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Computing First Zagreb and Forgotten Indices of Certain Dominating Transformation Graphs of Kragujevac Trees

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ABSTRACT

In the mathematical literature several transformation graphs have been considered and constructed. Let $\mathcal{G}$ denote the set of simple undirected graphs. Various important results in graph theory have been obtained by considering some functions $\mathcal{F}: \mathcal{G} \to \mathcal{G}$ or $\mathcal{F}_s: \mathcal{G}_1 \times \ldots \times \mathcal{G}_s \to \mathcal{G}$ called transformations or operations (here each $\mathcal{G}_i = \mathcal{G}$) and by establishing how these operations affect certain properties or parameters of graphs. The same concept of transformation has been applied to dominating sets and defined a variety of *dominating transformation(d-transformation)* graphs. In this paper we compute the first Zagreb index, coindex and forgotten index of certain d-transformation graphs.

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Keywords: degree; Zagreb indices; Zagreb coindices; forgotten index; d-transformation graphs.

1. INTRODUCTION

The graphs in this paper are simple and undirected. The terminology not defined here can be found in[14]. Let $G = (V, E)$ be such a graph. The number of vertices of G we denote by n and the number of edges we denote by m, thus $|V(G)| = n$ and $|E(G)| = m$. By the *open neighborhood* of a vertex v of G we mean the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The *degree of a vertex* v, denoted by $d_G(v)$, is the cardinality of its open neighborhood. Let S be a finite set and let $F = \{S_1, S_2, ..., S_n\}$ be a partition of S. Then the *intersection graph*

$\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices S_i and S_j are adjacent if and only if $S_i \cap S_j \neq \phi$.

A graph invariant is any function on a graph that does not depend on a labeling of its vertices. Such quantities are also called topological indices. Hundreds of different invariants have been employed to date (with unequal success)in QSAR/QSPR studies. Among more useful of them appear two that are known under various names, but mostly as Zagreb indices. Due to their chemical relevance they have been subject to numerous papers in chemical literature[11,30]. There are two invariants called the first Zagreb index and second Zagreb index[11] defined as respectively.

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2 \quad and \quad M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v),$$

In fact, one can rewrite the first Zagreb index as

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

In view of above equations the first and second Zagreb coindices were recently introduced and defined as respectively[1].

$$\overline{M}_1(G) = \sum_{uv \notin E(G)} [d_G(u) + d_G(v)] \quad and \quad \overline{M}_2(G) = \sum_{uv \notin E(G)} d_G(u) d_G(v),$$

Also, another topological index namely forgotten index is[7] defined as the sum of cubes of vertex degrees and is given by

$$F(G) = \sum_{u \in V(G)} d_G(u)^3$$

In fact, one can rewrite the forgotten index as

$$F(G) = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2].$$

Readers interested in more information on computing topological indices of graph operations can be referred to[1, 2, 3, 4, 6, 8, 9, 12, 17, 18, 20, 21, 22, 23, 29, 31].

2. KRAGUJEVAC TREES

A connected acyclic graph is called a *tree.* A *rooted tree* is a tree in which one particular vertex is distinguished; this vertex is referred to as the root. The vertex of degree one is a *pendant vertex.* The vertex adjacent to pendant vertex is called *support vertex.* The formal definition of a Kragujevac tree was introduced in[10].

Definition 1.[10] *Let P_3 be the 3-vertex tree,rooted at one of its terminal vertices.For $k = 2,3,\ldots$, construct the rooted tree B_k by identifying the roots of k copies of P_3. The vertex obtained by identifying the roots P_3-trees is the root of B_k.*

Examples illustrating the structure of the rooted tree B_k depicted in Figure 1.

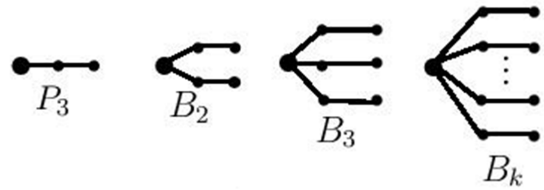


Figure 1.

Definition 2.[10] *Let* $d \geq 2$ *be an integer. Let* $\beta_1, \beta_2, \beta_3 ... \beta_d$ *be rooted trees specified in Definition 1, i.e.,* $\beta_1, \beta_2, \beta_3 ..., \beta_d \in \{B_2, B_3, ...\}$. *A Kragujevac tree T is a tree possessing a vertex of degree d, adjacent to the roots of* $\beta_1, \beta_2, \beta_3 ..., \beta_d$ *. This vertex is said to be the central vertex of T, where d is the degree of T. The subgraphs* $\beta_1, \beta_2, \beta_3 ..., \beta_d$ *are the branches of T. Recall that some (or all) branches of T may be mutually isomorphic.*

A typical Kragujevac tree of degree d=5 is depicted in Figure 2.

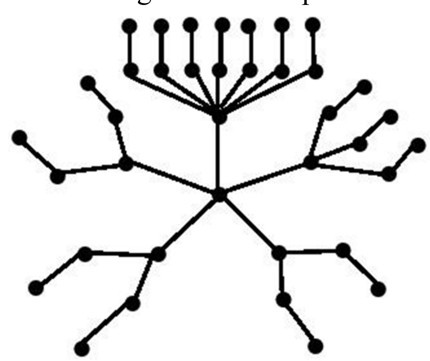


Figure 2

Clearly, the branch B_k has 2k+1 vertices. Therefore, the vertex set and the edge set of the Kragujevac tree T are:

$$V(T) = 1 + \sum_{i=1}^{d} (2k_i + 1) \quad and \quad E(T) = \sum_{i=1}^{d} (2k_i + 1)$$

We denote the vertices of Kragujevac trees as follows:

- pendant vertices by x_i
- support vertices by w_i
- vertices belongs to the set $N(w_i) - \{x_i\}_{i=1}^{k}$ by v_i

- the central vertex by u

Recent information on Kragujevac trees can be found in[5,10,19].

3. d-TRANSFORMATION GRAPHS

The theory of domination has emerged as one of the most studied area in graph theory and its allied branch in mathematics. The wide variety of domination parameters have been defined and studied their applications in various fields[15]. A subset $D \subseteq V(G)$ is a *dominating set* of G if every vertex of $V(G) \setminus D$ has a neighbor in D. The *domination number* of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G. The dominating set D is called a *minimal dominating* set if no proper subset of D is a dominating set. For a comprehensive survey of domination in graphs, see[16].

In the mathematical literature several transformation graphs have been considered and constructed. Let $\mathcal{G}$ denote the set of simple undirected graphs. Various important results in graph theory have been obtained by considering some functions $\mathcal{F}: \mathcal{G} \rightarrow \mathcal{G}$ or $\mathcal{F}_s$: $\mathcal{G}_1 \times \ldots \times \mathcal{G}_s \rightarrow \mathcal{G}$ called transformations or operations (here each $\mathcal{G}_i = \mathcal{G}$) and by establishing how these operations affect certain properties or parameters of graphs. The complement, the k-th power of graph, line graph and the total graph are well known examples of such transformations or operations. The same concept of transformation has been applied to dominating sets and defined a variety of *dominating transformation(d-transformation)* graphs by Prof. V.R.Kulli and his students[24, 25, 26, 27, 28]. In what follows we define the best known representatives of such graphs.

The *minimal dominating graph MD(G)* of a graph G is the intersection graph defined on the family of all minimal dominating sets of vertices in G[24].

The *common minimal dominating graph CD(G)* of a graph G is the graph having same vertex set as G with two vertices are adjacent if and only if there exist a minimal dominating set in G containing them[25].

The *vertex minimal dominating graph* $M_vD(G)$ of a graph $G = (V, E)$ is a graph with $V(M_vD(G)) = V' = V \cup S$, where S is the collection of all minimal dominating sets of G with two vertices $u, v \in V'$ are adjacent if and only if they are adjacent in G or $v = D$ is a minimal dominating set of G containing u[26].

The *dominating graph D(G)* of a graph $G = (V, E)$ is a graph with $V(D(G)) = V \cup S$, where S is the set of all minimal dominating set of G and with two vertices $u, v \in V(D(G))$ are adjacent if $u \in V$ and $v = D$ is a minimal dominating set of G containing u[27].

In Figure 3, a graph G and its d-transformation graphs are shown.

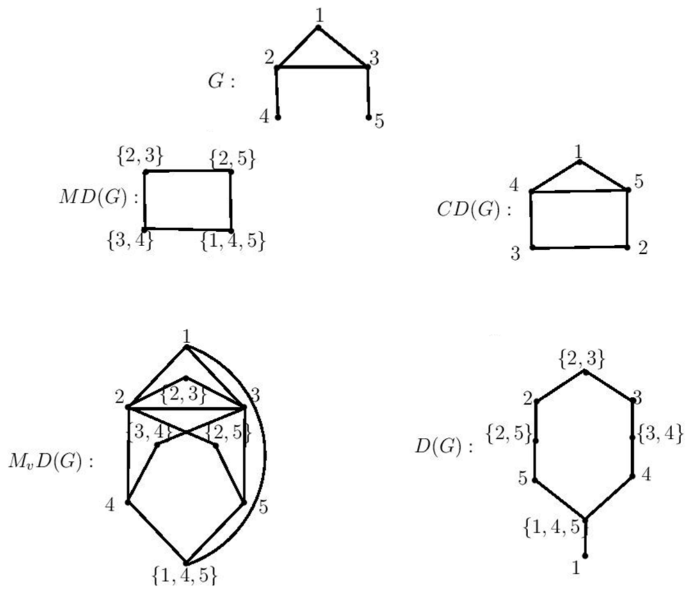
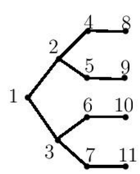


Figure 3 : A graph G and its d − transformation graphs $MD(G)$, $CD(G)$, $M_vD(G)$ and $D(G)$

In Figure 4, Kragujevac tree T and its d-transformation graphs $MD(T), CD(T), M_vD(T)$ and $D(T)$ are given.

$T :$



pendant vertex set $\{x_i\} = \{8, 9, 10, 11\}$

support vertex set $\{w_i\} = \{4, 5, 6, 7\}$

vertices belonging to set $\{N(w_i) - \{x_i\}\} = \{v_i\} = \{2, 3\}$

central vertex $\{u\} = \{1\}$

Dominating sets :

$D_1 = \{1, 4, 5, 6, 7\}$ $D_2 = \{1, 8, 9, 10, 11\}$

$D_3 = \{2, 3, 4, 5, 6, 7\}$ $D_4 = \{2, 3, 8, 9, 10, 11\}$

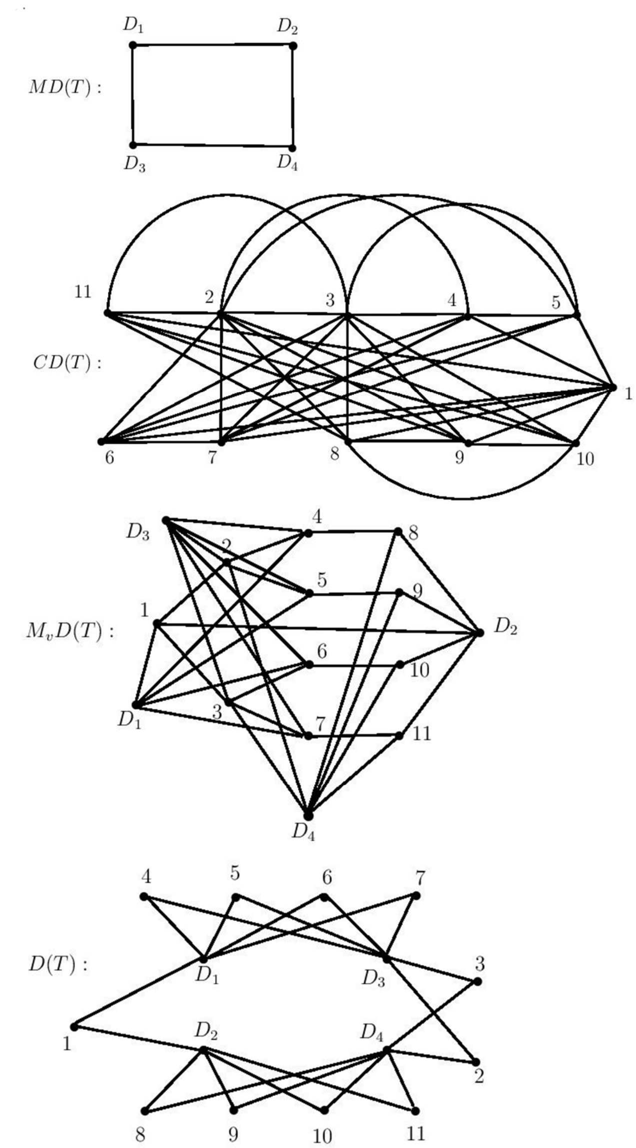


Figure 4 : Kragujevac tree T and its d − transformation graphs $MD(T), CD(T), M_vD(T)$ and $D(T)$

In this paper, expressions for the first Zagreb index , first Zagreb coindex and forgotten index are obtained for Kragujevac tree T of these graphs.

4. Computing first Zagreb index and first Zagreb coindex for MD(T), CD(T), $M_vD(T)$ and D(T) of Kragujevac tree T.

We begin with the following straightforward, previously known auxiliary result.

Lemma 1 *[1] Let G be any nontrivial graph of order n and size m. Then*

$$M_1(G)+\overline{M}_1(G)=2m(n-1).$$

Lemma 2 *The dominating sets of the Kragujevac tree T are:*

$$D_1=\{u,\sum_{i=1}^{k}w_i\}$$

$$D_2=\{u,\sum_{i=1}^{k}x_i\}$$

$$D_3=\{\sum_{i=1}^{d}v_i,\sum_{i=1}^{k}w_i\}$$

$$D_4=\{\sum_{i=1}^{d}v_i,\sum_{i=1}^{k}x_i\}.$$

Theorem 3 *The first Zagreb index of the minimal dominating graph of Kragujevac tree T is* $M_1(MD(T))=16.$

Proof. Let T be any Kragujevac tree T of degree $d\geq 2$. Then by Lemma 2, there exist four minimal dominating sets namely D_1,D_2,D_3 *and* D_4. Hence by the definition of $MD(G)$, the vertex set of the minimal dominating graph of Kragujevac tree T is 4,i.e., $V(MD(T))=4$ and observe that each vertex of $MD(T)$ is intersect with exactly two vertices. Therefore, $d_{MD(T)}(v_i)=2$. Now by employing definition of first Zagreb index we get $M_1(MD(T))=16$.

Before going to prove our next result we need the following lemma.

Lemma 4 *The degree of each vertex in the common minimal dominating graph of Kragujevac tree T is:*

1. $d_{CD(T)}(u)=2k$
2. $d_{CD(T)}(w_i)=(k+d)$
3. $d_{CD(T)}(x_i)=(k+d)$
4. $d_{CD(T)}(v_i)=(2k+d-1)$.

Theorem 5 *The first Zagreb index of the common minimal dominating graph of Kragujevac tree T is*

$$M_1(CD(T)) = 2k[k^2 + 3d^2 + 4kd + 2k - 2d] + d[d^2 - 2d + 1].$$

Proof. By the definition of $CD(G)$ it is clear that $V(CD(T)) = V(T)$. By using the definition of first Zagreb index and Lemma 4, we have

$$\begin{aligned} M_1(CD(T)) &= \sum_{i=1}^{n} d_{CD(T)}(u)^2 \\ &= d_{CD(T)}(u)^2 + [\sum_{i=1}^{k} d_{CD(T)}(w_i)]^2 + [\sum_{i=1}^{k} d_{CD(T)}(x_i)]^2 + [\sum_{i=1}^{d} d_{CD(T)}(v_i)]^2 \\ &= (2k)^2 + k(k+d)^2 + k(k+d)^2 + d(2k+d-1)^2 \end{aligned}$$

$$M_1(CD(T)) = 2k[k^2 + 3d^2 + 4kd + 2k - 2d] + d[d^2 - 2d + 1].$$

By the definition of vertex minimal dominating graph $M_vD(G), V(M_vD(T)) = V \cup S$, where $S = \{D_1, D_2, D_3, D_4\}$. Hence the following lemma gives the information about the degree of each vertex of $M_vD(T)$ of Kragujevac tree T.

Lemma 6 *The degree of each vertex in the vertex minimal dominating graph of Kragujevac tree T is:*

1. $d_{M_vD(T)}(u) = (d+2)$
2. $d_{M_vD(T)}(w_i) = 4$
3. $d_{M_vD(T)}(x_i) = 3$
4. $d_{M_vD(T)}(v_i) = d_T(v_i) + 2$
5. $d_{M_vD(T)}(D_1) = (1+k)$
6. $d_{M_vD(T)}(D_2) = (1+k)$
7. $d_{M_vD(T)}(D_3) = (k+d)$
8. $d_{M_vD(T)}(D_4) = (k+d)$.

Theorem 7 *The first Zagreb index of the vertex minimal dominating graph of Kragujevac tree T is*

$$M_1(M_vD(T)) = 4k^2 + 3d^2 + 29k + 4kd + d[d_T(v_i)^2 + 4\, d_T(v_i) + 8] + 6.$$

Proof. By the definition of $M_vD(G), V(M_vD(T)) = V \cup S$, where $S = \{D_1, D_2, D_3, D_4\}$. By using the definition of first Zagreb index and Lemma 6, we have,

$$M_1(M_vD(T)) = \sum_{i=1}^{n} d_{M_vD(T)}(u)^2$$
$$= [d_{M_vD(T)}(u)]^2 + [d_{M_vD(T)}(w_i)]^2 + [d_{M_vD(T)}(x_i)]^2$$
$$+ [d_{M_vD(T)}(v_i)]^2 + [d_{M_vD(T)}(D_1)]^2 + [d_{M_vD(T)}(D_2)]^2$$
$$+ [d_{M_vD(T)}(D_3)]^2 + [d_{M_vD(T)}(D_4)]^2$$
$$= (d+2)^2 + k(4)^2 + k(3)^2 + d(d_T(v_i)+2)^2 + (1+k)^2$$
$$+ (1+k)^2 + (k+d)^2 + (k+d)^2$$
$$M_1(M_vD(T)) = 4k^2 + 3d^2 + 29k + 4kd + d[d_T(v_i)^2 + 4\, d_T(v_i) + 8] + 6.$$

By the definition of dominating graph $D(T), V(D(T)) = V \cup S$,
where $S = \{D_1, D_2, D_3, D_4\}$. Hence the following lemma gives the information about the degree of each vertex of $D(T)$ of Kragujevac tree T.

Lemma 8 *The degree of each vertex in the dominating graph of Kragujevac tree T is*

1. $d_{D(T)}(u) = 2$
2. $d_{D(T)}(w_i) = 2$
3. $d_{D(T)}(x_i) = 2$
4. $d_{D(T)}(v_i) = 2$
5. $d_{D(T)}(D_1) = (1+k)$
6. $d_{D(T)}(D_2) = (1+k)$
7. $d_{D(T)}(D_3) = (k+d)$
8. $d_{D(T)}(D_4) = (k+d)$.

Theorem 9 *The first Zagreb index of the dominating graph of Kragujevac tree T is*
$$M_1(D(T)) = 4k^2 + 2d^2 + 4k(d+3) + 4d + 6.$$

Proof. By the definition of $D(G), V(D(T)) = V \cup S$, where $S = \{D_1, D_2, D_3, D_4\}$. By using the definition of first Zagreb index and Lemma 8, we have,

$$M_1(D(T)) = \sum_{i=1}^{n} d_{D(T)}(u)^2$$
$$= [d_{D(T)}(u)]^2 + k\,[d_{D(T)}(w_i)]^2 + k\,[d_{D(T)}(x_i)]^2$$

$$+d\,[d_{D(T)}(v_i)]^2+[d_{D(T)}(D_1)]^2+[d_{D(T)}(D_2)]^2$$
$$+[d_{D(T)}(D_3)]^2+[d_{D(T)}(D_4)]^2$$
$$=2^2+k(2)^2+k(2)^2+d(2)^2+(1+k)^2+(1+k)^2+(k+d)^2+(k+d)^2$$
$$M_1(D(T))=4k^2+2d^2+4k(d+3)+4d+6.$$

Applying Lemma 1, from the results of Theorems 3, 5, 7 and 9 we can deduce expressions for the first Zagreb coindex of the minimal dominating graph,common minimal dominating graph, vertex minimal dominating graph and dominating graphs of Kragujevac tree T. These are collected in the following.

Corollary 10 *Let T be a Kragujevac tree of order n and size m. Then*

$$\overline{M}_1(MD(T))=2[\sum_{i=1}^{d}(2k_i+1)]^2-16.$$
$$\overline{M}_1(CD(T))=2[\sum_{i=1}^{d}(2k_i+1)]^2-2k[k^2+3d^2+4kd+2k-2d]+d[d^2-2d+1].$$
$$\overline{M}_1(M_vD(T))=2[\sum_{i=1}^{d}(2k_i+1)]^2-[4k^2+3d^2+29k+4kd$$
$$+d(d_T(v_i)^2+4\,d_T(v_i)+8)+6].$$
$$\overline{M}_1(D(T))=2[\sum_{i=1}^{d}(2k_i+1)]^2-[4k^2+2d^2+4k(d+3)+4d+6].$$

5. Computing forgotten index for MD(T) ,CD(T), M$_v$D (T) and D(T) of Kragujevac tree T.

Using the definition of Forgotten index and Lemma 2,4,6 and 8 we can deduce the expression for the Forgotten index of $MD(T)$, $CD(T)$, $M_vD(T)$ and $D(T)$ of Kragujevac tree T .

Theorem 11 *The forgotten index of the minimal dominating graph of Kragujevac tree T is* $F(MD(T))=32$.

Proof. Let T be any Kragujevac tree T of degree $d\geq 2$. Then by Lemma 2, there exist four minimal dominating sets namely D_1, D_2, D_3 *and* D_4. Hence by the definition of $MD(G)$, the vertex set of the minimal dominating graph of Kragujevac tree T is 4, i.e., $V(MD(T))=4$ and observe that each vertex of $MD(T)$ is intersect with exactly two vertices. Therefore, $d_{MD(T)}(v_i)=2$. Now by employing definition of forgotten index we get $F(MD(T))=32$.

By the definition of common minimal dominating graph $CD(G)$ and Lemma 4, we can deduce forgotten index for $CD(T)$ for Kragujevac tree in the following theorem.

Theorem 12 *The forgotten index of the common minimal dominating graph of Kragujevac tree T is*

$F(CD(T)) = 2k^3[k+7d+4]+d^3[d+8k-3]+6kd[3kd-2k-2d+1]+d[3d-1].$

Proof. By the definition of $CD(G)$ it is clear that $V(CD(T)) = V(T)$. By using the definition of Forgotten index and Lemma 4, we have

$$\begin{aligned}F(CD(T)) &= \sum_{i=1}^{n} d_{CD(T)}(u)^3\\ &= d_{CD(T)}(u)^3 + [\sum_{i=1}^{k} d_{CD(T)}(w_i)]^3 + [\sum_{i=1}^{k} d_{CD(T)}(x_i)]^3 + [\sum_{i=1}^{d} d_{CD(T)}(v_i)]^3\\ &= (2k)^3 + k(k+d)^3 + k(k+d)^3 + d(2k+d-1)^3\end{aligned}$$

$F(CD(T)) = 2k^3[k+7d+4]+d^3[d+8k-3]+6kd[3kd-2k-2d+1]+d[3d-1].$

By the definition of vertex minimal dominating graph $M_vD(G), V(M_vD(T)) = V \cup S$, where $S = \{D_1, D_2, D_3, D_4\}$. By using the definition of forgotten index and Lemma 6 we deduce forgotten index for $M_vD(T))$ of Kragujevac tree T in next theorem.

Theorem 13 *The forgotten index of the vertex minimal dominating graph of Kragujevac tree T is*

$F(M_vD(T)) = d[d_T(v_i)^3 + 6d\ d_T(v_i)^2 + 12d_T(v_i) + 20] + 2k^2[2k+3d+3] + 3d^2[d+2k+2] + 97k + 10.$

Proof. By the definition of forgotten index and Lemma 6, we have

$$\begin{aligned}F(M_vD(T)) &= \sum_{i=1}^{n} d_{M_vD(T)}(u)^3\\ &= [d_{M_vD(T)}(u)]^3 + [d_{M_vD(T)}(w_i)]^3 + [d_{M_vD(T)}(x_i)]^3\\ &\quad + [d_{M_vD(T)}(v_i)]^3 + [d_{M_vD(T)}(D_1)]^3 + [d_{M_vD(T)}(D_2)]^3\\ &\quad + [d_{M_vD(T)}(D_3)]^3 + [d_{M_vD(T)}(D_4)]^3\end{aligned}$$

$= (d+2)^3 + k(4)^3 + k(3)^3 + d(d_T(v_i)+2)^3 + (1+k)^3 + (1+k)^3 + (k+d)^3 + (k+d)^3$

$F(M_vD(T)) = d[d_T(v_i)^3 + 6d\ d_T(v_i)^2 + 12d_T(v_i) + 20] + 2k^2[2k+3d+3] + 3d^2[d+2k+2] + 97k + 10.$

In the following theorem we deduce the forgotten index of dominating graph $D(T)$ of Kragujevac tree T by using Lemma 8 and definition of forgotten index.

Theorem 14 *The forgotten index of the dominating graph of Kragujevac tree T is*

$$F(D(T)) = 2k[2k^2 + 3d^2 + 3kd + 3k + 11] + 2d[d^2 + 4] + 10.$$

Proof. By the definition of forgotten index and Lemma 8, we have

$$\begin{aligned}F(D(T)) &= \sum_{i=1}^{n} d_{D(T)}(u)^3\\ &= [d_{D(T)}(u)]^3 + k\,[d_{D(T)}(w_i)]^3 + k\,[d_{D(T)}(x_i)]^3\end{aligned}$$

$$+d\,[d_{D(T)}(v_i)]^3+[d_{D(T)}(D_1)]^3+[d_{D(T)}(D_2)]^3$$
$$+[d_{D(T)}(D_3)]^3+[d_{D(T)}(D_4)]^3$$
$$=2^3+k(2)^3+k(2)^3+d(2)^3+(1+k)^3+(1+k)^3+(k+d)^3+(k+d)^3$$
$$F(D(T))=2k[2k^2+3d^2+3kd+3k+11]+2d[d^2+4]+10.$$

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