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Special Types of Generalized $\mathcal{B}P -$ Recurrent Spaces

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ABSTRACT

In this paper, we introduced a Finsler space which Cartan's second curvature tensor P^i_{jkh} satisfies a generalized recurrence property the sense of Berwald, i.e. characterized by the following condition

$$\mathcal{B}_l P^i_{jkh} = \lambda_l P^i_{jkh} + \mu_l\left(\delta^i_j g_{kh} - \delta^i_k g_{jh}\right), \quad P^i_{jkh} \neq 0,$$

where $\mathcal{B}_l$ is Berwald's covariant differential operator with respect to x^l, λ_l and μ_l are known as recurrence vectors, such space is called *a generalized* $\mathcal{B}P$ *−recurrent space.*

The purpose of the present paper to prove the above space can't be an affinely connected space and we proved the above space can't be a Landsberg space. Also we used properties of $P2$ −Like space in above space and we got a new space it called $P2$ −Like generalized $\mathcal{B}P$ −recurrent space. Also we obtained different theorems for some tensors which satisfy in above space.

Keywords: a generalized $\mathcal{B}$P −recurrent space, an affinely connected space, a Landsberg space, P2 −Like space.

1. INTRODUCTION

Pande and Single[8] discussed the recurrence property in an affinely connected space, Hussien[13] obtained certain identities in a $K^h -$ recurrent affinely connected spaces, Qasem and Abdallah[5] introduced and studied a generalized $\mathcal{B}R$ − recurrent affinely connected space, Al – qashbari[3] introduced and studied a generalized $H^h -$ recurrent affinely connected space.

Qasem and Baleedi[7] introduced and studied a property of Landsberg space in a generalized $\mathcal{B}K$ −recurrent space.

Verma[14] obtained some results when the $R^h -$ recurrent and $\mathrm{C} -$concircularly spaces are $P2 -$Like spaces, Qasem[4] introduced and studied $P2 -$Like space, Mohammed[1] introduced and studied $P2 -$Like spaces in $P^h -$ recurrent space, Dikshit[16] studied $P2 -$Like space, Awed[2] introduced and studied $P2 -$Like spaces in generalized $P^h -$ recurrent space, Qasem and Abdallah[6] introduced and studied $P2 -$Like spaces in generalized $\mathcal{B}R -$ recurrent.

Let F_n be an n – dimensional Finsler space equipped with the metric function *F(x,y)* satisfying the request conditions[10].

The vector y_i is defined by

$$y_i = g_{ij}(x,y)y^j \ . \tag{1.1}$$

The two sets of quantities g_{ij} and its associative g^{ij}, which are components of a metric tensor connected by

$$g_{ij}g^{ik} = \delta_j^k = \begin{cases} 1 \ \ if \ \ j = k \, , \\ 0 \ \ if \ \ j \neq k \, . \end{cases} \tag{1.2}$$

In view of (1.1) and (1.2), we have

$$\delta_j^i g_{ir} = g_{jr} \tag{1.3}$$

The tensor C_{ijk} is defined by[12]

$$C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij} \tag{1.4}$$

which called $(h)hv -$*torsion tensor*[12] and its associative $C_{jk}^{\ i}$ is symmetric in its lower indices and called $(v)hv -$ *torsion tensor*. These tensors satisfy the following:

$$\text{a) } C_{ijk}g^{jk} = C_i \ , \qquad \text{b) } C_{ik}^h = C_{ijk}g^{hj} \tag{1.5}$$

$$\text{c) } C_{ri}^i = C_r \quad \text{and} \quad d) \ C_{ijk} := g_{hj}C_{ik}^h$$

$$C_{ijk}\, y^i = C_{kij}\, y^i = C_{jki}\, y^i = 0, \tag{1.6}$$

É. Cartan $h -$ *covariant differentiation* (*Cartan*'s *second kind covariant differentiation*) with respect to x^k is given by [10]

$$X_{|k}^i := \partial_k X^i - \left(\dot{\partial}_r X^i\right)G_{\ k}^{r} + X^r \Gamma_{rk}^{*\,i} \, .$$

the metric tensor g_{ij} is covariant constant with respect to $h -$covariant derivative, *i.e.*

$$g_{\ |k}^{ij} = 0 \tag{1.7}$$

The $h -$ covariant derivative of the vector y^i vanish identically $i.e.$

$$y^i_{\ |k} = 0 \, . \tag{1.8}$$

Berwald covariant derivative $\mathcal{B}_k T_j^i$ of an arbitrary tensor field T_j^i with respect to x^k is given by

$$\mathcal{B}_k T_j^i \coloneqq \partial_k T_j^i - \left(\dot{\partial}_r T_j^i\right)G_k^r + T_j^r G_{rk}^i - T_r^i G_{jk}^r .$$

Berwald covariant derivative of the metric tensor g_{ij} does not vanish and given by

$$\mathcal{B}_k g_{ij} = -2C_{ijk|h}y^h = -2y^h \mathcal{B}_h C_{ijk} \ . \tag{1.9}$$

The tensor R_{jkh}^i called $h -$*curvature tensor (Cartan's third curvature tensor)* defined by[10]

$$R^i_{jkh} := \partial_h \Gamma^{*i}_{jk} + (\dot{\partial}_\ell \Gamma^{*i}_{jh}) G^\ell_k + G^i_{jm}(\partial_h G^m_k - G^{\;m}_{h\ell} G^\ell_k) + \Gamma^{*i}_{mh}\Gamma^{*m}_{jk} - h/k.$$

and satisfies the relations

$$R^i_{jkh}\, y^j = H^i_{kh} \tag{1.10}$$

the curvature tensor R^i_{jkh} and its associative tensor R_{ijhk} satisfies the following identities known as *Bianchi identity* [10]

a) $R^r_{ijk|h} + R^r_{ihj|k} + R^r_{ikh|j} + y^m(R^s_{mkh}P^r_{ijs} + R^s_{mjk}P^r_{ihs} + R^s_{mhj}P^r_{iks}) = 0,$ (1.11)

* h/k means the subtraction from the former terms by interchanging the indices k and h .

$b)$ $R^i_{hjk} + R^i_{jkh} + R^i_{khj} - (C^i_{hr}H^r_{jk} + C^i_{jr}H^r_{kh} + C^i_{kr}H^r_{jh}) = 0$

and

$c)$ $R_{ijhk} + R_{ihkj} + R_{ikjh} + C_{ijs}H^s_{hk} + C_{ihs}H^s_{kj} + C_{iks}H^s_{jh} = 0,$

Where the tensor P^i_{jkh} called hv *− curvature tensor* (*Cartan's second curvature tensor*) defined by [10]

$$P^i_{jkh} := \dot{\partial}_h \Gamma^{*i}_{jk} + C^i_{jr}P^r_{kh} - C^i_{jh|k} \tag{1.12}$$

satisfies the relations

$$P^i_{jkh}\, y^j = \Gamma^{*i}_{jkh} y^j = P^i_{kh} = C^i_{\;kh|r} y^r, \tag{1.13}$$

The $P -$ Ricci tensor P_{jk} is given by

$$P^i_{jki} = P_{jk}\,. \tag{1.14}$$

The associate tensor P_{ijkh} of the hv −curvature tensor P^i_{jkh} is given by[10]

$a)$ $P_{ijkh} = g_{ir}\, P^r_{jkh}$ and $b)$ $P^r_{jkh} = g^{ir} P_{ijkh}$ (1.15)

The tensor $(P_{ij} - P_{ji})$ is given by

$$P_{ijkh}\, g^{kh} = P_{ij} - P_{ji}\,. \tag{1.16}$$

2. A GENERALIZED $\mathcal{B}P-$ RECURRENT SPACE

Let us consider a Finsler space F_n which Cartan's second curvature tensor P^i_{jkh} satisfies the generalized recurrence property with respect to Berwald's connection parameter G^i_{kh} , i.e. characterized by the following condition:

$$\mathcal{B}_l P^i_{jkh} = \lambda_l P^i_{jkh} + \mu_l(\delta^i_j g_{kh} - \delta^i_k g_{jh}) \quad , \quad P^i_{jkh} \neq 0 \;^* \;, \tag{2.1}$$

where $\mathcal{B}_l$ is Berwald's covariant differential operator with respect to x^l, λ_l and μ_l are called recurrence vectors.

∗ In Rund's book, P^i_{jkh} defined here, is denoted by P^i_{hkj} . This difference must be noted.

Definition 2.1. A Finsler space F_n which Cartan's second curvature tensor P^i_{jkh} satisfies the condition (2.1), where λ_l and μ_l are non-zero covariant vectors field. Such space satisfying the condition (2.1) will be called a generalized $\mathcal{B}$P − recurrent space and denoted it briefly by G($\mathcal{B}$P) − RF_n .

Let us consider a $G(\mathcal{B}P) - RF_n$ which is characterized by the condition (2.1).

Transvecting the condition (2.1) by g_{im} , using (1.15a), (1.9) and (1.3), we get

$$\mathcal{B}_l P_{mjkh} = \lambda_l P_{mjkh} + \mu_l \big(g_{jm} g_{kh} - g_{km} g_{jh} \big) + 2 P^i_{jkh} y^t \mathcal{B}_t C_{iml} \, . \tag{2.2}$$

Contracting the indices i and h in the condition (2.1), using (1.14) and (1.3), we get

$$\mathcal{B}_l P_{jk} = \lambda_l P_{jk} \, . \tag{2.3}$$

Transvecting the condition (2.2) by g^{kh}, using (1.16), (1.2), (1.3) and put ($g^{kh} g_{kh} = 1$), we get

$$\mathcal{B}_l \, (P_{mj} - P_{jm}) = \lambda_l \, (P_{mj} - P_{jm}) + 2 g^{kh} \, P^i_{jkh} y^t \mathcal{B}_t C_{iml} + P_{mjkh} \, \mathcal{B}_l \, g^{kh}$$

This shows that

$$\mathcal{B}_l \, (P_{mj} - P_{jm}) = \lambda_l \, (P_{mj} - P_{jm}) \tag{2.4}$$

If and only if

$$2 g^{kh} \, P^i_{jkh} y^t \mathcal{B}_t C_{iml} + P_{mjkh} \, \mathcal{B}_l \, g^{kh} = 0 \tag{2.5}$$

3. AN AFFINELY CONNECTED SPACE

Definition 3.1. A Finsler space whose connection parameter G^i_{jk} is independent of y^i is called an affinely connected or Berwald's space. Thus, an affinely connected or Berwald's space is characterized by any one of the equivalent conditions[10]

a) $G^i_{jkh} = 0$ and b) $C_{ijk|h} = 0$. (3.1)

Remark 2.1. The connection parameters Γ^{*i}_{kh} of Cartan and G^i_{kh} of Berwald coincide in affinely connected space and there are independent of directional argument[10], i.e. the conditions

a) $\dot{\partial}_j G^r_{kh} = 0$ and b) $\dot{\partial}_j \Gamma^{*i}_{kh} = 0$ (3.2)

are satisfied.

Definition 3.2. The generalized $\mathcal{B}P$ −recurrent which is an affinely connected space [satisfies any one of the conditions (3.1a), (3.1b), (3.2a) and (3.2b)], will be called *a generalized* $\mathcal{B}P -$ *recurrent affinely connected space* and we shall denote it briefly by $G(\mathcal{B}P) - R -$ *affinely connected space.*

Transvecting the condition (3.1b) by g^{ir}, using (1.7) and (1.5b), we get

$$C^r_{jk|h} = 0 \tag{3.3}$$

S. B*á*sc*ó*, *et al*.[15] proved that in a generalized Berwald's space, the associate $hv -$ curvature tensor P_{ijkh} is vanishing. A particular case, in a Finsler space, we have the $hv -$ curvature tensor P^i_{jkh} given by relation (1.12).

using (3.2b) and (3.3) in (1.12) , we get

$$P^i_{jkh} = C^i_{jr} P^r_{kh} \tag{3.4}$$

Transvecting (3.4) by g_{im} , using (1.15a) and (1.5d), we get

$$P_{mjkh} = C_{jmr} \, P^r_{kh} \tag{3.5}$$

We know that, the tensor P_{mjkh} is skew- symmetric in the first two indices and the right –hand side is symmetric in the indices itself. Hence, we get

a) $P_{mjkh} = 0$ (3.6)

b) $C_{jmr}\, P^r_{kh} = 0$

Transvecting the condition (3.6a) by g^{im}, using (1.15b), we get

$$P^i_{jkh} = 0$$

But in $G(\mathcal{B}P) - RF_n$, we have $P^i_{jkh} \neq 0$

Thus, we conclude

Theorem 3.1. The space $\mathrm{G}(\mathcal{B}\mathrm{P}) - \mathrm{RF_n}$, can't be an affinely connected space or Berwald's space

4. A LANDSBERG SPACE

Definition 4.1. A Landsberg space is a Finsler space which is characterized by the condition

$$y_r G^r_{ijk} = -2C_{ijk|h}y^h = -2P_{ijk} = 0 . \quad (4.1)$$

Various authors denote the tensor $C_{ijk|h}y^h$ by P_{ijk} as H. Izumi[9].

Definition 4.2. The generalized $\mathcal{B}P$ −recurrent space which is a Landsberg space [satisfies the conditions (4.1)], will called *a generalized* $\mathcal{B}P$ *−recurrent Landsberg space* and we shall denote it briefly by $G(\mathcal{B}P) - R -$ *Landsberg space.*

M. Hashiguchi[11] proved that, in a Landsberg space, the $hv-$ curvature tensor is vanishing. Such that, in a Landsberg space, we have $P_{jkh} = 0$ which implies that associate it $P^r_{jk} = 0$. So that, the associate $hv-$ curvature tensor P_{ijkh} from the following formula is vanishing[11]

$$P_{ijkh} = \dot{\partial}_i P_{jkh} - \dot{\partial}_j P_{ikh} + C_{jhr} P^r_{ik} - C_{ihr} P^r_{jk}$$

But in $\mathrm{G}(\mathcal{B}P) - RF_n$, the associate $hv -$ curvature tensor $P_{ijkh} \neq 0$, so that, $\mathrm{G}(\mathcal{B}P) - RF_n$, can't be A Landsberg space

Thus, we conclude

Theorem4.1. *The space* $G(\mathcal{B}P) - RF_n$ *, can't be a Landsberg space.*

5. $P2$ −LIKE − GENERALIZED $\mathcal{B}P$ −RECURRENT SPACE

A $P2$ −Like space is characterized by the condition[9]

$$P^i_{jkh} = \varphi_j C^i_{kh} - \varphi^i C_{jkh} , \quad (5.1)$$

where φ_j and φ^i are non- zero covariant and contravariant vectors field, respectively.

Definition 5.1. The generalized $\mathcal{B}P$ −recurrent space which is $P2$ −Like space [satisfies the condition (5.1)], will called a $P2$ *−Like generalized* $\mathcal{B}P$ *−recurrent space* and will denote it briefly by a $P2 -Like\ - G(\mathcal{B}P) - RF_n$.

Let us consider a $P2-\text{Like}-G(\mathcal{B}P)-RF_n$.
Taking the covariant derivative for the condition (5.1) with respect to x^l in the sense of Berwald, we get

$$\mathcal{B}_l P^i_{jkh} = \mathcal{B}_l(\varphi_j C^i_{kh} - \varphi^i C_{jkh}) \tag{5.2}$$

Using (2.1) in (5.2), we get

$$\mathcal{B}_l(\varphi_j C^i_{kh} - \varphi^i C_{jkh}) = \lambda_l P^i_{jkh} + \mu_l(\delta^i_j g_{kh} - \delta^i_k g_{jh}) \tag{5.3}$$

Using (5.1) in (5.3), we get

$$\mathcal{B}_l(\varphi_j C^i_{kh} - \varphi^i C_{jkh}) = \lambda_l(\varphi_j C^i_{kh} - \varphi^i C_{jkh}) + \mu_l(\delta^i_j g_{kh} - \delta^i_k g_{jh}) \tag{5.4}$$

Thus, we conclude

Theorem 5.1. In $P2-\text{Like}-G(\mathcal{B}P)-RF_n$, the tensor $(\varphi_j C^i_{kh} - \varphi^i C_{jkh})$ is a generalized recurrent.

Transvecting (5.1) by g_{im} , using (1.15a) and (1.5d), we get

$$P_{mjkh} = \varphi_j C_{mkh} - \varphi_m C_{jkh} \tag{5.5}$$

Where $\varphi_m = \varphi^i g_{im}$

Taking the covariant derivative for (5.5) with respect to x^l in the sense of Berwald, we get

$$\mathcal{B}_l P_{mjkh} = \mathcal{B}_l(\varphi_j C_{mkh} - \varphi_m C_{jkh}) \tag{5.6}$$

Using (2.2) in (5.6), we get

$$\mathcal{B}_l(\varphi_j C_{mkh} - \varphi_m C_{jkh}) = \lambda_l P_{mjkh} + \mu_l(g_{jm} g_{kh} - g_{km} g_{jh}) + 2P^i_{jkh} y^t \mathcal{B}_t C_{iml} \tag{5.7}$$

Using (5.5) in (5.7), we get

$$\mathcal{B}_l(\varphi_j C_{mkh} - \varphi_m C_{jkh}) = \lambda_l(\varphi_j C_{mkh} - \varphi_m C_{jkh}) + \mu_l(g_{jm} g_{kh} - g_{km} g_{jh}) + 2P^i_{jkh} y^t \mathcal{B}_t C_{iml} \tag{5.8}$$

This shows that

$$\mathcal{B}_l(\varphi_j C_{mkh} - \varphi_m C_{jkh}) = \lambda_l(\varphi_j C_{mkh} - \varphi_m C_{jkh}) + \mu_l(g_{jm} g_{kh} - g_{km} g_{jh}) \tag{5.9}$$

if and only if

$y^t \mathcal{B}_t C_{iml} = 0$

Since $P^i_{jkh} \neq 0$

Thus, we conclude

Theorem 5.2. In $P2-\text{Like}-G(\mathcal{B}P)-RF_n$,the tensor $(\varphi_j C_{mkh} - \varphi_m C_{jkh})$ is a generalized recurrent if and only if $y^t \mathcal{B}_t C_{iml} = 0$.

Contracting the indices i and h in (5.1), using (1.14) and (1.5c), we get

$$P_{jk} = \varphi_j C_k - \varphi^i C_{jki} \tag{5.10}$$

Taking the covariant derivative for (5.10) with respect to x^l in the sense of Berwald, we get

$$\mathcal{B}_l P_{jk} = \mathcal{B}_l(\varphi_j C_k - \varphi^i C_{jki}) \tag{5.11}$$

Using (2.3) in (5.11), we get

$$\mathcal{B}_l(\varphi_j C_k - \varphi^i C_{jki}) = \lambda_l P_{jk} \tag{5.12}$$

Using (5.10) in (5.12), we get

$$\mathcal{B}_l(\varphi_j C_k - \varphi^i C_{jki}) = \lambda_l(\varphi_j C_k - \varphi^i C_{jki}) \tag{5.13}$$

Thus, we conclude

Theorem 5.3. In $P2-\text{Like}-G(\mathcal{B}P)-RF_n$, the tensor $(\varphi_j C_k - \varphi^i C_{jki})$ behaves as recurrent.
Transvecting (5.5) by g^{kh}, using (1.16) and (1.5a), we get

$$P_{mj} - P_{jm} = \varphi_j C_m - \varphi_m C_j \tag{5.14}$$

Taking the covariant derivative for (5.14) with respect to x^l in the sense of Berwald, we get

$$\mathcal{B}_l\,(P_{mj} - P_{jm}) = \mathcal{B}_l(\varphi_j C_m - \varphi_m C_j) \tag{5.15}$$

Using (2.4) in (5.15), we get

$$\mathcal{B}_l(\varphi_j C_m - \varphi_m C_j) = \lambda_l\,(P_{mj} - P_{jm}) \tag{5.16}$$

Using (5.14) in (5.16), we get

$$\mathcal{B}_l(\varphi_j C_m - \varphi_m C_j) = \lambda_l(\varphi_j C_m - \varphi_m C_j) \tag{5.17}$$

Thus, we conclude

Theorem 5.4. In $P2-\text{Like}-G(\mathcal{B}P)-RF_n$,the tensor $(\varphi_j C_m - \varphi_m C_j)$ behaves as recurrent [provided (2.5) hold] .
Using (5.1) and (1.10) in (1.11a), we get

$$R^r_{ijk|h} + R^r_{ihj|k} + R^r_{ikh|j} = -\{H^s_{kh}(\varphi_i C^r_{js} - \varphi^r C_{ijs}) + H^s_{jk}((\varphi_i C^r_{hs} - \varphi^r C_{ihs}) + H^s_{hj}(\varphi_i C^r_{ks} - \varphi^r C_{iks})\},$$

Or

$$R^r_{ijk|h} + R^r_{ihj|k} + R^r_{ikh|j} = -\varphi_i\big(H^s_{kh}C^r_{js} + H^s_{jk}C^r_{hs} + H^s_{hj}C^r_{ks}\big) + \varphi^r\big(H^s_{kh}C_{ijs} + H^s_{jk}C_{ihs} + H^s_{hj}C_{iks}\big) \tag{5.18}$$

Using (1.11b) and (1.11c) in (5.18), we get

$$R^r_{ijk|h} + R^r_{ihj|k} + R^r_{ikh|j} = -\varphi_i(R^r_{jkh} + R^r_{hjk} + R^r_{khj}) + \varphi^r(R_{jikh} + R_{hijk} + R_{kihj}) \tag{5.19}$$

Transvecting (5.18) by y^i, using (1.8), (1.10) and (1.6), we get

$$H^r_{jk|h} + H^r_{hj|k} + H^r_{kh|j} = -\varphi\,(H^s_{kh}C^r_{js} + H^s_{jk}C^r_{hs} + H^s_{hj}C^r_{ks}) \tag{5.20}$$

Where $\varphi_i y^i = \varphi$
Using (1.11b) in (5.20), we get

$$H^r_{jk|h} + H^r_{hj|k} + H^r_{kh|j} = -\varphi\,(R^r_{jkh} + R^r_{hjk} + R^r_{khj}) \tag{5.21}$$

Thus, we conclude

Theorem 5.5. *In $P2-Like-G(\mathcal{B}P)-RF_n$, we have identities (5.19), (5.20), (5.21).*

6. CONCLUSION

(6.1) The space which defined by the condition (2.1) we called a generalized $\mathcal{B}P-$ recurrent Finsler space.
(6.2) The generalized $\mathcal{B}P-$ recurrent Finsler space can't be an affinely connected space or Berwald's space.
(6.3) The generalized $\mathcal{B}P-$ recurrent Finsler space can't be a Landsberg space.
(6.4) We used properties of $P2-$Like space in the main space and we got a new space it called $P2-$Like generalized $\mathcal{B}P-$recurrent space
(6.5) In $P2-$Like generalized $\mathcal{B}P-$recurrent space the tensor $(\varphi_j C^i_{kh} - \varphi^i C_{jkh})$ is a generalized recurrent.

(6.6) In $P2$ −Like generalized $\mathcal{B}P$ −recurrent space the tensor $(\varphi_j C_{mkh} - \varphi_m C_{jkh})$ is a generalized recurrent if and only if $y^t \mathcal{B}_t C_{iml} = 0$.

(6.7) In $P2$ −Like generalized $\mathcal{B}P$ −recurrent space the tensor $(\varphi_j C_k - \varphi^i C_{jki})$ and the tensor $(\varphi_j C_m - \varphi_m C_j)$ behave as recurrent.

(6.8) We obtained certain identities in $P2$ − Like generalized $\mathcal{B}P$ −recurrent space.

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