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Quadrant Fuzzy Number and its Arithmetic Operations

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ABSTRACT

In this research paper a new fuzzy number called quadrant fuzzy number is introduced. Also its arithmetic operations are defined to convert the crisp model to fuzzy model. Signed distance method and graded mean integration representation method for defuzzifying the quadrant fuzzy numbers are discussed. Numerical examples are given to explain the definitions and operations.

Keywords: Quadrant fuzzy number, Triangular Quadrant fuzzy number, Trapezoidal quadrant fuzzy number, Pentagonal quadrant fuzzy number.

1. INTRODUCTION

Fuzzy sets have been introduced by Lotfi. A. Zadeh (1965)[11]. Fuzzy set theory permits the gradual assessment of the membership of elements in a set whose value is defined in the interval [0, 1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer[5] in 1965. By means of Zadeh's extension principle, the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. D. Dubois and H. Prade[4] in 1978 have defined any of the fuzzy numbers as a fuzzy subset of the real line. Many researchers introduced various fuzzy numbers and defined its arithmetic operations. Among the various shapes of fuzzy numbers, Triangular fuzzy number and Trapezoidal fuzzy number are the most commonly used fuzzy numbers. In this research paper a new fuzzy number called quadrant fuzzy number is introduced. Also its arithmetic operations are defined to convert the crisp model to fuzzy model.

This paper is organised as follows: Section 2 defines the new concept triangular quadrant fuzzy number, trapezoidal quadrant fuzzy number, pentagonal quadrant fuzzy number with examples. Section 3 explains the arithmetic operations on quadrant fuzzy numbers with examples. Section 4 introduces defuzzification methods for quadrant fuzzy numbers. Section 5 concludes the research work.

2. QUADRANT FUZZY NUMBERS AND ITS ARITHMETIC OPERATIONS

Definition 2.1 (Triangular quadrant Fuzzy Number – T_{quad}FN)

A fuzzy number $A_{q=}$ (a_{1q}, a_{2q}, a_{3q}) is called triangular quadrant fuzzy number if its membership function is given by

$$«_{A_q}(x) = \begin{cases} 1^{\frac{1}{4}}, & x < a_{1q} \\ \left(\dfrac{a_{2q} - x}{a_{2q} - a_{1q}}\right)^{\frac{1}{4}}, & a_{1q} \le x \le a_{2q} \\ 0, & x = a_{2q} \\ \left(\dfrac{x - a_{2q}}{a_{3q} - a_{2q}}\right)^{\frac{1}{4}}, & a_{2q} \le x \le a_{3q} \\ 1^{\frac{1}{4}}, & x > a_{3q} \end{cases}$$

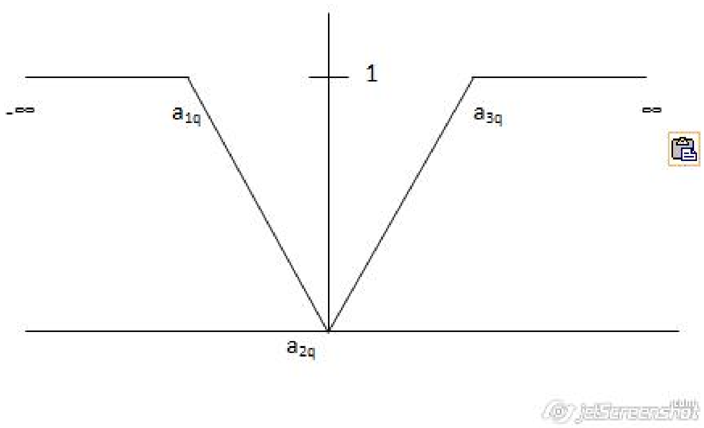

Figure 1 Triangular quadrant fuzzy number

Example 2.1

If A_q = (1,2,3) then its membership function is defined as

$$\mu_{A_q}(x)=\begin{cases}1^{\frac{1}{4}}, & x<1\\ (2-x)^{\frac{1}{4}}, & 1\le x\le 2\\ 0, & x=2\\ (x-2)^{\frac{1}{4}}, & 2\le x\le 3\\ 1^{\frac{1}{4}}, & x>3\end{cases}$$

Definition 2.2.(**α-cut of T_{quad}FN)**

The α-cut of triangular quadrant fuzzy number A_q is the closed interval $A_{q_\alpha}=\left[a_{2q}-\alpha^4\left(a_{2q}-a_{1q}\right),\ a_{2q}+\alpha^4\left(a_{3q}-a_{2q}\right)\right],\quad \alpha\in(0,1]$.

Example 2.2

If A_q = (1,2,3) is a triangular quadrant fuzzy number then the α cut A_{q_α} is (1.94,2.06) if α =0.5.

Definition 2.3 (Trapezoidal quadrant fuzzy numbers- Tr_{quad}FN)

A fuzzy number $A_q=(a_{1q}, a_{2q}, a_{3q}, a_{4q})$ is called trapezoidal quadrant fuzzy number if its membership function is given by

$$\mu_{A_q}(x)=\begin{cases}1^{\frac{1}{4}}, & x<a_{1q}\\ \left(\dfrac{a_{2q}-x}{a_{2q}-a_{1q}}\right)^{\frac{1}{4}}, & a_{1q}\le x\le a_{2q}\\ 0, & a_{2q}\le x\le a_{3q}\\ \left(\dfrac{x-a_{3q}}{a_{4q}-a_{3q}}\right)^{\frac{1}{4}}, & a_{3q}\le x\le a_{4q}\\ 1^{\frac{1}{4}}, & x>a_{4q}\end{cases}$$

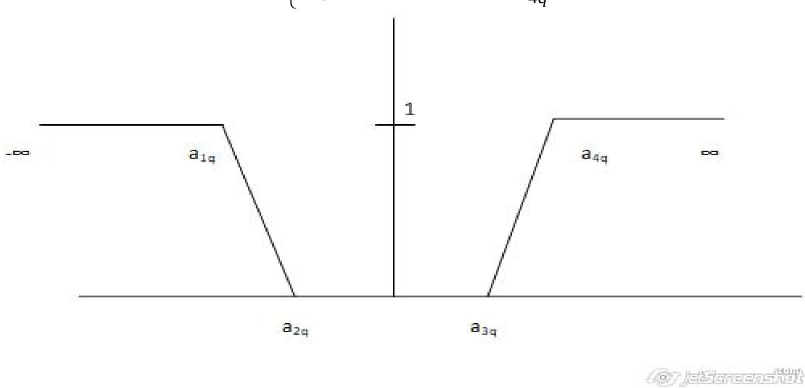

Figure 2 Trapezoidal quadrant fuzzy number

Example 2.3

If A_q =(1,2,3,4) is trapezoidal quadrant fuzzy number then its membership function is defined as

$$«_{A_q}(x) = \begin{cases} 1^{\frac{1}{4}}, & x < 1 \\ (2-x)^{\frac{1}{4}}, & 1 \le x \le 2 \\ 0, & 2 \le x \le 3 \\ (x-3)^{\frac{1}{4}}, & 3 \le x \le 4 \\ 1^{\frac{1}{4}}, & x > 4 \end{cases}$$

Definition 2.4 (α-cut of $\mathbf{Tr_{quad}}$FN)

The α-cut of **$\mathbf{Tr_{quad}}$FN** is defined as

$$A_{q_\sim} = \left[a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right), a_{3q} + \sim^4\left(a_{4q} - a_{3q}\right)\right], \alpha \in [0,1].$$

Example 2.4

If (1,2,3,4) is **$\mathbf{Tr_{quad}}$FN** then α-cut $A_{q_1} = (1,4)$. If α = 0.5 then $A_{q_{0.5}} = (1.94, 3.06)$

Definition 2.5 (Pentagonal quadrant fuzzy number- $\mathbf{P_{quad}}$FN)

A fuzzy number A_q=(a_{1q}, a_{2q}, a_{3q}, a_{4q}, a_{5q}) is called **$\mathbf{P_{quad}}$FN** if its membership function is given by

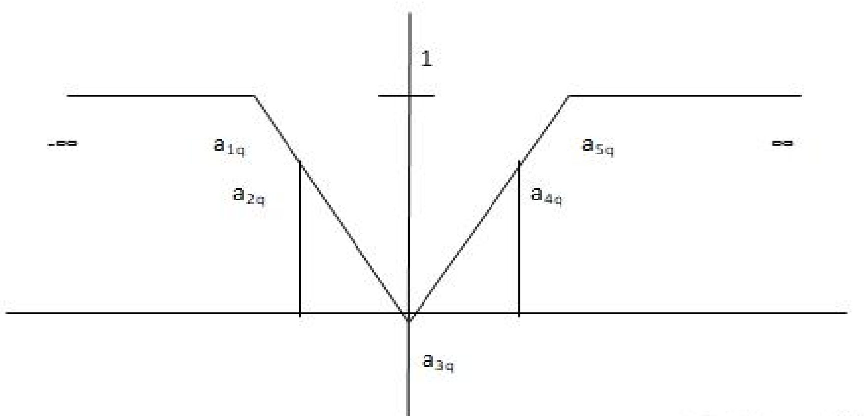

Figure 3 Pentagonal Quadrant Fuzzy Number

$$
« _{A_q}(x) = \begin{cases} (1)^{\frac{1}{4}}, & x < a_{1q} \\ \left(\dfrac{a_{2q} - x}{a_{2q} - a_{1q}}\right)^{\frac{1}{4}}, & a_{1q} \le x \le a_{2q} \\ \left(\dfrac{a_{3q} - x}{a_{3q} - a_{2q}}\right)^{\frac{1}{4}}, & a_{2q} \le x \le a_{3q} \\ 0 & , \quad x = a_{3q} \\ \left(\dfrac{x - a_{3q}}{a_{4q} - a_{3q}}\right)^{\frac{1}{4}}, & a_{3q} \le x \le a_{4q} \\ \left(\dfrac{x - a_{4q}}{a_{5q} - a_{4q}}\right)^{\frac{1}{4}}, & a_{4q} \le x \le a_{5q} \\ 1^{\frac{1}{4}} & , \quad x > a_{5q} \end{cases}
$$

Definition 2.6 (α-cut of $\mathbf{P_{quad}FN}$)

The α-cut of $\mathbf{P_{quad}FN}$ is defined as

$A_{q_\sim}$ =[$a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right),\ a_{4q} + \sim^4\left(a_{5q} - a_{4q}\right)$] for α ϵ [0,0.5]

$A_{q_\sim}$ =[$a_{3q} - \sim^4\left(a_{3q} - a_{2q}\right), a_{3q} + \sim^4\left(a_{4q} - a_{3q}\right)$] for α ϵ [0.5,1]

Example 2.5

Let A = (1,2,3,4,5) then the α cut for α=0 is $A_{q_0} = (2,4)$, for α=0.5 $A_{q_{0.5}} = (1.96,4.06)$ and for α=1 $A_{q_1} = (2,4)$.

3. ARITHMETIC OPERATIONS ON QUADRANT FUZZY NUMBERS

3.1 Addition of two quadrant fuzzy numbers

Definition 3.1

Let A_q =(a_{1q}, a_{2q}, a_{3q}) and B_q = (b_{1q}, b_{2q}, b_{3q}) be two triangular quadrant fuzzy numbers then the addition of two T_{quad}FNs is defined as $A_q \oplus B_q$ = ($a_{1q}+b_{1q}$, $a_{2q}+b_{2q}$, $a_{3q}+b_{3q}$).

Example 3.1: If A_q = (1,2,3) and B_q = (3,5,6) then $A_q \oplus B_q$ =(4,7,9)

Definition 3.2

Let $A_q = (a_{1q}, a_{2q}, a_{3q}, a_{4q})$ and $B_q = (b_{1q}, b_{2q}, b_{3q}, b_{4q})$ be two trapezoidal quadrant fuzzy numbers then the addition of two Tr_{quad}FNs is defined as $A_q \oplus B_q = (a_{1q}+b_{1q}, a_{2q}+b_{2q}, a_{3q}+b_{3q}, a_{4q}+b_{4q})$.

Example 3.2: If A_q = (1,2,3,4) and B_q = (3,5,6,8) then $A_q \oplus B_q$ =(4,7,9,12).

Definition 3.3

Let $A_q = (a_{1q}, a_{2q}, a_{3q}, a_{4q}, a_{5q})$ and $B_q = (b_{1q}, b_{2q}, b_{3q}, b_{4q}, b_{5q})$ be two pentagonal quadrant fuzzy numbers then the addition of two P_{quad}FNs is defined as $A_q \oplus B_q = (a_{1q}+b_{1q}, a_{2q}+b_{2q}, a_{3q}+b_{3q}, a_{4q}+b_{4q}, a_{5q}+b_{5q})$.

Example 3.3: If A_q = (1,2,3,4,5) and B_q = (3,5,6,8,10) then $A_q \oplus B_q$ =(4,7,9,12,15).

3.2 Difference of two Quadrant Fuzzy numbers

Definition 3.4

Let $A_q = (a_{1q}, a_{2q}, a_{3q})$ and $B_q = (b_{1q}, b_{2q}, b_{3q})$ be two triangular quadrant fuzzy numbers then the difference of two T_{quad}FNs is defined as $A_q \ominus B_q = (a_{1q}-b_{3q}, a_{2q}-b_{2q}, a_{3q}-b_{1q})$.

Example3.4: If A_q = (1,2,3) and B_q = (3,5,6) then $A_q \ominus B_q$ =(-5,-3,0)

Definition 3.5

Let $A_q = (a_{1q}, a_{2q}, a_{3q}, a_{4q})$ and $B_q = (b_{1q}, b_{2q}, b_{3q}, b_{4q})$ be two trapezoidal quadrant fuzzy numbers then the difference of two Tr_{quad}FNs is defined as $A_q \ominus B_q = (a_{1q}-b_{4q}, a_{2q}-b_{3q}, a_{3q}-b_{2q}, a_{4q}-b_{1q})$.

Example 3.5: If A_q = (1,2,3,4) and B_q = (3,5,6,8) then $A_q \ominus B_q$ =(-7,-4,-2,1).

Definition 3.6

Let $A_q = (a_{1q}, a_{2q}, a_{3q}, a_{4q}, a_{5q})$ and $B_q = (b_{1q}, b_{2q}, b_{3q}, b_{4q}, b_{5q})$ be two Pentagonal quadrant fuzzy numbers then the difference of two P_{quad}FNs is defined as $A_q \ominus B_q = (a_{1q}-b_{5q}, a_{2q}-b_{4q}, a_{3q}-b_{3q}, a_{4q}-b_{2q}, a_{5q}-b_{1q})$.

Example 3.6: If A_q = (1,2,3,4,5) and B_q = (3,5,6,8,10) then $A_q \ominus B_q$ =(-9,-6,-3,-3,-2).

3.3 Multiplication of two quadrant fuzzy numbers

Definition 3.7

Let $A_q = (a_{1q}, a_{2q}, a_{3q})$ and $B_q = (b_{1q}, b_{2q}, b_{3q})$ be two triangular quadrant fuzzy numbers then the product of two T_{quad}FNs is defined as $A_q \otimes B_q = (a_{1q}.b_{1q}, a_{2q}.b_{2q}, a_{3q}.b_{3q})$.

Example3.7: If A_q = (1,2,3) and B_q = (3,5,6) then $A_q \otimes B_q$ =(3,10,18)

Definition 3.8

Let $A_q = (a_{1q}, a_{2q}, a_{3q}, a_{4q})$ and $B_q = (b_{1q}, b_{2q}, b_{3q}, b_{4q})$ be two trapezoidal quadrant fuzzy numbers then the product of two Tr_{quad}FNs is defined as $A_q \otimes B_q = (a_{1q}.b_{1q}, a_{2q}.b_{2q}, a_{3q}.b_{3q}, a_{4q}.b_{4q})$.

Example 3.8: If A_q = (1,2,3,4) and B_q = (3,5,6,8) then $A_q \otimes B_q$ =(3,10,18,32).

Definition 3.9

Let A_q =(a_{1q}, a_{2q}, a_{3q}, a_{4q}, a_{5q}) and B_q = (b_{1q}, b_{2q}, b_{3q}, b_{4q}, b_{5q}) be two pentagonal quadrant fuzzy numbers then the product of two P_{quad}FNs is defined as $A_q \otimes B_q$ = ($a_{1q}.b_{1q}$, $a_{2q}.b_{2q}$, $a_{3q}.b_{3q}$, $a_{4q}.b_{4q}$, $a_{5q}.b_{5q}$).

Example 3.9: If A_q = (1,2,3,4,5) and B_q = (3,5,6,8,10) then $A_q \otimes B_q$ =(3,10,18,32,50).

3.4 Division of two Quadrant fuzzy numbers

Definition 3.10

Let A_q =(a_{1q}, a_{2q}, a_{3q}) and B_q = (b_{1q}, b_{2q}, b_{3q}) be two triangular quadrant fuzzy numbers then the division of two T_{quad}FNs is defined as $A_q \oslash B_q$ = (a_{1q}/b_{3q} , a_{2q}/b_{2q} , a_{3q}/b_{1q}).

Example 3.10: If A_q = (1,2,3) and B_q = (3,5,6) then $A_q \oslash B_q$ =(1/6, 2/5, 3/3)

Definition 3.11

Let A_q =(a_{1q} ,a_{2q}, a_{3q}, a_{4q}) and B_q = (b_{1q} , b_{2q}, b_{3q}, b_{4q}) be two trapezoidal quadrant fuzzy numbers then the division of two Tr_{quad}FNs is defined as $A_q \oslash B_q$ = (a_{1q}/b_{4q} , a_{2q}/b_{3q} , a_{3q}/b_{2q} , a_{4q}/b_{1q}).

Example 3.11: If A_q = (1,2,3,4) and B_q = (3,5,6,8) then $A_q \oslash B_q$ =(1/8, 2/6, 3/5, 4/3).

Definition 3.12

Let A_q =(a_{1q}, a_{2q}, a_{3q}, a_{4q}, a_{5q}) and B_q = (b_{1q}, b_{2q}, b_{3q}, b_{4q}, b_{5q}) be two pentagonal quadrant fuzzy numbers then the division of two P_{quad}FNs is defined as $A_q \oslash B_q$ = (a_{1q}/b_{5q} , a_{2q}/b_{4q} , a_{3q}/b_{3q} , a_{4q}/b_{2q}, a_{5q}/b_{1q}).

Example 3.12: If A_q = (1,2,3,4,5) and B_q =(3,5,6,8,10) then $A_q \oslash B_q$ =(1/10, 2/8, 3/6, 4/5,5/3).

4. DEFUZZIFICATION METHODS

Defuzzification is the process of producing a quantifiable result in Crisp value, given fuzzy sets and corresponding membership degrees. It is the process that maps a fuzzy set to a crisp set. There are many defuzzification methods available. In this research paper signed distance method and graded mean integration methods are determined.

4.1 Signed Distance Method

a) The formula for defuzzifying triangular quadrant fuzzy number can be derived from

$d_{FS}(\tilde{A}_q) = \frac{1}{2}\int_0^1 (P_L(\sim) + P_R(\sim))\, d\sim$ where P_L (α) = $a_{2q} - \sim^4 \left(a_{2q} - a_{1q}\right)$ and

P_R(α)= $a_{2q} + \sim^4 \left(a_{3q} - a_{2q}\right)$.

$$d_F(\tilde{A}_q) = \frac{1}{2}\int_0^1 (a_{2q} - \sim^4 (a_{2q} - a_{1q}) + a_{2q} + \sim^4 (a_{3q} - a_{2q}))\; d\sim$$

$$d_F(\tilde{A}_q) = \frac{a_{1q} - 8a_{2q} + a_{3q}}{10} \tag{1}$$

b) The formula for defuzzifying trapezoidal quadrant fuzzy number can be derived from

$d_{FS}(\tilde{A}_q) = \frac{1}{2}\int_0^1 (P_L(\sim) + P_R(\sim))\, d\sim$ where $P_L(\alpha) = a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right)$ and $P_R(\alpha) = a_{3q} + \sim^4\left(a_{4q} - a_{3q}\right)$.

$$d_F(\tilde{A}_q) = \frac{1}{2}\int_0^1 (a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right) + a_{3q} + \sim^4\left(a_{4q} - a_{3q}\right))\; d\sim$$

$$d_F(\tilde{A}_q) = \frac{a_{1q} + 4a_{2q} + 4a_{3q} + a_{4q}}{10} \tag{2}$$

c) The formula for defuzzifying pentagonal quadrant fuzzy number can be derived from

$d_{FS}(\tilde{A}_q) = \frac{1}{2}\int_0^1 (P_L(\sim) + P_R(\sim) + Q_L(\sim) + Q_R(\sim))\, d\sim$ where $P_L(\alpha) = a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right)$, $P_R(\alpha) = a_{4q} + \sim^4\left(a_{5q} - a_{4q}\right)$, $Q_L(\alpha) = a_{3q} - \sim^4\left(a_{3q} - a_{2q}\right)$ and $Q_R(\alpha) = a_{3q} + \sim^4\left(a_{4q} - a_{3q}\right)$.

$$d_F(\tilde{A}_q) = \frac{1}{2}\int_0^1 \begin{pmatrix} a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right) + a_{4q} + \sim^4\left(a_{5q} - a_{4q}\right) + \\ a_{3q} - \sim^4\left(a_{3q} - a_{2q}\right) + a_{3q} + \sim^4\left(a_{4q} - a_{3q}\right) \end{pmatrix} d\sim$$

$$d_F(\tilde{A}_q) = \frac{a_{1q} + 5a_{2q} + 8a_{3q} + 5a_{4q} + a_5 q}{10} \tag{3}$$

4.2 Graded Mean Integration Method

a) The graded mean integration representation method for defuzzifying T_{quad}FN is

$d_{FG}(\tilde{A}_q) = \frac{1}{2}\int_0^1 (P_L(\sim) + P_R(\sim)) \sim\; d\sim$ where $P_L(\alpha) = a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right)$ and $P_R(\alpha) = a_{2q} + \sim^4\left(a_{3q} - a_{2q}\right)$.

$$d_{FG}(\tilde{A}_q) = \int_0^1 \left(a_{2q} - \sim^4\left(a_{2q} - a_{1q}\right) + a_{2q} + \sim^4\left(a_{3q} - a_{2q}\right)\right) \sim\; d\sim$$

$$d_{FG}(\tilde{A}_q) = \frac{a_{1q} + 6a_{2q} + a_{3q}}{12} \tag{4}$$

b) The graded mean integration representation method for defuzzifying $Tr_{quad}FN$ is

$d_{FG}(\tilde{A}_q) = \frac{1}{2}\int_0^1 (P_L(\sim) + P_R(\sim))\sim\ d\sim$ where $P_L(\alpha) = a_{2q} - \sim^4(a_{2q} - a_{1q})$ and

$P_R(\alpha) = a_{3q} + \sim^4(a_{4q} - a_{3q})$.

$$d_{FG}(\tilde{A}_q) = \int_0^1 \left(a_{2q} - \sim^4(a_{2q} - a_{1q}) + a_{3q} + \sim^4(a_{4q} - a_{3q})\right)\sim d\sim$$

$$d_{FG}(\tilde{A}_q) = \frac{a_{1q} + 2a_{2q} + 2a_{3q} + a_{4q}}{12} \qquad (5)$$

c) The graded mean integration representation method for defuzzifying $P_{quad}FN$ is

$d_{FG}(\tilde{A}_q) = \frac{1}{2}\int_0^1 (P_L(\sim) + P_R(\sim) + Q_L(\sim) + Q_R(\sim))\,d\sim$ where

$P_L(\alpha) = a_{2q} - \sim^4(a_{2q} - a_{1q})$, $P_R(\alpha) = a_{4q} + \sim^4(a_{5q} - a_{4q})$

$Q_L(\alpha) = a_{3q} - \sim^4(a_{3q} - a_{2q})$ and $Q_R(\alpha) = a_{3q} + \sim^4(a_{4q} - a_{3q})$.

$$d_F(\tilde{A}_q) = \frac{1}{2}\int_0^1 \begin{pmatrix} a_{2q} - \sim^4(a_{2q} - a_{1q}) + a_{4q} + \sim^4(a_{5q} - a_{4q}) + \\ a_{3q} - \sim^4(a_{3q} - a_{2q}) + a_{3q} + \sim^4(a_{4q} - a_{3q}) \end{pmatrix} \sim d\sim$$

$$d_{FG}(\tilde{A}_q) = \frac{a_{1q} + 3a_{2q} + 4a_{3q} + 3a_{4q} + a_{5q}}{12} \qquad (6)$$

5. CONCLUSION

In this research paper, triangular, trapezoidal and pentagonal quadrant fuzzy numbers and its α- cuts are defined. The arithmetic operations of the above mentioned quadrant fuzzy numbers are also discussed. The defuzzification methods such as signed distance method and graded mean integration method for the quadrant fuzzy numbers are formulated in this research work.

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