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Sum Divisor Cordial Labeling on Some Classes of Graphs

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ABSTRACT

A sum divisor cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f:V(G)\rightarrow \{1,2,...,|V(G)|\}$ such that each edge uv assigned the label 1 if 2 divides $f(u)+f(v)$ and 0 otherwise. Further, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which satisfying sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that the graphs such as Drums $D_n(n\geq 3)$, Twig T_m, Fire crackers $P_n \square S_n$ (n is even) and Double arrow DA_m^n (n is even), $|m-n|\leq 1$ are sum divisor cordial graphs.

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Keywords: Divisor Cordial, Sum divisor cordial labeling, Drums graph.

1. INTRODUCTION

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory, we refer to Harary[2]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers.

Varatharajan *et al*.[11] introduced the concept of divisor cordial labeling. For dynamic survey of various graph labeling, we refer to Gallian.[1] Lourdusamy and Patrick[4] introduced the concept of sum divisor cordial labeling. Sugumaran and Rajesh[5] have proved that the Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of

Swastik graph Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph are sum divisor cordial graphs. Sugumaran and Rajesh[6] have proved that the Theta graph and some graph operations in Theta graph are sum divisor cordial graphs. Sugumaran and Rajesh[7] proved that the Herschel graph and some graph operations in Herschel graph are sum divisor cordial graphs. Sugumaran and Rajesh[8] proved that H_n (n is odd), $C_3 @ K_{1,n}$, $< F_n^1 \Delta F_n^2 >$, open star of Swastik graph $S(t.Sw_n)$, when t is odd are sum divisor cordial graphs. Sugumaran and Rajesh[9] proved that some graph operations related to H- graph are sum divisor cordial graphs. Sugumaran and Rajesh[10] have proved that plus graph, umbrella graph, path union of odd cycles, kite and complete binary tree are sum divisor cordial graphs. In this paper we investigate sum divisor cordial labeling concept for Drums $D_n (n \geq 3)$, Twig T_m, Fire crackers P_n □ S_n (n is even) and Double arrow DA_m^n (n is even), $|m-n| \leq 1$.

Definition 1.1: If the vertices are assigned values subject to certain conditions then it is known as *graph labeling*.

Definition 1.2: A mapping $f : V(G) \to 0,1$ is called the *binary vertex* labeling of G and $f(v)$ is called the label of the vertex v of G under f.

Notation. If for an edge $e = uv$, the induced edge labeling $f^* : E(G) \to 0,1$ is given by $f^*(e) = |f(u) - f(v)|$, then we denote

$v_f(i) =$ number of vertices of G having label i under f,

$e_f(i) =$ number of edges of G having label i under f^*, where $i = 0,1$.

Definition 1.3: A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits a cordial labeling.

Definition 1.4: Let $G = (V(G), E(G))$ be a simple graph and let $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $2 \mid (f(u) + f(v))$ and the label 0 otherwise. The function f is called *a sum divisor cordial labeling* if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 1.5: The *drums* graph $D_n (n \geq 3)$ is obtained by joining two cycles and two paths of same length, sharing a common vertex. i.e., $D_n = 2C_n + 2P_n$

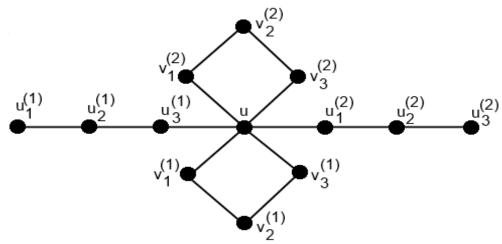

Figure 1: Drums graph D_4

Definition 1.6: A graph G is obtained from a path P_{n+2} by attaching exactly two pendant edges to each internal vertices of the path (except the terminal vertices) is called a twig. A twig T_n contains n internal vertices and it has $3n+2$ vertices , $3n+1$ edges.

Definition 1.7: Fire crackers graph is a graph obtained by attaching a star graph S_m at each pendant vertices of the path graph P_n, Such a fire cracker graph is denoted as $P_n \square S_m$. Note that the fire cracker graph $P_n \square S_m$ is of order $n+2m$ and of size $n+2m-1$

Definition 1.8: Consider a *grid* graph $P_m \times P_n$ which contains mn vertices. The first row vertices $V_{1,1}, V_{1,2}, ..., V_{1,n}$ and the last row vertices $V_{m,1}, V_{m,2}, ..., V_{m,n}$ are called the initial and final row vertices of the grid $P_m \times P_n$ respectively.

Definition 1.9: A *double arrow* graph DA_m^n with width n and length m is obtained from the grid graph $P_m \times P_n$ by adding two new vertices u and v such that each of the initial row vertices of $P_m \times P_n$ are connected by an edge from u. Similarly each of the final row vertices of $P_m \times P_n$ are connected by an edge from v.

Note that DA_m^n contains $mn+2$ vertices and $n(2m+1)-m$ edges.

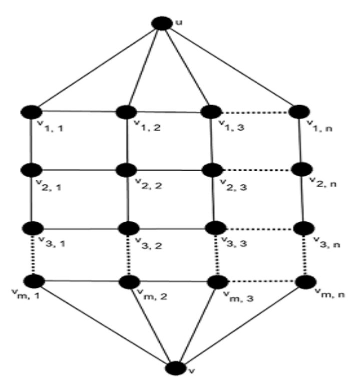

Figure 2: Double Arrow graph DA_m^n

2. MAIN RESULTS

Theorem 2.1: The drums graph $D_n(n \geq 3)$ is a sum divisor cordial labeling, where n is any positive integer.

Proof: Let $G = D_n$. Let $V(G) = \{u, u_i^j, v_i^j; 1 \leq i \leq n-1, 1 \leq j \leq 2\}$, where u is an apex vertex of G, u_i^j are the vertices of j^{th} copy of the path P_n and v_i^j are the vertices of j^{th} copy of cycle C_n, where $1 \leq i \leq n-1$, $1 \leq j \leq 2$. Here $|V(G)| = 4n-3$. For labeling of vertices of G, we follow the order as mentioned in Fig. 1.

We define a vertex labeling $f : V(G) \rightarrow 1, 2, ..., 4n-3$ into four distinguish cases.

Case 1: when $n \equiv 1 (\bmod 4)$

$f(u) = 2n-1,$

$f(u_i^{(1)}) = i$ for $i \equiv 1 (\bmod 4)$.

$f(u_i^{(1)}) = i+1$ for $i \equiv 2 (\bmod 4)$.

$f(u_i^{(1)}) = i-1$ for $i \equiv 3 (\bmod 4)$.

$f(u_i^{(1)}) = i$ for $i \equiv 0 (\bmod 4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 1 (\bmod 4)$.

$f(u_i^{(2)}) = n+i$ for $i \equiv 2 (\bmod 4)$.

$f(u_i^{(2)}) = n-2+i$ for $i \equiv 3 (\bmod 4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 0 (\bmod 4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 1 (\bmod 4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 2 (\bmod 4)$.

$f(v_i^{(1)}) = 2n+i$ for $i \equiv 3 (\bmod 4)$.

$f(v_i^{(1)}) = 2n-2+i$ for $i \equiv 0 (\bmod 4)$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 1 (\bmod 4)$.

$f(v_i^{(2)}) = 3n-1+i$ for $i \equiv 2 (\bmod 4)$.

$f(v_i^{(2)}) = 3n-3+i$ for $i \equiv 3 (\bmod 4)$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 0 (\bmod 4)$.

Case 2: when $n \equiv 2 (\bmod 4)$

$f(u) = 2n-1,$

$f(u_i^{(1)}) = i$ for $i \equiv 1 (\bmod 4)$.

$f(u_i^{(1)}) = i$ for $i \equiv 2(\text{mod}\,4)$.

$f(u_i^{(1)}) = i+1$ for $i \equiv 3(\text{mod}\,4)$.

$f(u_i^{(1)}) = i-1$ for $i \equiv 0(\text{mod}\,4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 1(\text{mod}\,4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 2(\text{mod}\,4)$.

$f(u_i^{(2)}) = n+i$ for $i \equiv 3(\text{mod}\,4)$.

$f(u_i^{(2)}) = n-2+i$ for $i \equiv 0(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 1(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n+i$ for $i \equiv 2(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n-2+i$ for $i \equiv 3(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 0(\text{mod}\,4)$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 1(\text{mod}\,4)$.

$f(v_i^{(2)}) = 3n-1+i$ for $i \equiv 2(\text{mod}\,4)$.

$f(v_i^{(2)}) = 3n-3+i$ for $i \equiv 3(\text{mod}\,4)$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 0(\text{mod}\,4)$.

Case 3: when $n \equiv 3(\text{mod}\,4)$

$f(u) = 2n-1,$

$f(u_i^{(1)}) = i$ for $i \equiv 1(\text{mod}\,4)$.

$f(u_i^{(1)}) = i+1$ for $i \equiv 2(\text{mod}\,4)$.

$f(u_i^{(1)}) = i-1$ for $i \equiv 3(\text{mod}\,4)$.

$f(u_i^{(1)}) = i$ for $i \equiv 0(\text{mod}\,4)$.

$f(u_i^{(2)}) = n-2+i$ for $i \equiv 1(\text{mod}\,4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 2(\text{mod}\,4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 3(\text{mod}\,4)$.

$f(u_i^{(2)}) = n+i$ for $i \equiv 0(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n+i$ for $i \equiv 1(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n-2+i$ for $i \equiv 2(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 3(\text{mod}\,4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 0(\text{mod}\, 4)$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 1(\text{mod}\, 4)$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 2(\text{mod}\, 4)$.

$f(v_i^{(2)}) = 3n-1+i$ for $i \equiv 3(\text{mod}\, 4)$.

$f(v_i^{(2)}) = 3n-3+i$ for $i \equiv 0(\text{mod}\, 4)$.

Case 4: when $n \equiv 0(\text{mod}\, 4)$

$f(u) = 2n-1,$

$f(u_i^{(1)}) = i$ for $i \equiv 1(\text{mod}\, 4)$.

$f(u_i^{(1)}) = i$ for $i \equiv 2(\text{mod}\, 4)$.

$f(u_i^{(1)}) = i+1$ for $i \equiv 3(\text{mod}\, 4)$.

$f(u_i^{(1)}) = i-1$ for $i \equiv 0(\text{mod}\, 4)$.

$f(u_i^{(2)}) = n-2+i$ for $i \equiv 1(\text{mod}\, 4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 2(\text{mod}\, 4)$.

$f(u_i^{(2)}) = n-1+i$ for $i \equiv 3(\text{mod}\, 4)$.

$f(u_i^{(2)}) = n+i$ for $i \equiv 0(\text{mod}\, 4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 1(\text{mod}\, 4)$.

$f(v_i^{(1)}) = 2n+i$ for $i \equiv 2(\text{mod}\, 4)$.

$f(v_i^{(1)}) = 2n-2+i$ for $i \equiv 3(\text{mod}\, 4)$.

$f(v_i^{(1)}) = 2n-1+i$ for $i \equiv 0(\text{mod}\, 4)$.

$f(v_{n-1}^{(2)}) = 4n-3$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 1(\text{mod}\, 4)$.

$f(v_i^{(2)}) = 3n-2+i$ for $i \equiv 2(\text{mod}\, 4)$.

$f(v_i^{(2)}) = 3n-1+i$ for $i \equiv 3(\text{mod}\, 4)$ and $i \neq n-1$.

$f(v_i^{(2)}) = 3n-3+i$ for $i \equiv 0(\text{mod}\, 4)$.

From all the above cases, we obtain $e_f(0) = e_f(1) = \frac{|E(G)|}{2} = 2n-1$.

Therefore $|e_f(0) - e_f(1)| \leq 1$.

Thus G is a sum divisor cordial graph.

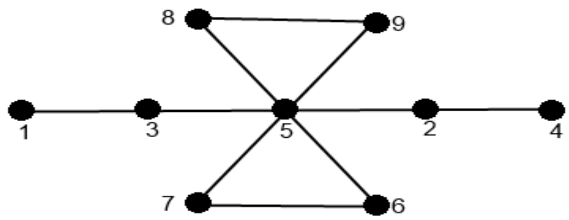

Figure 3: Sum divisor cordial labeling of Drums graph D_3

Theorem 2.2: Twig T_n is a sum divisor cordial labeling for all n.

Proof: Let G be a twig graph T_n. By the definition of twig, the order and size of G are $3n+2$ and $3n+1$ respectively. Let $V(G)= u_i, u_j : 1\le i\le n, 1\le j\le 2n$ where u_i are the vertices of path P_n and v_j are the pendant vertices attached to each internal vertex of path P_{n+2}. We define a vertex labeling $f: V(G)\to 1,2,...,3n+2$ as follows:

Case 1: when n is odd

$f(u_i)=i$ for $i=1(\bmod 4)$.

$f(u_i)=i+1$ for $i=2(\bmod 4)$.

$f(u_i)=i-1$ for $i=3(\bmod 4)$.

$f(u_i)=i$ for $i=0(\bmod 4)$.

$f(v_j)=n+2+j$ for $1\le j\le 2n$.

Case 2: when n is even

$f(u_i)=i$ for $i=1(\bmod 4)$.

$f(u_i)=i$ for $i=2(\bmod 4)$.

$f(u_i)=i+1$ for $i=3(\bmod 4)$.

$f(u_i)=i-1$ for $i=0(\bmod 4)$.

$f(v_j)=n+2+j$ for $1\le j\le 2n$.

From the above two cases, we obtain

$e_f(0)=e_f(1)=\frac{3n+1}{2}$, when n is odd.

$e_f(0)=\frac{3n}{2}+1$, $e_f(1)=\frac{3n}{2}$, when n is even.

Thus $|e_f(0)-e_f(1)|\le 1$.

Hence G is a sum divisor cordial graph.

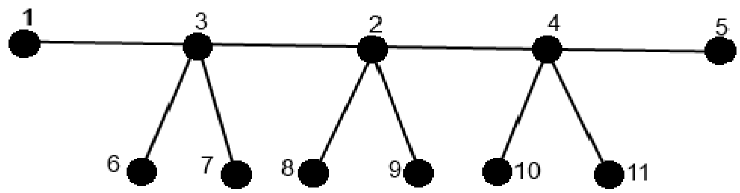

Figure 4: Sum divisor cordial labeling of Twig graph T_3

Theorem 2.3: All fire crackers graph $P_n \square S_n$ is a sum divisor cordial labeling, where n is even.

Proof: Consider $G = P_n \square S_n$. Let $V(G) = \{u_i, v_j : 1 \le i \le n, 1 \le j \le 2n\}$ where u_i are the vertices of path P_n and v_j are the vertices of star S_n. Then G has $3n$ vertices and $3n-1$ edges. We define a vertex labeling $f : V(G) \to 1, 2, ..., 3n$ as follows:

$f(u_i) = i$ for $i = 1(\bmod 4)$.

$f(u_i) = i$ for $i = 2(\bmod 4)$.

$f(u_i) = i+1$ for $i = 3(\bmod 4)$.

$f(u_i) = i-1$ for $i = 0(\bmod 4)$.

$f(v_j) = n + j$ for $1 \le j \le 2n$.

From the above labeling pattern, we have

$e_f(0) = \frac{3n}{2}$ and $e_f(1) = \frac{3n}{2} - 1$.

Thus $|e_f(0) - e_f(1)| \le 1$.

Hence G is a sum divisor cordial graph.

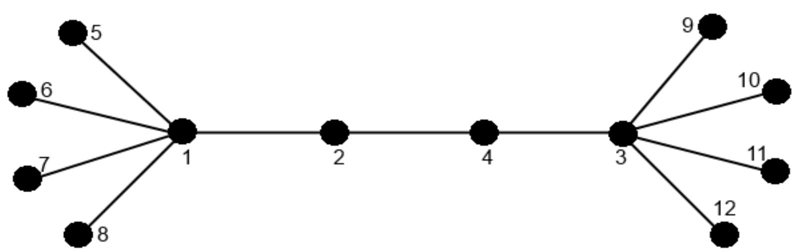

Figure 5: Sum divisor cordial labeling of fire crackers $P_4 \square S_4$

Theorem 2.4: The double arrow graph DA_m^n is a sum divisor cordial labeling, where n is an even integer and $|m-n| \le 1$.

Proof: Let G be a double arrow graph DA_m^n. Let $V(G)=\{u,u_i,v_{i,j}:1\le i\le m,1\le j\le n\}$. Then $|V(G)|=mn+2$ and $|E(G)|=n(2m+1)-m$. For the labeling of vertices of G, we follow the order as mentioned in Fig. 2.

We define a vertex labeling $f:V(G)\to 1,2,...,mn+2$ as follows:

$f(u)=1$,

$f(v)=mn+2$,

$f(v_{i,j})=k(n-1)+i+j$ for $1\le i\le m$, $1\le j\le n$, $k=i-1$.

We observe that each row edge label are 0, since addition of two consecutive integers is always odd. Similarily, each column edges labels are 1, since the column vertices are all odd or even alternatively. The edges connected from u and v are alternatively takes the value 0 and 1.

From the above labeling pattern, we obtain

$e_f(0)=e_f(1)=\dfrac{n(2m+1)-m}{2}$, when $n=m$.

$e_f(0)=\dfrac{n(2m+1)-m+1}{2}$, $e_f(1)=\dfrac{n(2m+1)-m-1}{2}$, if $n>m$.

$e_f(0)=\dfrac{n(2m+1)-m-1}{2}$, $e_f(1)=\dfrac{n(2m+1)-m+1}{2}$, if $n<m$.

Therefore we conclude that $|e_f(0)-e_f(1)|\le 1$.

Hence G is a sum divisor cordial graph.

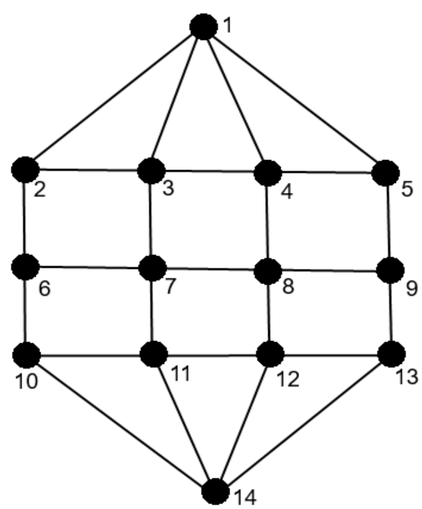

Figure 6: Sum divisor cordial labeling of Double Arrow DA_3^4

3. CONCLUSION

In this paper, we have proved that the graphs such as drums $D_n (n \geq 3)$, Twig T_m, Fire crackers $P_n \odot S_n$ (n is even) and Double arrow DA_m^n (n is even), $|m-n| \leq 1$ are sum divisor cordial graphs. It is interesting open area of research to identify new class of graphs which admit sum divisor cordial labeling.

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