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Model of Rayleigh Wave Propagation in a Microstretch Thermoelastic Half-space with Two-temperatures

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ABSTRACT

In this research article, the effect of two temperatures on the Rayleigh wave propagation in a generalized microstretch thermoelastic solid half-space with two-temperature has been investigated. The Green- Naghdi (GN) theory of thermoelasticity is adopted in the present research. The medium is subjected to stress free, isothermal boundary. After developing a mathematical model, the dispersion curve in the form of polynomial equation is obtained. Phase velocity and attenuation coefficient of Rayleigh wave are computed numerically. The numerically simulated results are depicted graphically.

Keywords: Rayleigh wave, Two-temperatures, Green-Naghdi theory, Microstretch thermoelastic, Phase velocity, Attenuation coefficient.

INTRODUCTION

Eringen[1] developed the theory of thermo-microstretch elastic solids. Microstretch continuum is a model for Bravais lattice with basis on the atomic level and two-phase dipolar solids with a core on the macroscopic level. Composite materials reinforced with chopped elastic fibers, porous media whose pores are filled with gas or inviscid liquid, asphalt or other elastic inclusions and solid-liquid crystals etc. are examples of microstretch solids. Bofill and Quintanilla[2] verified the existence theorem and some uniqueness theorems for the microstretch thermoelastic materials in the context of linear theory of these materials. Singh and Kumar[3] investigated the results concerning reflection and transmission in microstretch thermoelastic materials. Mechanical interactions due to the mechanical forces in microstretch mass diffusive half-space were studied by Kumar and Kumar[4]. The Elastodynamic interactions due to inclined mechanical forces was investigated by Kumar and Kumar[5]. Thermomechanical interactions of ultra-short laser pulse in generalized microstretch thermoelastic solid were investigated by Kumar *et al*.[6]. Marin and Vlase[7] studied the effect of internal state variables in microstretch thermoelasticity. Kumar[8] investigated the solution of a problem in magneto-microstretch thermoelasticity subjected to inclined loads. Kumar *et al*.[9] discussed the pulsed laser heating effect in a dual phase lag mass diffusion thermoelastic medium.

Chen and Gurtin[10] developed a theory for elastic solids in which the equation of heat conduction involved two distinct temperatures names as thermodynamic temperature and conductive temperature. The difference between these two temperatures is directly proportional to the amount of heat supplied. If no heat is supplied, then these two temperatures are identical. This two-temperature model is significantly applicable to find the electron and photon temperature distribution in laser processing of metals. Youssef[11] proved the uniqueness and existence relations in theory of two-temperature generalized thermoelasticity. Youssef and Bassiouny[12] discussed a boundary value problem in piezo-electric thermoelastic half-space using state space approach method. Ezzat and Bary[13] derived solutions of one-dimensional problem in magneto-thermoelasticity with two temperatures. Kumar and Mukhopadhyay[14] discussed the effects of cylindrical cavity in the generalized theory of two temperatures. A theory of thermoelasticity with two distinct temperatures and without energy dissipation was presented by Youssef and Elsibai[15]. Al-Lehaibi and Eman[16] derived the generalized solutions of thermal shock problem of Nano beam resonator in generalized thermoelasticity with two temperatures. Deswal *et al*.[17] presented thermal and mechanical interactions in a micropolar thermoelastic dual phase lag medium with two temperatures. They also studied the effect of gravity on thermal stresses in the considered medium. Sur and Kanoria[18] presented a 3-dimensional deformation problem in thermoelasticity with two temperatures.

This present research deals with study of Rayleigh waves in a microstretch thermoelastic half-space including the two-temperature effect. The normal mode analysis technique is used to obtain the expressions for the thermal stresses and the temperature change.

BASIC EQUATIONS

The basic equations for homogeneous microstretch thermoelastic medium with two temperatures in the absence of body force and body couple is:

Stress equation of motion:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\nabla T=\rho\ddot{\boldsymbol{u}}, \quad (1)$$

Couple stress equation of motion:

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}}, \quad (2)$$

Equation of balance of stress moments:

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.\boldsymbol{u}+\nu_1 T=\frac{\rho j_0}{2}\ddot{\phi}^*, \quad (3)$$

Equation of heat conduction:

$$K\nabla^2\chi+K^*\nabla^2\dot{\chi}=\rho c^*\frac{\partial^2 T}{\partial t^2}+\beta_1 T_0\frac{\partial^2}{\partial t^2}(\nabla.\boldsymbol{u})+\nu_1 T_0\dot{\phi}^*, \quad (4)$$

The constitutive relations are:

$$t_{ij} = \left(\lambda_0\phi^* + \lambda u_{r,r}\right)\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \epsilon_{ijk}\phi_k\right) - \beta_1\delta_{ij}T, \quad (5)$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} + b_0\epsilon_{mji}\phi^*_{,m} \quad , \quad (6)$$

$$\lambda^*_i = \alpha_0\phi^*_{,i} + b_0\epsilon_{ijm}\phi_{j,m} \,, \quad (7)$$

$$T = (1 - \kappa\nabla^2)\chi, \quad (8)$$

Here $\lambda,\ \mu,\ \alpha,\ \beta,\ \gamma,\ K, \lambda_0, \lambda_1, \alpha_0, b_0$, are material constants, ρ is mass density, $\boldsymbol{u} = (u_1, u_2, u_3)$is the displacement vector and $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$is the microrotation vector, $\phi^*$is the scalar microstretch function, T is temperature and T_0 is the reference temperature of the body chosen, Q is the input heat source,j is the microinertia, $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t1}$,$\nu_1 = (3\lambda + 2\mu + K)\alpha_{t2}$, α_{t1}, α_{t2} are coefficients of linear thermal expansion.j_0 is the microinertia for the microelements,t_{ij} are components of stress, m_{ij} are components of couple stress, λ^*_i is the microstress tensor, e_{ij} are components of strain, e_{kk} is the dilatation,δ_{ij} is Kronecker's delta function.

FORMULATION OF THE PROBLEM

We consider a microstretch thermoelastic medium with two temperatures with rectangular Cartesian coordinate system $OX_1X_2X_3$ with x_3-axis pointing vertically downward the medium. The geometry of this problem is illustrated in Fig. 4.

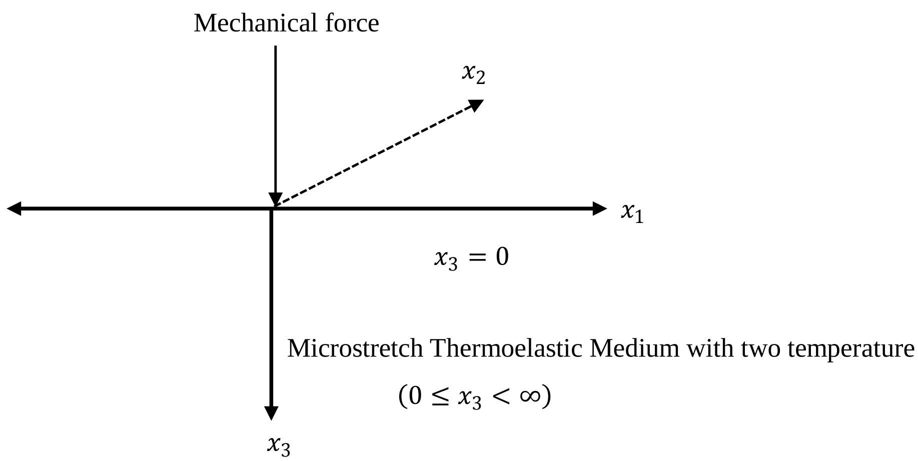

Fig. 1. Geometry of the problem

Here we write the following form of displacement vector and micro-rotational vector for the two-dimensional problem:

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \quad (9)$$

For further consideration it is convenient to introduce in equations (1)-(4) the dimensionless quantities defined by:

$u_i' = \frac{\rho\omega^* c_1}{\beta_1 T_0} u_i$, $x_i' = \frac{\omega^*}{c_1} x_i$, $t' = \omega^* t$, $T' = \frac{T}{T_0}$, $\tau_1' = \omega^* \tau_1$, $\tau_0' = \omega^* \tau_0$,$\gamma_1' = \omega^* \gamma_1$, $t_{ij}' = \frac{1}{\beta_1 T_0} t_{ij}$, $\omega^* = \frac{\rho c^* c_1^2}{K^*}$, $\phi_i' = \frac{\rho c_1^2}{\beta_1 T_0} \phi_i$, $\tau^{1'} = \omega^* \tau^1$, $c_1^2 = \frac{\lambda + 2\mu + k}{\rho}$, $c_2^2 = \frac{\mu + k}{\rho}$, $c_3^2 = \frac{\gamma}{\rho j}$, $c_4^2 = \frac{2\alpha_0}{\rho j_0}$,
$\varepsilon = \frac{\gamma^2 T_0}{\rho^2 c^* c_1}$,$m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij}$, $\phi^{*\prime} = \frac{\rho c_1^2}{\beta_1 T_0} \phi^*$, (10)

By Helmholtz representation of a vector into scaler and vector potentials the displacement components u_1 and u_3 are related to non- dimensional potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{11}$$

Switching the values of $u_1 \& u_3$ from (11) in (1)-(4) and with utility of (9) & (10), we yield after suppressing the primes, we obtain:

$$\nabla^2\phi - \ddot{\phi} + a_1\phi^* - T = 0, \tag{12}$$
$$(\nabla^2 - a_6)\phi^* - a_7\nabla^2\phi + a_8 T = 0, \tag{13}$$
$$(a_9\nabla^2\chi + \nabla^2\dot{\chi}) - [\ddot{T} + a_{10}\nabla^2\ddot{\phi} + a_{11}\ddot{\phi}^*] = 0, \tag{14}$$
$$a_2\nabla^2\psi - \ddot{\psi} + a_3\phi_2 = 0, \tag{15}$$
$$\nabla^2\phi_2 - 2a_4\phi_2 - a_4\nabla^2\psi = a_5\ddot{\phi}_2, \tag{16}$$

a_i are mentioned in the appendix 1.

$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ is Laplace operator.

SOLUTION OF THE PROBLEM

The solution of the considered physical variables can be decomposed in terms of the normal modes as expressed in the subsequent equation:

$$\{\phi, \psi, \chi, \phi_2, \phi^*\}(x_1, x_3, t) = \{\bar{\phi}, \bar{\psi}, \bar{\chi}, \overline{\phi_2}, \overline{\phi^*}\}(x_3) e^{i(kx_1 - \omega t)} \tag{17}$$

Here ω is the angular frequency and k is wave number.

Making use of (17), equations (12) - (16) reduces to the following relations:

$$(D^2 - k^2 - \omega^2)\bar{\phi} + a_1\overline{\phi^*} - \bar{T} = 0, \tag{18}$$
$$(a_2 D^2 - a_2 k^2 - \omega^2)\bar{\psi} + a_3\overline{\phi_2} = 0, \tag{19}$$
$$(D^2 - k^2 - g a_4 - a_5\omega^2)\overline{\phi_2} - a_4 (D^2 - k^2)\bar{\psi} = 0, \tag{20}$$
$$(D^2 - k^2 - a_6)\overline{\phi^*} - a_7 (D^2 - k^2)\bar{\phi} + a_8\bar{T} = 0, \tag{21}$$
$$(a_9 + \omega)(D^2 - k^2)\bar{\chi} - (1 - \kappa D^2 - \kappa k^2)\bar{\chi} - a_{10}(D^2 - k^2)\bar{\phi} + a_{11}\omega\overline{\phi^*} = 0, \tag{22}$$

The above set of (18)-(22) equations after some simplifications yield:

$$[A\boldsymbol{D}^6 + B\boldsymbol{D}^4 + C\boldsymbol{D}^2 + E]\bar{\phi} = 0 \tag{23}$$
$$[\boldsymbol{D}^4 + F\boldsymbol{D}^2 + G]\bar{\psi} = 0 \tag{24}$$

Where $\boldsymbol{D} = \frac{d}{dx_3}$, where

$A = \frac{A_1 + A_2\chi}{k_5 + a_{10}'\chi}$, $A_1 = k_2 - k_4 k_5 - k_5\omega_{11} + a_1 a_7 k_5 + k_6 a_{10}'$, $A_2 = a_8 a_{11}\omega + a_1 a_8 a_{10}' - a_7 a_{11}\omega + a_{10}' k^2 + a_{10}' k_4$, $B = \frac{B_1 + B_2\chi}{k_5 + a_{10}'\chi}$, $B_1 = a_8 a_{11} a_6\omega - k_2 k_4 - \omega_{11} k_2 + \omega_{11} k_4 k_5 +$

$a_7k_2 + a_1a_8k_6a'_{10} - a_1a_7k_5k^2 - k_6(wa_7a_{11} + a'_{10}k^2 + a'_{10}k_4), B_2 = \omega a_7a_{11}k^2 + a'_{10}k^2k_4 - \omega_{11}a_8a_{11}\omega - a_1a_8a'_{10}k^2$

$C = \frac{k_6(\omega a_7a_{11}k^2+a'_{10}k^2k_4)-\omega_{11}(\omega a_6a_8a_{11}-k_2k_4)-a_1(a_7k_2k^2+a_8a'_{10}k_6k^2)}{k_5+a'_{10}\chi}, E = \frac{a_3a_4-a_2a_3-k_7}{a_2}, F = \frac{k_3k_7-a_3a_4k^2}{a_2}, \tau_{11} = (1+\tau_1 s), \xi_1 = \xi^2 + s^2,$

Also, a_i , $i = 1, \ldots.,12$are defined in appendix A.

The solution of the above system of equations (23)-(24) satisfying the radiation conditions that $(\bar{\phi}, \bar{\psi}, \bar{\chi}, \overline{\phi_2}, \overline{\phi^*}) \to 0$ as $x_3 \to \infty$ are given as following:

$$\bar{\phi} = \sum_{i=1}^{3} c_i e^{-m_i x_3}, \tag{25}$$

$$\overline{\phi^*} = \sum_{i=1}^{3} \alpha_{1i} c_i e^{-m_i x_3}, \tag{26}$$

$$\bar{\chi} = \sum_{i=1}^{3} \alpha_{2i} c_i e^{-m_i x_3}, \tag{27}$$

$$(\bar{\psi}, \overline{\phi_2}) = \sum_{i=4}^{5} (1, \delta_i) c_i e^{-m_i x_3}, \tag{28}$$

Here $m_i^2 (i = 1,2,3)$ are the roots of the equation (23) and $m_i^2 (i = 4,5)$ are the roots of characteristic equation (24).

$\alpha_{1i} = -\frac{\Delta_{2i}}{\Delta_{1i}}, \alpha_{2i} = \frac{\Delta_{3i}}{\Delta_{1i}}$, $i = 1,2,3$ & $\delta_i = \frac{a_3}{(a_2 m_i^2 - k_7)}$, $i = 4,5$

Here, $\Delta_{1i}, \Delta_{2i}, \Delta_{3i}$ are defined in Appendix B.

Substituting the values of $(\bar{\phi}, \bar{\psi}, \bar{\chi}, \overline{\phi_2}, \overline{\phi^*})$ from the equations (25)-(28) in the (5)-(7), and using (9)-(11) & (17) and then solving the resulting equations, we obtain:

$$\bar{t}_{33} = \sum_{i=1}^{5} G_{1i} e^{-m_i x_3} e^{i(kx_1-\omega t)}, \tag{29}$$

$$\bar{t}_{31} = \sum_{i=1}^{5} G_{2i} e^{-m_i x_3} e^{i(kx_1-\omega t)}, \tag{30}$$

$$\bar{m}_{32} = \sum_{i=1}^{5} G_{3i} e^{-m_i x_3} e^{i(kx_1-\omega t)}, \tag{31}$$

$$\lambda_3^* = \sum_{i=1}^{5} G_{4i} e^{-m_i x_3} e^{i(kx_1-\omega t)}, \tag{32}$$

$$\bar{T} = \sum_{i=1}^{5} G_{5i} e^{-m_i x_3} e^{i(kx_1-\omega t)}, \tag{33}$$

$G_{mi} = g_{mi} C_i$, $i, m = 1,2, \ldots,5$. G_{rs} , $M_r, (r, s = 1,2, \ldots,5)$, are described in Appendix C.

BOUNDARY CONDITIONS

We consider the mechanical and thermal stress-free boundary at the surface $x_3 = 0$, i.e. vanishing of normal stress, tangential stress, couple stress in addition to thermal stress-free boundaries considered at $x_3 = 0$. Mathematically this can be written as:

$$t_{33} = 0, t_{31} = 0, m_{32} = 0, \lambda_3^* = 0, \frac{\partial T}{\partial x_3} = 0 \tag{34}$$

DERIVATION OF THE SECULAR EQUATIONS

Making use of (6)-(10), (19) in boundary conditions (33), we obtain a system of seven simultaneous homogeneous linear equations:

$$\sum_{j=1}^{7} Q_{ij} C_j = 0 \tag{34}$$

The system of equations (34) has a nontrivial solution if the coefficient determinant of amplitudes, C_j , $j = 1,2, \ldots,7$ vanishes. Mathematically this can be written in the following way:

$$\left|Q_{ij}\right| = 0\,, \qquad i,j = 1,2,\dots,7 \tag{35}$$

where

$$Q_{1j} = \begin{cases} b_1\alpha_{1j} + b_2(m_j^2 - k^2) + b_3 m_j^2 - \beta_1\left(1 - \kappa\left(m_j^2 - k^2\right)\right)\alpha_{2j},\ j = 1,2,3 \\ -\iota b_3 k m_j,\ j = 4,5 \end{cases},$$

$$Q_{2j} = \begin{cases} \iota k b_3 m_j,\ j = 1,2,3 \\ b_6 m_j^2 + b_5 k^2 - b_7\alpha_{3j},\ j = 4,5 \end{cases},$$

$$Q_{3j} = \begin{cases} -\iota b_9 k\alpha_{1j},\ j = 1,2,3 \\ -b_8\alpha_{3j} m_j,\ j = 4,5 \end{cases}, \quad Q_{4j} = \begin{cases} -\alpha_0 b_{10} m_j \alpha_{1j},\ j = 1,2,3 \\ -ikb_0 b_{10}\alpha_{3j},\ j = 4,5 \end{cases},$$

$$Q_{5j} = \begin{cases} \alpha_{2j},\ j = 1,2,3 \\ 0,\ j = 4,5 \end{cases},$$

b_i are defined in Appendix 4.

DISCUSSION ON THE SECULAR EQUATIONS

(a) Microstretch thermoelastic solid:

If we neglect the two-temperature effect in (35), we obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.

(b) Micropolar thermoelastic solid with two temperatures:

If we neglect the microstretch effect in (35), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic medium with two temperatures solid.

(c) Micropolar thermoelastic solid:

If we neglect the two-temperature effect and the stretch effect in the final results, we yield the expressions of thermal stresses for the micropolar thermoelastic medium.

Specific loss:

The specific loss is defined as the ratio of energy ($\Delta\overline{W}$) dissipated in taking specimen through cycle, to elastic energy ($\Delta\overline{W}$) stored in a specimen when the strain is at maximum. The specific loss is the most direct way of defining internal friction for a material (Puri and Cowin[20]). For a sinusoidal surface wave of small amplitude the specific loss $W/\Delta W$ equals 4π times the absolute value of the imaginary part of k to the real part of k.

Appendix A:

$a_1 = \frac{\lambda_0}{\rho c_1^2}$, $a_2 = \frac{\mu+K}{\beta_1 T_0}$, $a_3 = \frac{K}{\rho c_1^2}$, $a_4 = \frac{K{c_1}^2}{\gamma\omega^{*2}}$, $a_5 = \frac{\rho j {c_1}^2}{\gamma}$, $a_6 = \frac{\lambda_1 {c_1}^2}{\alpha\omega^{*2}} + \frac{\rho j_0 {c_1}^2}{2\alpha}$, $a_7 = \frac{\lambda_0 {c_1}^2}{\alpha\omega^{*2}}$, $a_8 = \frac{\gamma_1 {c_1}^2}{\alpha\beta_1\omega^{*2}}$, $a_9 = \frac{K}{\omega^* K^*}$, $a_{10} = \frac{{\beta_1}^2 T_0 c_1}{\rho K^*\omega^{*2}}$, $a_{11} = \frac{\gamma_1\beta_1 T_0}{\rho K^*\omega^{*2}}$,

$k_1 = a_9 + \omega$, $k_2 = \chi k^2 - 1 - k_1 k^2$, $k_3 = k^2 + 2a_4 + a_5\omega^2$, $k_4 = k^2 + a_6$, $k_5 = K + k_1$, $k_6 = k^2 - 1$, $\omega_{11} = k^2 + \omega^2$, $a'_{10} = \omega^2 a_{10}$, $k_7 = \omega^2 + a_2 k^2$, $k_8 = k - k_4 k_5 + \omega a_8 a_{11}\chi$, $k_9 = \omega a_8 a_{11} k_6 - k_2 k_4$, $k_{10} = a_1(a_7 k_5 + a_8 a'_{10}\chi)$, $k_{11} = a_1(a_7 k_2 + a_7 k_5 c - a_8\chi\, k^2 a'_{10} + a_8 k_6 a'_{10})$, $k_{12} = k^2 a_1(a_7 k_2 + a_8 k_6 a'_{10})$, $k_{13} = \omega a_{11} a_7 + k_4 a'_{10} + k^2$, $k_{14} = k^2(a_7\omega a_{11} + a'_{10} k_4)$

Appendix B:

$\Delta_{1i}=\begin{vmatrix} m_i^2-k_4 & -a_8(\chi\omega_i^2+k_6) \\ \omega a_{11} & k_5 m_i^2+k_2 \end{vmatrix}$, $\Delta_{2i}=\begin{vmatrix} -a_7(m_i^2-k^2) & -a_8(\chi\omega_i^2+k_6) \\ -a'_{10}(m_i^2-k^2) & k_5 m_i^2+k_2 \end{vmatrix}$,

$\Delta_{3i}=\begin{vmatrix} -a_7(m_i^2-k^2) & m_i^2-k_4 \\ -a'_{10}(m_i^2-k^2) & \omega a_{11} \end{vmatrix}$,

Appendix C:

$b_1=\frac{\lambda_0}{\rho c_1^2}$, $b_2=\frac{\lambda}{\rho c_1^2}$, $b_3=\frac{2\mu+K}{\rho c_1^2}$, $b_5=\frac{\mu+K}{\rho c_1^2}$, $b_6=\frac{\mu}{\rho c_1^2}$, $b_7=\frac{K}{\rho c_1^2}$, $b_8=\frac{\omega^{*2}\gamma}{\rho c_1^4}$, $b_9=\frac{\omega^{*2}b_0}{\rho c_1^4}$, $b_{10}=\frac{\omega^{*2}}{\rho c_1^4}$

CONCLUSIONS

The propagation of Rayleigh waves in a homogeneous isotropic microstretch thermoelastic medium with two temperatures subjected to stress free, thermally insulated boundaries have been studied. Secular equation for Rayleigh wave propagation in the context of GN theory of thermoelasticity has been derived.

The current research has important practical usefulness. Since valuable organic and inorganic deposits beneath the earth's surface are difficult to detect by drilling randomly, the wave propagation technique is the simplest and most economical and does not require any drilling through the earth.

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