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Some New Results on Prime Cordial Labeling for Theta Graph

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ABSTRACT

A prime cordial labeling of a graph G with the vertex set $V(G)$ is bijection to $f:V(G)\rightarrow\{1,2,\ldots|V(G)|\}$ such that for each edge *uv* is assigned the label 1 if $\gcd(f(u),f(v))=1$ and 0 if $\gcd(f(u),f(v))>1$; then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which admits a prime cordial labeling is called a prime cordial graph. In this paper, we have prove that $P_n^t(tn\cdot T_\alpha), S(t\cdot T_\alpha)$ and path union of even copies of Theta graph admits prime cordial labeling.

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Keywords: Prime cordial labeling, open star of graphs and one point union of path graphs, path union.

1. INTRODUCTION

Graph labeling is an active area of research in graph theory which has rigorous applications in coding theory, communication network, to determine optimal circuits and layouts. The recent survey on graph labeling we refer to Gallian[2]. Vast amount of literature is availabile on different types of graph labeling. For all terminology and notations we follow Harary[3]. We shall give a brief summary of definitions which are useful in this paper.

Definition 1.1: Let G be a graph. A mapping $f:V(G)\rightarrow\{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.

For an edge $e=uv$, the induced edge labeling $f^*:E(G)\rightarrow\{0,1\}$ is given by $f^*(e)=|f(u)-f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0

and 1 respectively under f while $e_f(0), e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 1.2: A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called cordial if it admits cordial labeling.
The concept of cordial labeling was introduced by Cahit[1].

Definition 1.3: A prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f: V(G) \to \{1,2,3,\ldots,|V(G)|\}$ defined by

$$f(e = uv) = \begin{cases} 1 & if \quad gcd(f(u), f(v)) = 1; \\ 0 & otherwise. \end{cases}$$

further $|e_f(0) - e_f(1)| \leq 1$.

A graph which admits prime cordial labeling is called a prime cordial graph. The concept of prime cordial labeling was introduced by Sundaram *et al*.[4].
Now let us recall the definitions of Theta graph, open star of graphs, one point union of path of graphs and the graph operations path union of a graph.

Definition 1.4: A Theta graph is a block with two non-adjacent vertices of degree 3 and all other vertices of degree 2 is called a Theta graph.

Definition 1.5: A graph obtained by replacing each vertex of $K_{1,n}$ except the apex vertex by the graphs $G_1, G_2, \ldots, G_n$ is known as open star of graphs. We shall denote such graph by $S(G_1, G_2, \ldots, G_n)$.
If we replace each vertices of $K_{1,n}$ except the apex vertex by a graph G. i.e. $G_1 = G_2 = \ldots = G_n = G$, such open star of a graph, we shall denote such graph by $S(n \cdot G)$.

Definition 1.6: A graph G is obtained by replacing each edge of $K_{1,t}$ by a path P_n of length n on n+1 vertices is called one point union for t copies of path P_n. Ww shall note such graph G by P_n^t.

Definition 1.7: A graph G obtained by replacing each vertex of P_n^t except the central vertex by the graphs $G_1, G_2, \ldots, G_{tn}$ is known as one point union for path of graphs. We shall denote such graph G by $P_n^t(G_1, G_2, \ldots, G_{tn})$, where P_n^t is the one point union of t copies of path P_n.
If we replace each vertex of P_n^t except the central vertex by a graph H. i.e. $G_1 = G_2 = \ldots = G_{tn} = H$, such one point union for path of graphs, we shall denote it by $P_n^t(tn \cdot H)$.

Definition 1.8. Let $G_1, G_2, G_3, \ldots G_n$, $n \geq 2$ be n copies of a fixed graph G. The graph obtained by adding an edge between G_i and $G_i + 1$ for $i = 1, 2, \ldots n-1$ is called the path union of G.

2. MAIN RESULTS

Theorem 2.1: $P_n^t(tn \cdot T_\alpha)$ a prime cordial graph, where n is even.

Proof: Let G be a graph obtained by replacing each vertex of P_n^t except the central vertex by the graph T_α. i.e. G = $P_n^t(tn \cdot T_\alpha)$, where t is any positive integer. Let u_0 be the central vertex for the graph G. We denote $v_{i,j}^k$ be the k^th^ copy of the j^th^ vertex in i^th^ row, where $1 \leq i \leq t$; $1 \leq j \leq 7$; $1 \leq k \leq n$. Then $|V(G)| = 7tn + 1$ and $|E(G)| = (8n+1)t + (n-1)t.$

We shall define labeling function $f : |V(G)| \rightarrow \{1, 2, ..., 7tn + 1\}$ as follows:

Let $f(u_0)=1$.

$f(v_{ij}^k) = 2j + 7[(i-1)n + (k-1)]$ *if* k *is odd*, $1 \leq i \leq t,\ 1 \leq j \leq 7,\ 1 \leq k \leq n$

$f(v_{ij}^k) = (2j+1) + 7[(i-1)n + (k-2)]$ *if* k *is even*, $1 \leq i \leq t,\ 1 \leq j \leq 7,\ 1 \leq k \leq n.$

Finally, we replace the vertex labelings $f(v_{i2}^k)$ by $f(v_{i7}^k)$ for k = 2,4,6,…n, $1 \leq i \leq t$

In view of the labeling pattern defined above, we have $e_f(0) = e_f(1).$

Thus we have $|e_f(0) - e_f(1)| \leq 1$.

Hence the above labeling pattern gives prime cordial labeling to the grpah G and so it is a prime cordial graph.

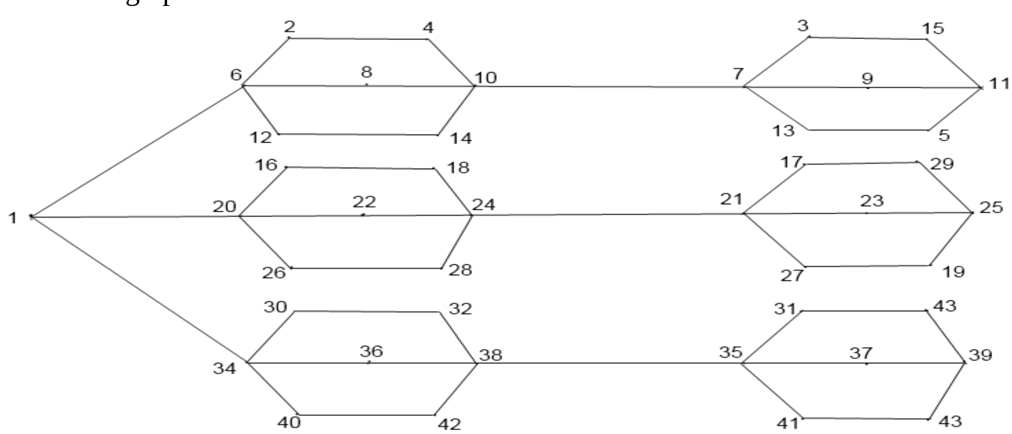

Figure 1:A graph obtained by one point for the path of T_α and its prime cordial labeling.

Theorem 2.2: $S(n \cdot T_\alpha)$ *is prime cordial graph, where n is even.*

Proof: Let *G* be a graph obtained by replacing each vertices of $K_{1,n}$ except the central vertex by the Theta graph T_α , where *n* is any positive integer, i.e. $G = S(n \cdot T_\alpha)$. Let u_0 be the central vertex of the graph *G* .i.e. u_0 is apex vertex of the orginal graph $K_{1,n}$.

Let u_i^k be the i^{th} vertices in the k^{th} copy of T_α , where $1 \le i \le 7$ and $1 \le k \le n$. Now we shall join each j^{th} vertex of T_α to the apex vertex u_0 , where *j* is any fixed number between 1 and 7. Then $|V(G)| = 7n+1$ and $|E(G)| = 9n$.

We shall define labeling function $f : |V(G)| \to \{1,2,...,7n+1\}$ as follows:

Let $f(u_0)=1$.

$$f(u_i^k) = \begin{cases} 2i + 14(k-1);\ 1 \le i \le 7,\ k = 1,2,..., \dfrac{n}{2} \\ 2i + 14\left(k - \dfrac{n}{2}\right) - 13, 1 \le i \le 7,\ k = \dfrac{n}{2} + 1, ..., n. \end{cases}$$

In view of the labeling pattern defined above we have $e_f(0) = e_f(1)$.

Thus we have $|e_f(0) - e_f(1)| \le 1$.

Hence the above labeling pattern gives prime cordial labeling to the grpah *G* and so it is a prime cordial graph.

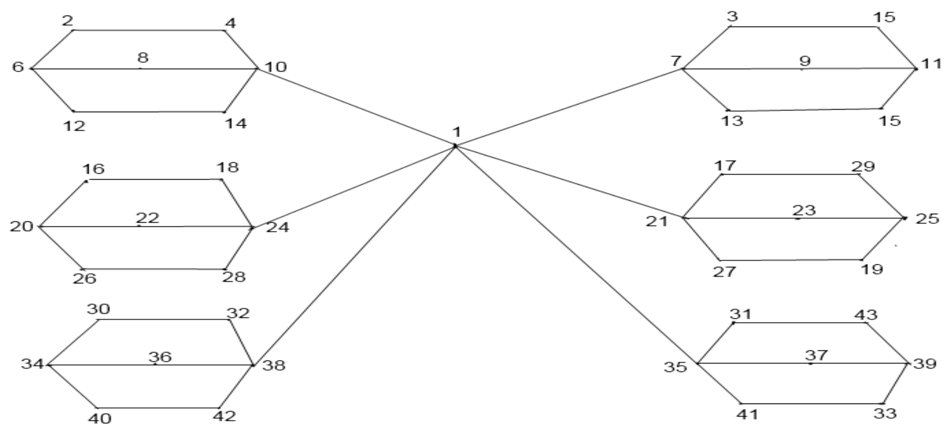

Figure 2: A open star of 6 copies of T_α and its prime cordial labeling.

Theorem 2.3. The path union of theta graph in a path P_n is prime cordial, where n is a positive even integer.

Proof: Let G be the graph obtained by joining n even copies of Theta graph by a path P_n .
We denote v_j^k be the of j[th] vertex in the k[th] copy, where $1 \le j \le 7$; $1 \le k \le n$.
Then $|V(G)| = 7n$ and $|E(G)| = 9n - 1$.
To define $f : |V(G)| \to \{1,2,...,7n\}$ as follows:
Let $f(u_0)=1$.

$f(v_j^k) = 2j + 7(k-1);\ j = 1,2,...,7;\ k = 1,3,5,...,n-1$

$f(v_j^k) = (2j-1) + 7(k-2);\ j = 1,2,...,7;\ k = 2,4,6,...,n.$

In view of the labeling pattern defined above we have $e_f(0) = e_f(1)$.
Thus we have $|e_f(0) - e_f(1)| \le 1$.
Hence the above labeling pattern gives prime cordial labeling to the grpah G and so it is a prime cordial graph.

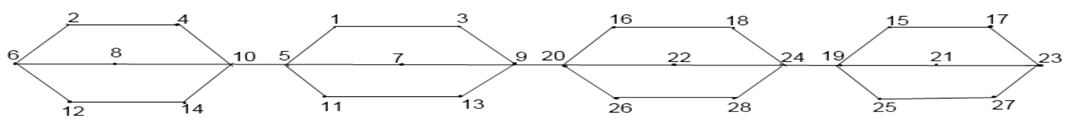

Figure 3: Prime cordial labeling of graph obtained by joining 4 copies of T_α by path

3. CONCLUDING REMARKS

In this paper, we proved that $P_n^t(tn \cdot .T_\alpha), S(t \cdot T_\alpha)$ and path union of even copies of Theta graph admits prime cordial labeling. The result obtained here are new and of very general nature. This work contributes three results to the families of prime cordial labeling. The labeling pattern is demonstrated by means of illustrations.

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