

# Heat and Mass Transfer on Visco-elastic Fluid through a Porous Medium Past an Infinite Oscillating Plate with Hall Effects

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## ABSTRACT

In this paper, we have considered the unsteady flow of an incompressible visco-elastic liquid of the Walter  $B'$  model with simultaneous heat and mass transfer near an oscillating porous plate in slip flow regime taking hall current into account. The governing equations of the flow field are solved by a regular perturbation method for small elastic parameter. The expressions for the velocity, temperature, concentration have been derived analytically and also its behaviour is computationally discussed with reference to different flow parameters with the help of graphs. The skin friction on the boundary, the heat flux in terms of the Nusselt number, and the rate of mass transfer in terms of the Sherwood number are also obtained and their behaviour is discussed.

**Keywords:** Heat and mass transfer, Hall effects, MHD flows, porous medium, unsteady flows and visco-elastic fluids.

## 1. INTRODUCTION

The heat and mass transfer in MHD flow of viscous fluids past a vertical plate under oscillatory suction velocity has been analysed by Singh *et al.*<sup>1</sup>. Sharma and Singh<sup>9</sup> have put-forth their reports regards the unsteady MHD-free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. As observed in practical applications, the particle at surface slips along the surface. As far the MHD Free convective flow of a viscous fluid through a porous medium is concerned as has been discussed by Singh and Gupta<sup>10</sup> that the flow is bounded by an oscillating porous plate in slip flow regime

with mass transfer. Khandelwal and Jain<sup>2</sup> have also in the same dimension analytically observed the unsteady MHD flow phenomenon with same characteristics. Das *et al.*<sup>16</sup> have studied the magneto hydrodynamic unsteady flow of a viscous stratified fluid through a porous medium past a porous flat moving plate in the slip flow regime with heat source. Mahapatra *et al.*<sup>7</sup> have studied effects of chemical reaction on a moving isothermal vertical surface with suction. Another reference here is that of Al-Odat and Al-Azab<sup>4</sup>. Both scientists and engineers alike have paid attention notably in this phenomenon of visco-elastic fluid flow through porous medium.

The effects of the Hall current and radiation on MHD mixed convection flow of a visco-elastic fluid past an infinite vertical plate have also been observed by Chaudhary and Jain<sup>14</sup>. Sahoo *et al.*<sup>17</sup> have studied the unsteady two-dimensional MHD flow and thereby the heat transfer of an elastic-viscous liquid past an infinite hot vertical porous surface. They have observed that the vertical porous surface is bounded by porous media with source/sink. Kumar and Chand<sup>15</sup> have studied the effect of slip conditions and the Hall current on unsteady MHD flow of a visco-elastic fluid past an infinite vertical porous plate through porous medium. Heat and mass transfer effect on MHD flow of a visco-elastic fluid through a porous medium bounded by an oscillating porous plate in slip flow regime discussed by Sahoo<sup>16</sup>.

Recently Animasaun<sup>3</sup> discussed the effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow with suction and  $n$ th order of chemical reaction. The behaviour of unsteady non-Darcian magnetohydrodynamic fluid flow past an impulsively started vertical porous surface is investigated by Motsa and Animasaun<sup>19</sup>. N. Ali *et al.*<sup>8</sup> discussed the Soret and Dufour effects on hydromagnetic flow of viscoelastic fluid over porous oscillatory stretching sheet with thermal radiation. Manjunatha *et al.*<sup>11</sup> investigated thermal analysis of conducting dusty fluid flow in a porous medium over a stretching cylinder in the presence of non-uniform source/sink. Prasannakumara *et al.*<sup>12</sup> discussed the effects of chemical reaction and nonlinear thermal radiation on Williamson nanofluid slip flow over a stretching sheet embedded in a porous medium. The effects of radiation and hall current on MHD free convection three dimensional flow in a vertical channel filled with a porous medium has been studied by VeeraKrishna and Chand Basha<sup>5</sup>. Veera Krishna<sup>6</sup> discussed the unsteady flow of an incompressible electrically conducting viscous fluid in a rotating porous media, with a variable pressure gradient and in the presence of hall current. Recently, Krishna and Swarnalathamma<sup>20</sup> discussed the peristaltic MHD flow of an incompressible and electrically conducting Williamson fluid in a symmetric planar channel with heat and mass transfer under the effect of inclined magnetic field. Krishna and M.G. Reddy<sup>21</sup> discussed MHD free convective rotating flow of visco-elastic fluid past an infinite vertical oscillating plate. Krishna and G.S. Reddy<sup>22</sup> discussed unsteady MHD convective flow of second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source. Swarnalathamma and Krishna<sup>23</sup> discussed the theoretical and computational study of peristaltic hemodynamic flow of couple stress fluids through a porous medium under the influence of magnetic field with wall slip condition.

After having gone through all the studies, we have observed, in this paper the unsteady flow of an incompressible visco-elastic liquid of the Walter  $B'$  model with all its characteristics.

## 2. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the unsteady flow of an incompressible visco-elastic liquid of Walter  $B'$  model through porous medium with simultaneous heat and mass transfer near an oscillating infinite porous plate in slip flow regime with heat source and chemical reaction under the influence of a uniform transverse magnetic field of strength  $H_0$  and taking Hall current into account. The  $z$ -axis is taken along the plate in vertical direction, and  $x$ -axis is perpendicular to it. A uniform magnetic field of strength  $B_0$  is applied in the direction of  $z$ -axis. Let  $u$  and  $v$  be the velocity components in  $x$ - and  $y$ -directions respectively. If the plate is extended to infinite length, then all the physical variables in the problem are functions of  $z$  and  $t$  alone. Initially, the plate and fluid are at rest, and then the plate is set to an oscillatory motion. The Reynolds number is assumed to be very small, and the induced magnetic field due to the flow is neglected with respect to the applied magnetic field. The pressure in the fluid is assumed to be constant. If  $w_0$  represents the suction or injection velocity at the plate, then governing equations are,

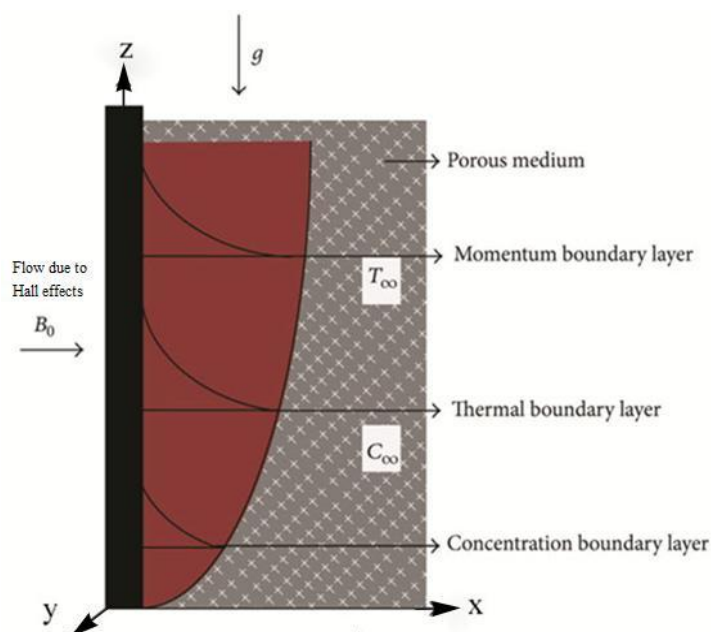


Figure 1: Physical configuration of the Problem

$$\frac{\partial w}{\partial z} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} = v \frac{\partial^2 u}{\partial z^2} - \frac{K_0}{\rho} \left( \frac{\partial^3 u}{\partial z^2 \partial t} + v_0 \frac{\partial^3 u}{\partial z^3} \right) + B_0 J_y - \frac{v}{k} u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{2.2}$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} = v \frac{\partial^2 v}{\partial z^2} - \frac{K_0}{\rho} \left( \frac{\partial^3 v}{\partial z^2 \partial t} + v_0 \frac{\partial^3 v}{\partial z^3} \right) - B_0 J_x - \frac{v}{k} v \tag{2.3}$$

$$\frac{\partial T}{\partial t} - w_0 \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} - S(T - T_\infty) \tag{2.4}$$

$$\frac{\partial C}{\partial t} - w_0 \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} - K_1(C - C_\infty) \tag{2.5}$$

When the strength of the magnetic field is very large, the generalized ohm's law is modified to include the hall current so that

$$J + \frac{\omega_e \tau_e}{B_0} (J \times B) = \sigma \left[ E + V \times B + \frac{1}{e\eta_e} \nabla P_e \right] \tag{2.6}$$

Where,  $\omega_e$  is the cyclotron frequency of the electrons,  $\tau_e$  is the electron collision time,  $\sigma$  is the electrical conductivity,  $e$  is the electron charge and  $P_e$  is the electron pressure. The ion-slip and thermo electric effects are not included in equation (2.6). Further it is assumed that  $\omega_e \tau_e \sim O(1)$  and  $\omega_i \tau_i \ll 1$ , where  $\omega_i$  and  $\tau_i$  are the cyclotron frequency and collision time for ions respectively. We also assume that the electric field  $E=0$  under assumptions reduces to

$$J_x + m J_y = \sigma B_0 v \tag{2.7}$$

$$J_y - m J_x = -\sigma B_0 u \tag{2.8}$$

where,  $m = \tau_e \omega_e$  is the Hall parameter.

On solving equations (2.7) and (2.8) we obtain

$$J_x = \frac{\sigma B_0}{1 + m^2} (v + mu) \tag{2.9}$$

$$J_y = \frac{\sigma B_0}{1 + m^2} (mv - u) \tag{2.10}$$

Substituting the equations (2.9) and (2.10) in (2.2) and (2.3) , we obtain

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} = v \frac{\partial^2 u}{\partial z^2} - \frac{K_0}{\rho} \left( \frac{\partial^3 u}{\partial z^2 \partial t} + v_0 \frac{\partial^3 u}{\partial z^3} \right) - \left( \frac{\sigma B_0^2}{\rho(1+m^2)} + \frac{v}{k} \right) u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{2.11}$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} - \frac{K_0}{\rho} \left( \frac{\partial^3 v}{\partial z^2 \partial t} + \nu_0 \frac{\partial^3 v}{\partial z^3} \right) - \left( \frac{\sigma B_0^2}{\rho(1+m^2)} + \frac{\nu}{k} \right) v \quad (2.12)$$

Combining the equations (2.11) and (2.12), let  $q = u + iv$ ,

$$\frac{\partial q}{\partial t} - w_0 \frac{\partial q}{\partial z} = \nu \frac{\partial^2 q}{\partial z^2} - \frac{K_0}{\rho} \left( \frac{\partial^3 q}{\partial z^2 \partial t} + w_0 \frac{\partial^3 q}{\partial z^3} \right) - \left( \frac{\sigma B_0^2}{\rho(1+m^2)} + \frac{\nu}{k} \right) q + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (2.13)$$

The first order velocity slip boundary conditions of the problem when the plate executes linear harmonic oscillations in its own plane are given by

$$q = U_0 e^{i\omega t} + L_1 \frac{\partial q}{\partial z}, T = T_w, C = C_w \quad \text{at } z = 0 \quad (2.14)$$

$$q = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{at } z \rightarrow \infty \quad (2.15)$$

where,  $L_1 = (2 - m_1)(L / m_1)$ ,  $L = \mu(\pi / 2p\rho)^{1/2}$  is the mean free path, and  $m_1$  is Maxwell's reflection coefficient.

On introducing the following non-dimensional quantities

$$u^* = \frac{u}{U_0}, v^* = \frac{v}{U_0}, t^* = \frac{tU_0^2}{\nu}, z^* = \frac{zU_0}{\nu}, \omega^* = \frac{\nu}{U_0^2} \omega,$$

$$w_0^* = \frac{w_0}{U_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty}$$

Making use of non-dimensional variables, the governing equations reduce to (Dropping asterisks)

$$\frac{\partial q}{\partial t} - w_0 \frac{\partial q}{\partial z} = \frac{\partial^2 q}{\partial z^2} - Rc \left( \frac{\partial^3 q}{\partial z^2 \partial t} + \nu_0 \frac{\partial^3 q}{\partial z^3} \right) - \left( \frac{M^2}{1+m^2} + \frac{1}{K} \right) q + Gr\theta + GmC \quad (2.16)$$

$$Pr \frac{\partial \theta}{\partial t} - w_0 \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial z^2} - Pr S\theta \quad (2.17)$$

$$Sc \frac{\partial C}{\partial t} - w_0 \frac{\partial C}{\partial z} = \frac{\partial^2 C}{\partial z^2} - Kc Sc C \quad (2.18)$$

Corresponding boundary conditions are,

$$q = e^{i\omega t} + R \frac{\partial q}{\partial z}, \theta = 1, C = 1 \quad \text{at } z = 0 \quad (2.19)$$

$$q = 0, \theta = 0, C = 0 \quad \text{at } z \rightarrow \infty \quad (2.20)$$

where,

$M^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2}$  is the Hartmann number (Magnetic field parameter),  $K = \frac{kU_0^2}{\nu^2}$  is the permeability

parameter (Darcy parameter),  $R = \frac{L_1 U_0}{\nu}$  is the Rarefaction parameter,  $Pr = \frac{\nu}{\alpha}$  is the Prandtl

number,  $Sc = \frac{\nu}{D}$  is the Schmidt number,  $Rc = \frac{U_0^2 K_0}{\rho \nu^2}$  is the elastic parameter,  $Kc = \frac{\nu K_1}{U_0^2}$  is the chemical reaction parameter,  $S^* = \frac{\nu S}{U_0^2}$  is the Heat source parameter,  $Gr = \frac{g \beta \nu (T_w - T_\infty)}{U_0^3}$  is the Grashof number,  $w_0^* = \frac{w_0}{U_0}$  is the suction/injection velocity and  $Gm = \frac{g \beta^* \nu (C_w - C_\infty)}{U_0^3}$  is the mass Grashof number..

Equation (2.9) is of third order, and two boundary conditions are available. Due to inadequate boundary condition, a perturbation method has been applied with  $Rc < 1$  as the perturbation parameter. This assumption is quite consistent as the model under consideration is valid only for slightly elastic fluid. We have considered the following,

$$q = q_0 + Rc q_1 + O(Rc)^2 \tag{2.21}$$

$$\theta = \theta_0 + Rc \theta_1 + O(Rc)^2 \tag{2.22}$$

$$C = C_0 + Rc C_1 + O(Rc)^2 \tag{2.23}$$

Substituting (2.21) to (2.23) in (2.16) - (2.18) and equating like powers of  $Rc$ , we get the following.

**Zeroth order:**

$$\frac{\partial q_0}{\partial t} - w_0 \frac{\partial q_0}{\partial z} = \frac{\partial^2 q_0}{\partial z^2} - \left( \frac{M^2}{1+m^2} + \frac{1}{K} \right) q_0 + Gr \theta_0 + Gm C_0 \tag{2.24}$$

$$Pr \frac{\partial \theta_0}{\partial t} - w_0 \frac{\partial \theta_0}{\partial z} = \frac{\partial^2 \theta_0}{\partial z^2} - Pr S \theta_0 \tag{2.25}$$

$$Sc \frac{\partial C_0}{\partial t} - w_0 \frac{\partial C_0}{\partial z} = \frac{\partial^2 C_0}{\partial z^2} - Kc Sc C_0 \tag{2.26}$$

**First order:**

$$\frac{\partial q_1}{\partial t} - w_0 \frac{\partial q_1}{\partial z} = \frac{\partial^2 q_1}{\partial z^2} - \left( \frac{\partial^3 q_0}{\partial z^2 \partial t} + \nu_0 \frac{\partial^3 q_1}{\partial z^3} \right) - \left( \frac{M^2}{1+m^2} + \frac{1}{K} \right) q_1 + Gr \theta_1 + Gm C_1 \tag{2.27}$$

$$Pr \frac{\partial \theta_1}{\partial t} - w_0 \frac{\partial \theta_1}{\partial z} = \frac{\partial^2 \theta_1}{\partial z^2} - Pr S \theta_1 \tag{2.28}$$

$$Sc \frac{\partial C_1}{\partial t} - w_0 \frac{\partial C_1}{\partial z} = \frac{\partial^2 C_1}{\partial z^2} - Kc Sc C_1 \tag{2.29}$$

The corresponding boundary conditions

$$q_0 = e^{i\omega t} + R \frac{\partial q_0}{\partial z}, q_1 = 1, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \quad \text{at } z = 0 \quad (2.30)$$

$$q_0 = 0, q_1 = 0, \theta_0 = 0, \theta_1 = 0, C_0 = 0, C_1 = 0 \quad \text{at } z \rightarrow \infty \quad (2.31)$$

In order to reduce the system of partial differential equations (2.27)-(2.29) to a system of ordinary differential equations, we further introduce

$$q_0(z, t) = q_{00}(z) + q_{01}(z) e^{i\omega t} \quad (2.32)$$

$$q_1(z, t) = q_{10}(z) + q_{11}(z) e^{i\omega t} \quad (2.33)$$

$$\theta_0(z, t) = \theta_{00}(z) + \theta_{01}(z) e^{i\omega t} \quad (2.34)$$

$$\theta_1(z, t) = \theta_{10}(z) + \theta_{11}(z) e^{i\omega t} \quad (2.35)$$

$$C_0(z, t) = C_{00}(z) + C_{01}(z) e^{i\omega t} \quad (2.36)$$

$$C_1(z, t) = C_{10}(z) + C_{11}(z) e^{i\omega t} \quad (2.37)$$

Substituting (2.32) to (2.37) into (2.27) - (2.29) and equating the harmonic and non-harmonic terms, we obtain

$$q_{00}'' + w_0 q_{00}' - \left( \frac{M^2}{1+m^2} + \frac{1}{K} \right) q_{00} = -Gr \theta_{00} - Gm C_{00} \quad (2.38)$$

$$q_{01}'' + w_0 q_{01}' - \left( \frac{M^2}{1+m^2} + \frac{1}{K} + i\omega \right) q_{01} = -Gr \theta_{01} - Gm C_{01} \quad (2.39)$$

$$q_{10}'' + w_0 q_{10}' - \left( \frac{M^2}{1+m^2} + \frac{1}{K} \right) q_{10} = -Gr \theta_{10} - Gm C_{10} + w_0 q_{00}''' \quad (2.40)$$

$$q_{11}'' + w_0 q_{11}' - \left( \frac{M^2}{1+m^2} + \frac{1}{K} + i\omega \right) q_{11} = -Gr \theta_{11} - Gm C_{11} + w_0 q_{01}''' + i\omega q_{01}'' \quad (2.41)$$

$$\theta_{00}'' + w_0 Pr \theta_{00}' - Pr S \theta_{00} = 0 \quad (2.42)$$

$$\theta_{01}'' + w_0 Pr \theta_{01}' - (i\omega + S) Pr \theta_{01} = 0 \quad (2.43)$$

$$\theta_{10}'' + w_0 Pr \theta_{10}' - Pr S \theta_{10} = 0 \quad (2.44)$$

$$\theta_{11}'' + w_0 Pr \theta_{11}' - (i\omega + S) Pr \theta_{11} = 0 \quad (2.45)$$

$$C_{00}'' + w_0 Sc C_{00}' - Kc Sc C_{00} = 0 \quad (2.46)$$

$$C_{01}'' + w_0 Sc C_{01}' - (i\omega + Kc) Sc C_{01} = 0 \quad (2.47)$$

$$C_{10}'' + w_0 Sc C_{10}' - Kc Sc C_{10} = 0 \quad (2.48)$$

$$C_{11}'' + w_0 Sc \theta_{11}' - (i\omega + Kc) Sc C_{11} = 0 \tag{2.49}$$

with boundary conditions

$$q_{00} = R \frac{\partial q_{00}}{\partial z}, q_{01} = 1 + R \frac{\partial q_{01}}{\partial z}, q_{10} = 0, q_{11} = 0, \theta_{00} = 1, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, C_{00} = 1, C_{01} = 0, C_{10} = 0, C_{11} = 0, \text{ at } z = 0 \tag{2.50}$$

$$q_{00} = 0, q_{01} = 0, q_{10} = 0, q_{11} = 0, \theta_{00} = 0, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, C_{00} = 0, C_{01} = 0, C_{10} = 0, C_{11} = 0, \text{ at } z \rightarrow \infty \tag{2.51}$$

The solutions of (2.35) to (2.38), applying boundary conditions (2.39) and (2.40) are

$$q_{00} = a_1 e^{-m_1 z} + a_2 e^{-m_2 z} + a_3 e^{-m_3 z}; q_{01} = a_4 e^{-m_4 z}; q_{10} = a_5 e^{-m_1 z} + a_6 e^{-m_2 z} + a_7 e^{-m_3 z}; q_{11} = a_8 e^{-m_4 z}; \theta_{00} = e^{-a_1 z}, \theta_{01} = \theta_{10} = \theta_{11} = 0; C_{00} = e^{-a_2 z}, C_{01} = C_{10} = C_{11} = 0 \tag{2.52}$$

Hence the velocity, temperature and concentration of the flow field are

$$q = a_1 e^{-m_1 z} + a_2 e^{-m_2 z} + a_3 e^{-m_3 z} + a_4 e^{-m_4 z} e^{i\omega t} + Rc \left( a_5 e^{-m_1 z} + a_6 e^{-m_2 z} + a_7 e^{-m_3 z} + a_8 e^{-m_4 z} e^{i\omega t} \right) \theta = e^{-a_1 z}, C = e^{-a_2 z} \tag{2.53}$$

The skin friction at the plate is given by

$$\tau = \frac{\tau_{xy}}{U_0^2} = \frac{\partial u}{\partial z} - Rc \left( \frac{\partial^2 q}{\partial t \partial z} + w_0 \frac{\partial^2 q}{\partial z^2} \right) \Bigg|_{z=0},$$

$$\text{where, } \tau_{xy} = \frac{\partial u}{\partial z} - \frac{K_0}{\rho} \left( \frac{\partial^2 q}{\partial t \partial z} + w_0 \frac{\partial^2 q}{\partial z^2} \right)$$

$$\tau = -a_1 m_1 - a_2 m_2 - a_3 m_3 + a_1 m_1^2 + a_2 m_2^2 + a_3 m_3^2 - (a_4 m_4 (1 + i\omega) - a_4 m_4^2) e^{i\omega t} Rc \left( (-a_5 m_1 - a_6 m_2 - a_7 m_3 + a_5 m_1^2 + a_6 m_2^2 + a_7 m_3^2) - (a_8 m_4 (1 + i\omega) - a_8 m_4^2) e^{i\omega t} \right)$$

The rate of heat transfer, that is, the heat flux at the plate in terms of the Nusselt number, is given by

$$Nu = - \frac{\partial \theta}{\partial z} \Bigg|_{z=0} = m_1 \tag{2.54}$$

The rate of mass transfer at the plate in terms of the Sherwood number is given by

$$Sh = - \frac{\partial C}{\partial z} \Bigg|_{z=0} = m_2 \tag{2.55}$$



### 3. RESULTS AND DISCUSSION

The effects of the non-dimensional parameters on the governing flow such as Hartmann number  $M$ , the porosity parameter  $K$ , Prandtl number  $Pr$ ; the elastic parameter  $Rc$ , the chemical reaction parameter  $Kc$ , the heat source parameter  $S$ ; thermal Grashof number  $Gr$ , the mass Grashof number  $Gm$ , the Schmidt number  $Sc$ ; the suction parameter  $w_0$ , rarefaction parameter  $R$  and the frequency of oscillation  $\omega$  on the velocity field have been studied analytically and presented with the help of time series profiles (2-7). The effects of the governing flow parameters on the temperature and concentration distributions have been presented in Figures (8-9) respectively. Further, the effects of the flow parameters on the skin friction, Nusselt number, and rate of mass transfer have been discussed with the help of Tables (1-3). For numerical computation, the values of  $Gr$  are taken positive. For computational purpose we are fixing the values  $M=2$ ,  $K=1$ ,  $Pr=0.71$ ,  $Rc=0.5$ ,  $Kc=1$ ,  $S=1$ ,  $Gr=5$ ,  $Gm=10$ ,  $Sc=0.22$ ,  $w_0=2$ ,  $R=2$ ,  $m=1$ ,  $\omega = \pi / 4$  and  $t = 0.1$ . From all the figures of velocity, it is seen that the velocity profiles decrease with increase in  $z$ . we observed that the velocity components  $u$  and  $v$  oscillate with time throughout the fluid region.

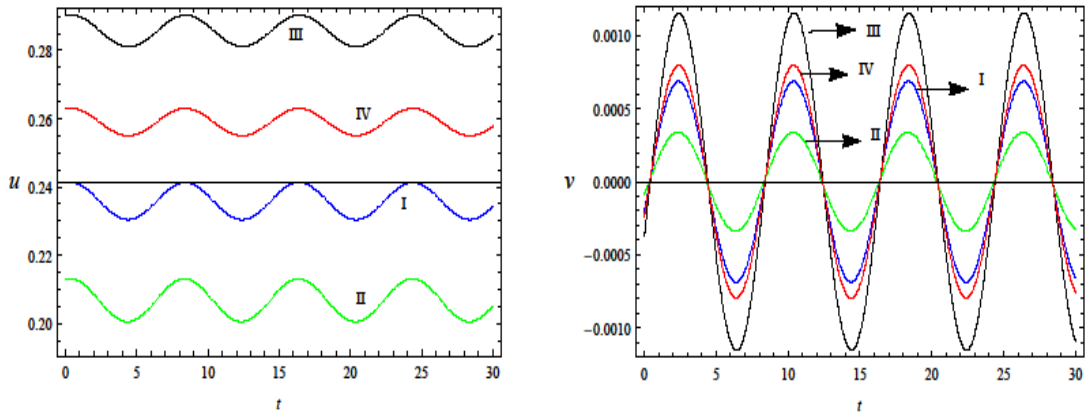
Figure 2 showed the effects of Hartmann number  $M$ , the Hall parameter  $m$ , the permeability parameter  $K$  on velocity profile. It is observed that the increase in the Hall parameter as well as permeability parameter increases both the velocity components of the flow field, whereas increase in Hartmann number decreases it. It is observed that the magnitude of the velocity components  $u$  and  $v$  decrease with increase in elastic parameter, chemical reaction parameter and heat source parameter. For  $Rc=S=Kc=0$ , the present work agrees with the work of Singh and Gupta<sup>3</sup>.

Figure 4 depicts the effect of the thermal Grashof number  $Gr$ , the mass Grashof number  $Gm$ , and the Schmidt number  $Sc$ , on the velocity components  $u$  and  $v$ . It is observed that, the velocity components  $u$  and  $v$  enhance with increasing thermal Grashof number  $Gr$  or mass Grashof number  $Gm$ , and reduces with increasing the Schmidt number  $Sc$  throughout the fluid region.

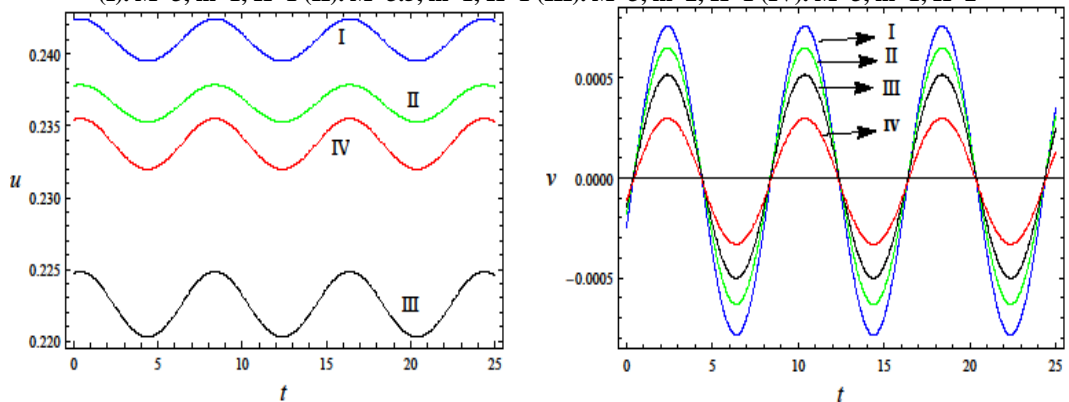
Figure 5 depicts the effect of suction parameter ( $w_0$ ), rarefaction parameter ( $R$ ) and Prandtl number  $Pr$  on velocity components  $u$  and  $v$ . It is noticed that the magnitude of the velocity components  $u$  and  $v$  decreases with increasing the suction parameter  $w_0$  throughout the flow field, whereas the velocity component  $u$  increases and the velocity component  $v$  experiences retardation with increasing rarefaction parameter  $R$  and Prandtl number  $Pr$ . As  $Pr$  increases, the kinematic viscosity of the fluid dominates the thermal diffusivity of the fluid which leads to decreasing the resultant velocity of the flow field.

We also noticed that, from the profile (6), the magnitude of the velocity components  $u$  and  $v$  reduce throughout the fluid region with increasing the frequency of oscillation ( $\omega$ ).

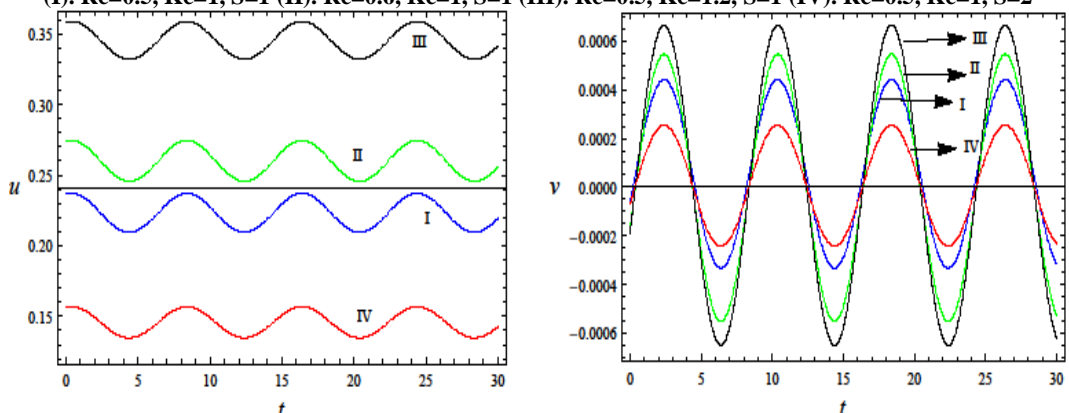
Also from the profile (7), it is seen that the velocity profiles for the components  $u$  and  $v$  decreases with increase in  $z$ .



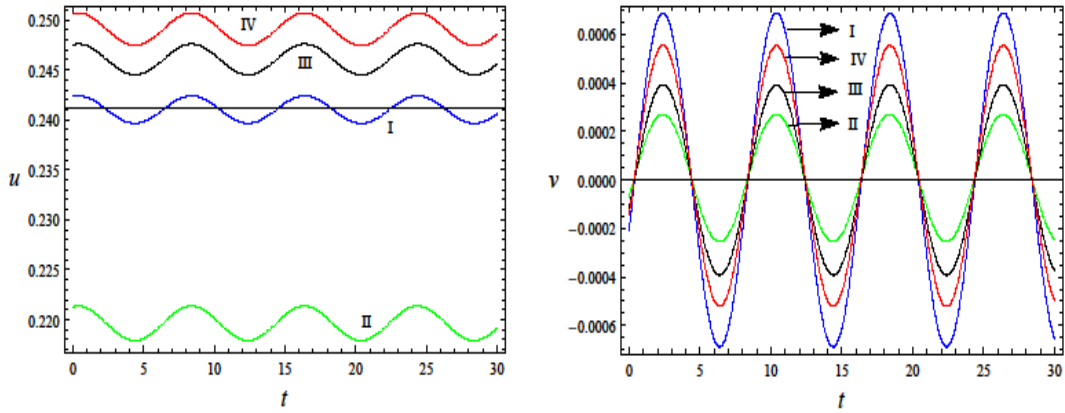
**Fig 2.** The profiles for the velocity components  $u$  &  $v$  against  $M$ ,  $m$  and  $K$   
 (I).  $M=3$ ,  $m=1$ ,  $K=1$  (II).  $M=3.5$ ,  $m=1$ ,  $K=1$  (III).  $M=3$ ,  $m=2$ ,  $K=1$  (IV).  $M=3$ ,  $m=1$ ,  $K=2$



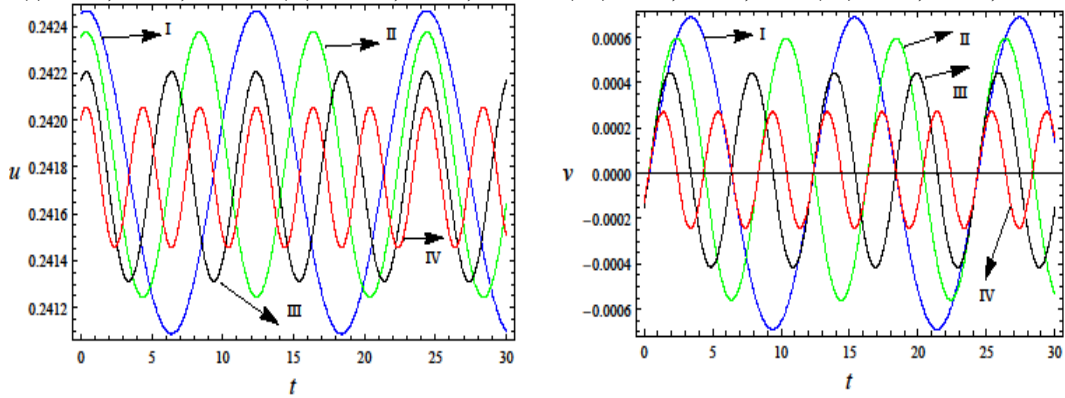
**Fig 3.** The profiles for the velocity components  $u$  &  $v$  against  $Re$ ,  $Kc$  and  $S$   
 (I).  $Re=0.5$ ,  $Kc=1$ ,  $S=1$  (II).  $Re=0.6$ ,  $Kc=1$ ,  $S=1$  (III).  $Re=0.5$ ,  $Kc=1.2$ ,  $S=1$  (IV).  $Re=0.5$ ,  $Kc=1$ ,  $S=2$



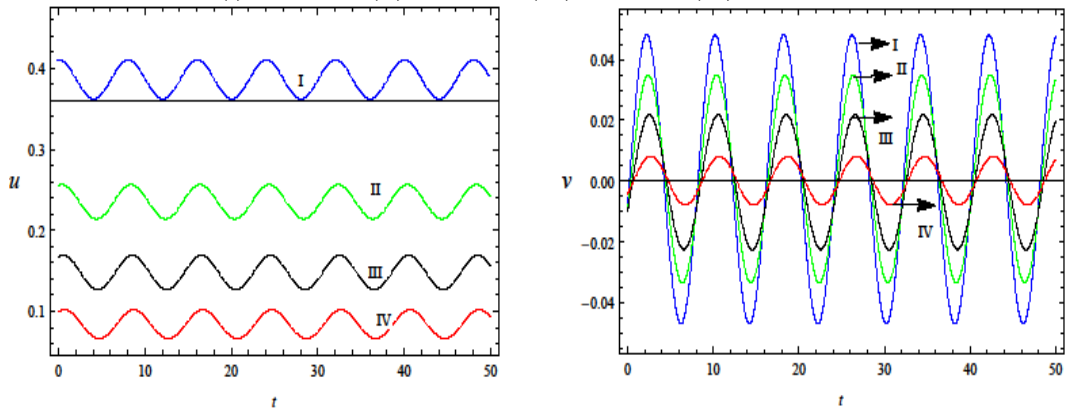
**Fig 4.** The profiles for the velocity components  $u$  &  $v$  against  $Gr$ ,  $Gm$  and  $Sc$   
 (I).  $Gr=5$ ,  $Gm=10$ ,  $Sc=0.22$  (II).  $Gr=10$ ,  $Gm=10$ ,  $Sc=0.22$  (III).  $Gr=5$ ,  $Gm=15$ ,  $Sc=0.22$  (IV).  $Gr=5$ ,  $Gm=10$ ,  $Sc=0.3$



**Fig 5. The profiles for the velocity components  $u$  &  $v$  against  $w_0$  and  $R$**   
 (I).  $w_0=2, R=0.2, Pr=0.71$  (II).  $w_0=2.2, R=0.2, Pr=0.71$  (III).  $w_0=5, R=0.5, Pr=7$  (IV).  $w_0=2, R=0.2, Pr=7$



**Fig 6. The profiles for the velocity components  $u$  &  $v$  against  $\omega$**   
 (I).  $\omega = \pi / 6$  (II).  $\omega = \pi / 4$  (III).  $\omega = \pi / 3$  (IV).  $\omega = \pi / 2$



**Fig 7. The profiles for the velocity components  $u$  &  $v$  against  $z$**   
 (I).  $z = 1$  (II).  $z = 2$  (III).  $z = 3$  (IV).  $z = 4$

Figures 8 showed the effect of the Prandtl number  $Pr$ , heat source parameter  $S$ , and suction parameter ( $w_0$ ) on the temperature of the flow field. We noted that the temperature of the flow field diminishes as the Prandtl number increases. With increasing heat source parameter reduces the temperature of the flow field. It is observed that temperature of the flow field diminishes as the suction parameter increases. Figure 9 depicts the effect of chemical reaction parameter  $Kc$ , the Schmidt number  $Sc$  and suction parameter ( $w_0$ ) on concentration distribution. It is observed that a destructive reaction ( $Kc > 0$ ) reduces the concentration distribution, whereas a generative reaction ( $Kc = 0$ ) enhances it. Also, it is observed that presence of suction parameter diminishes the concentration distribution.

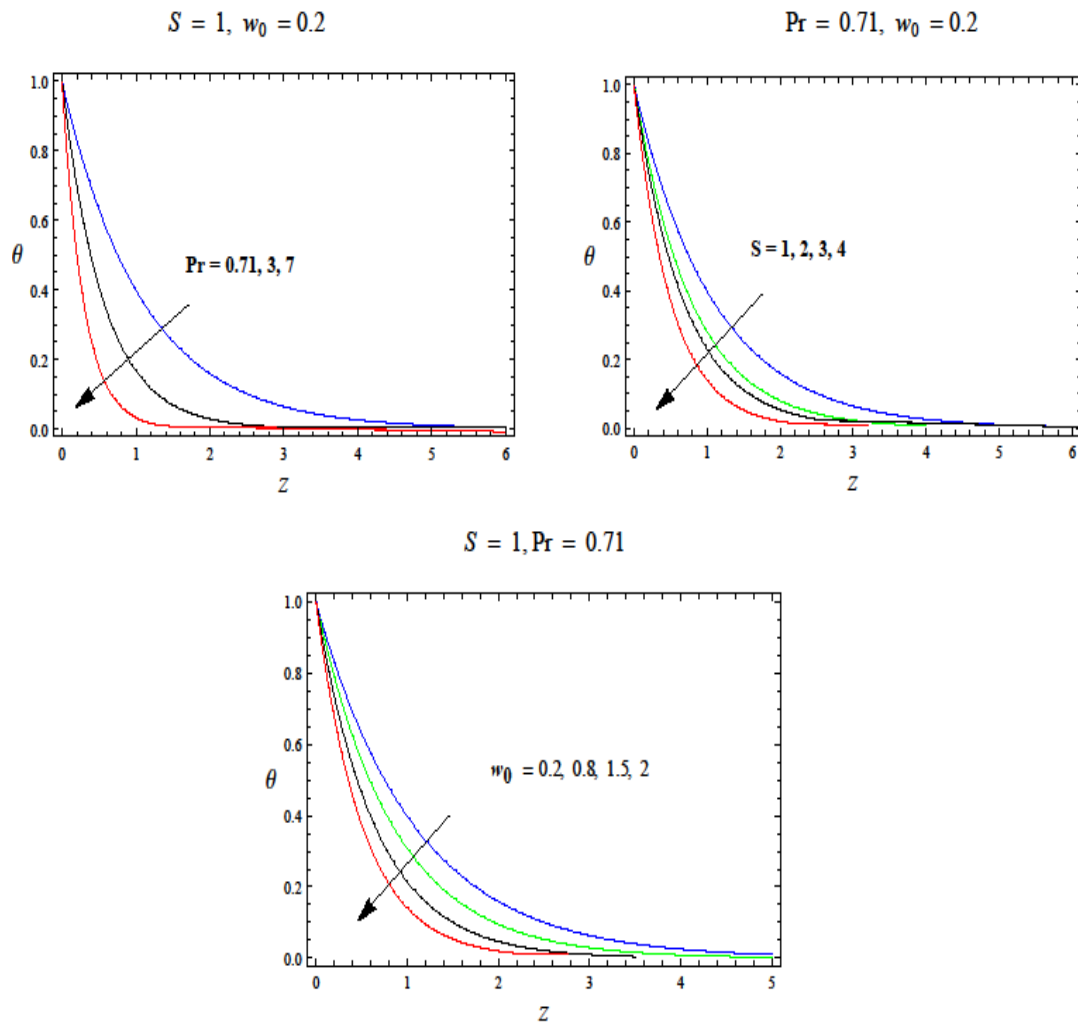


Figure 8. Temperature Profiles for  $Pr, S$  and  $w_0$

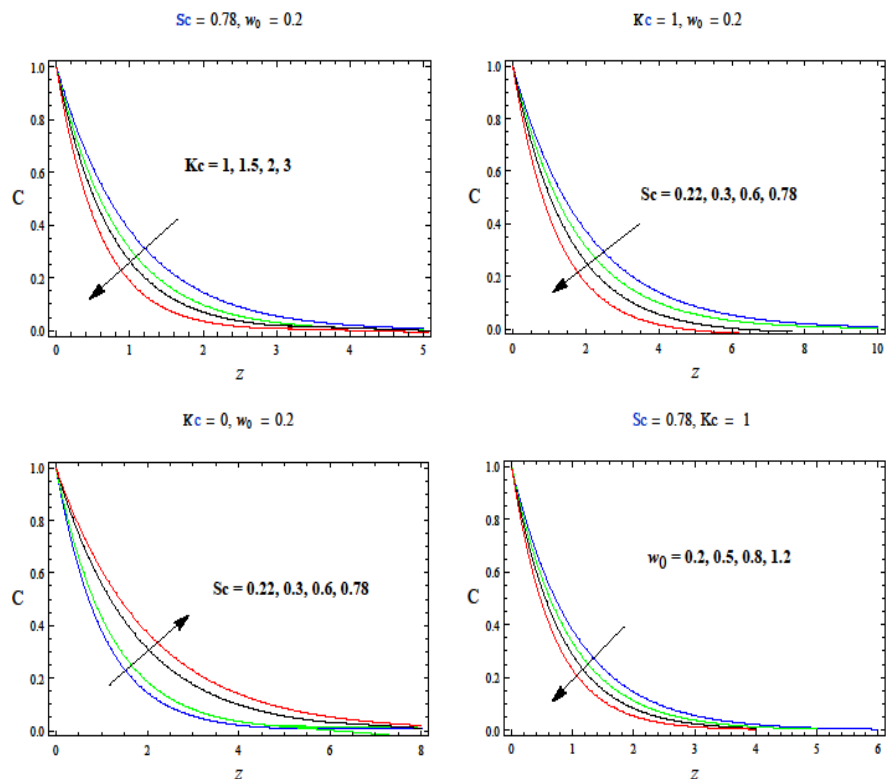


Figure 9. Concentration Profiles for  $Kc$ ,  $Sc$  and  $w_0$

#### 4. CONCLUSIONS

1. The resultant velocity decreases with increase in  $z$ , the velocity components  $u$  and  $v$  oscillates with time throughout the fluid region.
2. The magnitude of the velocity components increase with increasing  $m$  and  $K$ , whereas presence of magnetic field decreases it.
3. The resultant velocity of the flow field decreases suddenly near the plate due to the presence of elastic parameter.
4. The velocity of the flow field decreases due to the increase in the thermal Grashof number.
5. Thermal boundary layer thickness decreases with increasing the Prandtl number  $Pr$ .
6. Heavier diffusing species have a greater retarding effect on the concentration distribution.

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