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An Empirical Study for Comparison of Estimation Methodsfor Value-at-Risk and Expected Shortfall under Peaks over Threshold framework

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ABSTRACT

This paper aims to evaluate the performance of different value-at-risk (VaR) and expected shortfall (ES) computational methods under peaks over threshold setup. We considered measuring VaR and ES in the presence of additive outliers using several widely used methods including both robust and non-robust methods. To test our models' validity we used parametric bootstrap methods along with unconditional backtest procedures on daily returns of popular index S&P 500 historical data and a sample insurance claim dataset.We learned that generalized pareto distribution is a good choice for modeling exceedances, generated at appropriate threshold level, from empirical studies using R software. We found that robust methods are performing well in both situations (with outliers and without outliers), when exceedances is large. We believe this paper guides investors/practitioners in selecting appropriate method in calculating risk measures in presence of outliers.

Keywords: Value at Risk, Generalized Pareto Distribution, Peaks over Threshold.

1. INTRODUCTION

To measure and manage market risk, it is necessary to have an accurate model that describes asset returns. Value at risk (VaR) is one of the popular measures of risk used among all stake holders, both by investors and by regulators, which indicates the maximum amount that an investor may lose over a given time horizon and with a given probability. Various models and methodologies were developed over past few decades to calculate VaRand among them, one of the widely used methodis based on extreme value theory (EVT) (*McNeil, 2000*, *Embrechts,* 2000) and there are two approaches to analyze extreme data namely, the block-maxima approach *(Beirlant et al. 1996)* and the peaks-over-threshold (POT) approach *(Davison and Smith*, 1990). The POT model is generally considered to be the most useful for practical applications due to the more efficient use of the data for the extreme values.Both academics and practitioners realized that VaR lacks certain essential properties, e.g. coherence. As a result, expected shortfall (ES) was proposed, which is most popular alternative to VaR. The literature that compares these two measures is not as rich yet, especially under the fat-tails assumption, hence, we calculate VaR and ES using few recently proposed robust methods along with several classical methods under POT setup and compare their performance using back testing and parametric bootstrap procedures. We believe that the main contribution of our paper is comparison of various methods for estimating VaR and ES under POT setup, which will guide investors/practitioners in selecting appropriate method in calculating risk measures in presence of outliers. The paper is structured as follows. In section 2, necessary definitions and background information is covered. Section 3 covers empirical study made on S&Pmarket index data (with outlier), followed by another empirical study on a sample insurance claim data (without outlier) under section 4. Finally in section 5, we conclude the paper.

2. DEFINITIONS AND CONCEPTS

2.1 : Value at Risk & Expected Shortfall

Let X denotethe return random variable on a probability space$(x,\ F, P)$ with distributionfunction$F_X(x)$. Then, for each $\alpha\epsilon(0,1)$, the VaR with 100(1–α)% confidence level is defined as:

$$VaR_{(\alpha)}(X) = \inf(x\colon P(X \leq x) > \alpha) = \sup(x\colon P(X < x) \leq \alpha)$$

The main drawback with the use of VaR as a risk measure is that it does not respond to losses exceeding the confidence level. As a result, it cannot capture the risk associated with the shape of the distribution beyond the confidence level. Hence *Artzner et al. (1997)* propose the use of expected shortfall as an improvement on VaR.

$$ES_{(\alpha)}(X) = E\left(X \mid X > VaR_{(\alpha)}(X)\right)$$

It is observed that returns on stock markets generally follow normal distributions and as a result, under typical conditions, VaR is thought to be almost as effective as ES at capturing risk. However, the financial crisis in 2007 highlights the importance of measuring the risk associated with non-normal returns. In this connection Generalized Pareto Distribution can be used to model returns exceeding a sufficiently high threshold value in case of violation of normal assumption andfor heavy tailed distribution.

2.2: Extreme Value Theory: Peaks over Threshold Approach

The first approach considers the distribution of the maximum order statistic and a generalized extreme value (GEV) distribution is then fitted to the series of extremal observations which is known as Block Maxima Approach.The second approach extracts the peak values which exceed a certain threshold and the excess values over high threshold are modeled with Generalized Pareto Distribution (GPD) which is referred as Peaks over Threshold Approach. It has been shown that the limiting distribution of exceedances (or peaks) of a random variable X over a threshold u is Generalized Pareto Distribution (GPD) by Pickands(1975) and Balkemaand de Haan(1974).We restrict our attention to POT model as it yields better results in estimating expected shortfall.

2.3: Generalized Pareto Distribution

Let X be a random variable. The distribution function of the GPD with location$\mu(\mu \in R)$, scale$\sigma(\sigma > 0)$ and shape parameter$k$ $(k \in R)$, respectively - $GPD(\mu, \sigma, k)$, is defined as

$$F(X|\mu,\sigma,k) = \begin{cases} 1-\left(1-k\dfrac{x-\mu}{\sigma}\right)^{1/k} & k \neq 0 \\ 1-exp\left(-\dfrac{x-\mu}{\sigma}\right) & k = 0 \end{cases}$$

For k $fork \leq 0\ therangeis\mu \leq x \leq \infty, whilefork > 0,\ \ \mu \leq x \leq \mu + \frac{\sigma}{k}$

The GPD reduces to the Pareto distribution when k<0, infact, very 'large' values have a relatively higher probability when$k$< 0 heavy tailed,whereas very light can be produced for positive values of k. If X denotes the return random variable with distribution function$F_X(x)$, then, for each $\alpha\epsilon(0,1)$, the VaRand Expected Shortfall with 100(1–α)% confidence level aregiven as follows

$$VaR_{(\alpha)}(X) = \mu + \sigma F^{-1}_{k,\sigma}(\alpha)$$

$$VaR_{(\alpha)}(X) = u + \frac{\sigma}{k}\left[\left[\frac{n}{N_u}(1-\alpha)\right]^{-k} - 1\right]$$

$$ES(\alpha) = \frac{VaR(\alpha)}{1-k} + \frac{\sigma - ku}{1-k}$$

As value at risk is not a linear function of parameters of GPD, there is a need for studying estimation of VaR, especially in presence of outliers.

2.4: Threshold Selection Criterion

Under POT framework, we can estimate extremes for arbitrary distributions, if threshold value is sufficiently high. But the choice of threshold is critical, as a high threshold leads to high variance due to few exceedences (but not biased), and a low threshold would necessitate using samples that are no longer considered as being in the tails which leads to increased bias. Hence one has to balance between bias and precision in selecting threshold value u. Some of the suggested graphical techniques (*Embrechts et al. 1999b)*are

- Mean excess plot (*McNeil, et. al., 2005*)
- Mean residual life plot
- Parameter stability plot
- The dispersion index plot (*Ribatet, 2006*)
- Hill Plot, a non-parametric approach
- Zipf plot

2.5: Estimation Methods

Various parameter estimation methods have been studied for Generalized Pareto Distribution in literature and several methods have been compared under various conditions for estimating the GPD parameters, a detailed summary of results can be found in *P.Z Bermudeza & S. Kotz (2010, Part I & II)*. However, there are no universally accepted methods for estimating GPD parameters. Even if few methods are better than others over certain range of shape parameter k, they suffer from various constraints and convergence problems. Among all, maximum likelihood method (MLE) is more popular due to its asymptotic optimal properties *Davison (1984). Hosking and Wallis (1987)* compared MLE with method of moments (MOM) and probability weighted moment (PWM) estimates over small range of k, $|k| \leq ½$ and found that PWM performs well for $0 \leq k \leq 1$ and very good for $k \leq ½$.*Castillo and Hadi (1997)* introduced elemental percentile method (EPM) and compared with the MOM and the PWM methods, using root mean square error criterion, when $|k| \leq 2$ through simulation study and showed that the PWM performs good for $k \leq ½$ for small sample size. *Zhang and Stephens (2009)* developed empirical Bayes method (EBM) based on the likelihood and which uses a data-driven prior to estimate parameters of GPD and showed that it performs better when $– ½< k <½$. Recently *P.Chen et al. (2017)* proposed two new robust estimators for the GPD parameters using the minimum distance approach and M-estimation and they compared proposed methods with MLE, EPM and EBM and they claimed that the proposed methods are robust to outlier contamination as the distance measure is borrowed from robust estimation. It was found that many empirical studies have been conducted to compare various methods for estimating VaR(Ref: Fotios C. Harmantzis, Carlo M *et al*, 2006, Marco Bee & Luca Trapin, 2018). It is often seen that the number of exceedances are small and in such cases, a single abnormally large value may distort the estimates. Hence, there is a need for comparing all available methods under POT setup for one dataset including outliers. In this connection two empirical studies have been considered to compare few recently proposed robust methods with few classical methods in presence and absence of additive outliers.

Table 1: List of Estimation Methods Considered in the Study

Robust Methods	Non Robust Methods
1. Robust methods proposed by *P.Chen et..al* (2017), (PZ and WPZ) 2. Median Estimator, (MED), *Peng and Welsh, (2001)* 3. Minimum Density Power Divergence, (MDPD), *Juárez and Schucany (2004)* 4. Maximum Goodness-of-fit Estimators, (MGF), (Based on Anderson Darling statistic) *Alberto Luceño* (2006)	5. PICKANDS (Pickands) *Pickands, J. (1975)* 6. Maximum Likelihood Estimator, (MLE), *Smith (1984).* 7. Maximum Penalized Likelihood, (MPLE), *Cole Dixon (1999)* 8. Probability Weighted Moments,(PWMU, PWMB), *Hosking and Wallis (1987)* 9. Likelihood Moment Estimator, (LME), *Zhang (2007)* 10. Empirical Bayes Method, (*EBM or* ZJ),*Zhang (2010),*

3. EMPIRICAL STUDY – 1

3.1: S & P 500 Market Index

We have considered S&P 500 stock market index daily returns (log-differences) of the closing values between December 31, 2009 and February 15, 2018 from National Stock Exchange (NSE) which was downloaded from http://www.finance.yahoo.com. As our interest is in comparison of estimation methods in presence of outliers, we covered the period of financial crisis and summary of descriptive statistics are reported below

Table 2: Descriptive Statistics of S&P 500

Mean	1761.3
Standard Error	10.021
Median	1830.2
Mode	1178.1
Standard Deviation	453.29
Kurtosis	-1.0652
Skewness	0.1637
Count	2046

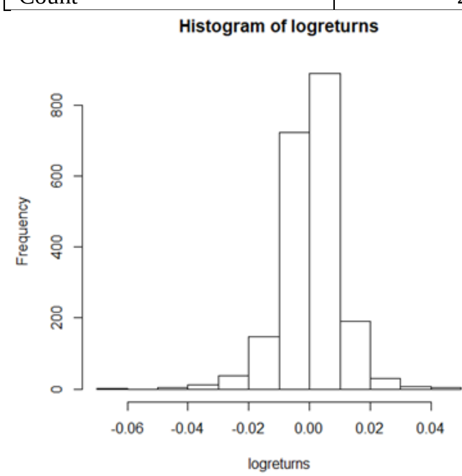


Figure 1: Density of daily log returns

3.2: Outlier Detection

In order to study the effect of outliers we applied outlier detection procedures using methods proposed by Chen & Liu (2012) and found that there are thirteen additive outliers and three among them occurs to the right end of the distribution,which confirms the presence of additive outliers among exeedances in our study. The summary of identified additive outliers is reported below along with graphical representation of outlier effects.

Table 3: Summary of additive outliers found in S&P 500 stock market index

Out-liers	log returns	Time	coefhat	t-stat
1	**0.043**	**2009:99**	**0.04244**	**4.887**
2	-0.04	2009:107	-0.0404	-4.65
3	-0.035	2009:117	-0.0356	-4.1
4	-0.049	2010:47	-0.0496	-5.71
5	-0.069	2010:49	-0.0696	-8.01
6	-0.046	2010:57	-0.0462	-5.32
7	-0.037	2010:115	-0.038	-4.37
8	**0.0424**	**2010:129**	**0.04181**	**4.814**
9	-0.04	2012:336	-0.0408	-4.7
10	**0.0383**	**2012:338**	**0.0377**	**4.34**
11	-0.037	2013:182	-0.0372	-4.28
12	-0.042	2014:223	-0.0424	-4.89
13	-0.038	2014:226	-0.0389	-4.47

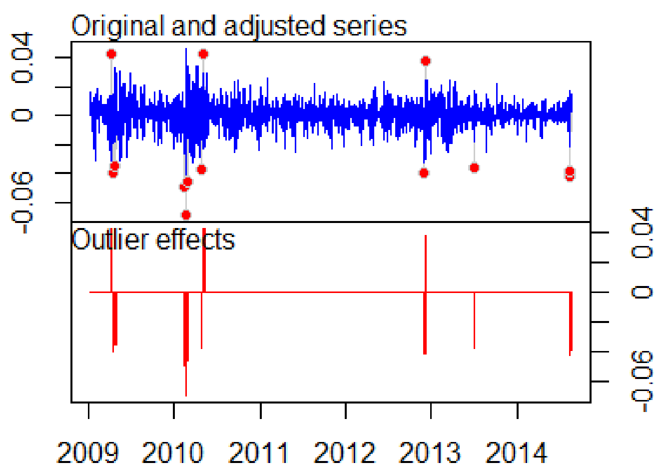


Figure 2: Plot of outlier effects

3.3: Choice of threshold value

It has been observed that the distribution is nearly symmetric and in order to generate exceedances, we applied mean excess plot and zipf plot to approximate threshold value.

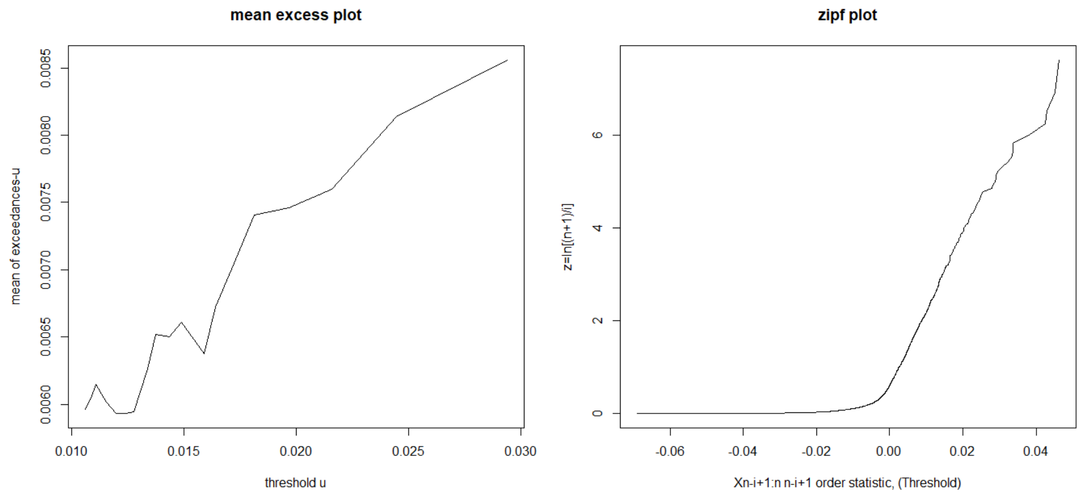
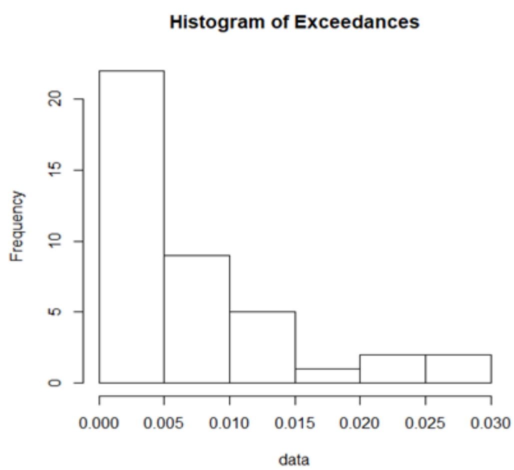


Figure 3: Threshold Selection Plots

From above plots, we have selected threshold at 98% percentile at t = 0.01967 and summary of exceedances(log returns – threshold) are generated are given below.

Table 4: Descriptive Statistics S&P 500 Exceedances	
Mean	0.00746
Standard Error	0.00114
Median	0.00479
Standard Deviation	0.00728
Kurtosis	0.95
Skewness	1.324
Count	41



3.4: Goodness of fit

We have considered Anderson-Darling statistic (A^2) and Cramer-von Mises statistic (W^2) to measure the goodness of fit

$$W^2 = \sum_{i=1}^{n} \left\{ F(x_{(i)}) - \frac{(2i-1)}{(2n)} \right\}^2 + \frac{1}{12n}$$

$$A^2 = -n - \left(\frac{1}{n}\right)\sum_{i=1}^{n}(2i-1)\left[\, log\, g\{F(x_{(i)})\} + \, log\, g\{1 - F(x_{(n+1-i)})\}\ \right]$$

Goodness of fit statistics and p-values are estimated using parametric bootstrap techniques to compare different methods of estimation in fitting GPD for exceedances and summary of the results are reported in table 5.

Table 5: Goodness of fit statistics

Methods	Robust (Yes/ No)	Parameters		Cramer Von statistics		Anderson Darling statistics	
		Shape	Scale	W^2	P-value	A^2	P-value
EBM (NR)	No	-0.0217	0.0073	0.03	0.942	0.252	0.95
PZ (R)	Yes	-0.1545	0.0068	0.0248	0.955	0.2393	**0.966**
WPZ (R)	Yes	-0.1279	0.0069	0.0251	0.93	0.2342	**0.966**
MLE (NR)	No	0	0.0075	13.6667	0	Inf	0
PWMU (NR)	No	0.0608	0.007	0.0528	0.73	0.5168	0.684
PWMB (NR)	No	0.0408	0.0072	0.0403	0.802	0.3816	0.792
PICK (NR)	No	0.0413	0.0068	0.0587	0.9517	0.5487	0.9336
MED (R)	Yes	0.1185	0.0066	0.1171	0.6709	1.2296	0.4815
MDPD (R)	Yes	0.01	0.0075	0.033	**0.9907**	0.2709	**0.9942**
LME (NR)	No	0.0071	0.0074	0.0323	**0.9999**	0.2705	**0.9989**
MPLE (NR)	No	0	0.0075	13.6667	0	Inf	0
MGF (R)	Yes	0	0.0078	0.0426	0.9544	0.2917	**0.979**

As p-values are high for all methods (Except for MLE and MPLE), it is clear that Generalized Pareto Distribution is good choice for modelling exceedances generated at t = 0.01967 (m=41, 98^{th} percentile) and it is observed that many of the robust methods (MDPD, PZ and WPZ) with higher p-values are performing better than non-robust methods in fitting GPD for exceedances in presence of additive outliers. This indicates that robust methods are appropriate for fitting GPD to exceedances in presence of outliers.

3.5: Estimation of Parameters, Value at Risk and Expected shortfall

After fitting Generalised Pareto Distribution to exceedances, estimates of shape, scale, VaR, ES & in-sample failure rate are computed and reported below at confidence level 99% and 99.9%.

Table 6: Estimates of Shape, Scale,Value at Risk, in-sample failure rate and expected shortfall, for S&P market index at, 99% and 99% confidence (at m = 41)

Methods	Parameters		99%			99.90%		
	Shape	Scale	Value at Risk	Failure Rate	Expected Shortfall	Value at Risk	Failure Rate	Expected Shortfall
EBM	-0.0217	0.0073	0.0247	**0.0156**	0.0318	0.0409	0.0024	0.0476
PZ	-0.1545	0.0068	0.0241	0.0161	0.0294	0.0359	0.0049	0.0396
WPZ	-0.1279	0.0069	0.0243	**0.0156**	0.0299	0.0369	0.0044	0.0410
MLE	0.0000	0.0075	0.0197	0.0274	0.0269	0.0197	0.0274	0.0269
PWMU	0.0608	0.0070	0.0242	**0.0156**	0.0321	0.0427	0.0020	0.0533
PWMB	0.0408	0.0072	0.0243	**0.0156**	0.0321	0.0425	0.0020	0.0524
PICK	0.0413	0.0068	0.0241	0.0161	0.0322	0.0430	0.0020	0.0550
MED	0.1185	0.0066	0.0241	**0.0156**	0.0349	0.0489	**0.0010**	0.0719
MDPD	0.0100	0.0075	0.0247	**0.0156**	0.0321	0.0417	0.0024	0.0492
LME	0.0071	0.0074	0.0245	**0.0156**	0.0321	0.0422	0.0020	0.0511
MPLE	0.0000	0.0075	0.0197	0.0274	0.0269	0.0197	0.0274	0.0269
MGF	0.0000	0.0078	0.0250	0.0147	0.0327	0.0426	0.0020	0.0503

It is observed that method of medians (MED) is performing better than others in estimating VaR as in-sample failure rate for MED is closer to expected rate of error for all confidence levels considered. Further for 99% confidence level the other robust methods (WPZ & MDPD) are also performing on par with MED. In above three cases we find that robust methods are better choice for estimating VaR in presence of outliers.

4. EMPIRICAL STUDY – 2

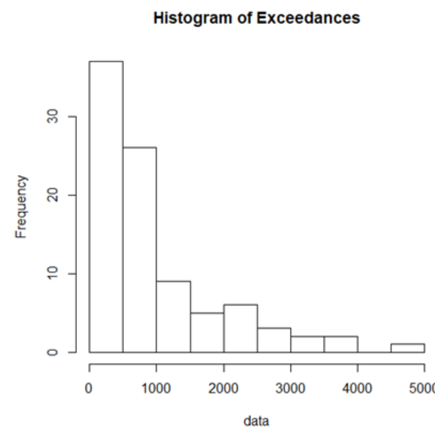
4.1: Sample Auto Insurance Claim Data

Another empirical study has been considered to examine how robust methods behave under non contaminated situation and a sample auto insurance claim data was downloaded from *Emcien: Predictive Software*, which includes location, policy type and claim amount. (*source: https://support.emcien.com /help/sample-data-sets*). Outlier detection has been carried out and outliers were not found in this dataset, which confirms that dataset is not contaminated.

4.2: Choice of threshold value

It has been observed that the data is positively skewed and threshold value is fixed at 99^{th} percentile at t= 3598.41 using mean excess plot and zipf plots. Summary of exceedances generated are given below.

Table 8 : Descriptive Statistics Exceedances: Auto Insurance Claim	
Mean	922.7876
Standard Error	101.9299
Median	580.3799
Standard Deviation	972.3491
Kurtosis	2.875714
Skewness	1.714958
Count	91



Generalized Pareto Distribution is fitted to the exeedances generated at t= 3598.41 (m=91, 99th percentile) using all methods. Estimates of shape, scale &goodness of fit statistics along with bootstrap p-values are reported in table 9.

Table 9: Goodness of fit statistics

Methods	Parameters		Cramer Von statistic		Anderson Darling statistic	
	Shape	Scale	W^2	P-value	A^2	P-value
EBM	-0.10027	833.7643	0.050133	**0.581**	0.321544	0.646
PZ	-0.14145	813.0319	0.049033	0.411	0.302085	**0.712**
WPZ	-0.17424	798.1057	0.049643	0.392	0.300917	**0.692**
MLE	0.027142	922.7791	0.078682	0.429303	0.557618	0.461066
PWMU	0.115903	815.8337	0.180406	0.168	2.224513	0.102
PWMB	0.106858	824.1802	0.156183	0.23	1.902527	0.149
PICK	0.178689	786.5276	NaN	NaN	NaN	NaN
MED	0.070704	816.9603	0.134568	0.528073	1.448855	0.373293
MDPD	0.040179	922.7875	0.078018	0.465116	0.589986	0.451972
LME	0.09214	795.1645	0.196452	0.222386	2.155878	0.106646
MPLE	0.02016	922.781	0.079314	0.419778	0.544259	0.45106
MGF	0.033998	922.7857	0.078251	0.544944	0.573403	0.58427

As p-values are not very high for all methods, except for PZ and WPZ, it is clear that Generalized Pareto Distribution is good choice for modelling exceedances when we use PZ and WPZ methods for estimating parameter of GPD. This indicates that robust methods proposed by Chen et.al (2017) are more appropriate for fitting GPD to exceedances when sample size is large.

4.3: Estimation of Value at Risk and Expected Shortfall

After fitting Generalized Pareto Distribution to the exceedances, estimates of VaR, ES and in-sample failure rate are calculated at 95% & 98% confidence levels which are reported below.

Table 10: Estimates of Shape, Scale,VaR, in-sample failure rate and ES based on sample auto insurance claim amountat 95% and 98% confidence (at m = 91)

Methods	Parameters		95%			98%		
	Shape	Scale	VaR	Failure Rate	ES	VaR	Failure Rate	ES
EBM	-0.10027	833.7643	2138.527	**0.05441**	3029.348	2996.6	**0.02080**	3809.226
PZ	-0.14145	813.0319	2125.152	**0.05506**	3020.001	3002.961	**0.02069**	3789.03
WPZ	-0.17424	798.1057	2111.78	**0.05506**	3012.052	3007.054	**0.02069**	3774.481
MLE	0.027142	922.7791	2135.732	0.05594	3047.196	2959.109	**0.02058**	3892.257
PWMU	0.115903	815.8337	2386.428	**0.05441**	3158.468	3047.774	0.02123	3903.616
PWMB	0.106858	824.1802	2365.149	0.04018	3148.657	3040.35	**0.01948**	3901.662
PICK	0.178689	786.5276	2532.215	0.04083	3231.442	3096.026	**0.01970**	3933.624
MED	0.070704	816.9603	2344.92	0.03426	3131.049	3043.499	0.01861	3869.478
MDPD	0.040179	922.7875	2150.832	0.04182	3055.464	2961.98	**0.01970**	3899.221
LME	0.09214	795.1645	2406.455	**0.05320**	3161.349	3061.695	**0.02123**	3883.090
MPLE	0.02016	922.781	2127.502	0.01007	3042.741	2957.552	0.01007	3888.577
MGF	0.033998	922.7857	2144.257	**0.05485**	3051.849	2960.726	0.02145	3896.159

It is observed that robust methods (PZ, WPZ, MDPD) and non-robust methods (EBM, PWM, PICK, LME) are performing equally well in estimating value at risk in absence of outliers when exceedances is large. Hence we learned that robust methods are performing well in both situations (with outliers and without outliers), when exceedances is large.

5. CONCLUSION

The main objective of this study was investigation of the performances of some estimators in estimation of Value at Risk and Expected shortfall under POT framework. However, outliers are expected to affect estimates of VaR and Expected shortfall, especially when sample size is small and moderately large. Hence, we considered two empirical studies for comparison of few methods of estimations under peaks over threshold framework.

S&P500 market index data was considered after verifying the presence of additive outliers. It is observed that MDPD, PZ and WPZ (robust methods) were performing better than others, which indicates that robust methods are appropriate for fitting GPD to exceedances in presence of outliers. Method of medians (MED) was performing better than others in estimating VaRbased on in-sample failure rate at all confidence levels considered in the study. So we finally conclude that robust methods are better choice for estimating VaR in presence of outliers.From another empirical study based on sample auto insurance claim data, we learned that robust methods (PZ, WPZ, MDPD) and non-robust methods (EBM, PWM, PICK, LME) were performing equally well in estimating VaR in absence of outliers when exceedances is large. Hence we conclude that robust methods are performing well in both situations (with outliers and without outliers), when exceedances is large. Hence, we conclude that one can use robust methods for estimating Value at Risk (extreme quantiles) without looking for outliers when the size of exceedances is large under peaks over threshold framework.

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