soc. Transl*. 78, 103-118 (1968).
19. M. Vigneshwaran and R. Devi, On GαO-kernel in the digital plane, *International Journal of Mathematical Archive*-3(6), 2358-2373 (2012).
20. M. Vigneshwaran and A. Singaravelan, Applications of **gα-closed sets in Topological Spaces, *IJMA*-5(10), 139-150 (2014).