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Further Results on Sum Divisor Cordial Labeling

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ABSTRACT

A sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1,2,...,|V(G)|\}$ such that each edge uv assigned the label 1 if 2 divides $f(u)+f(v)$ and 0 otherwise. Further, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that H_n (n is odd), $C_3 @ K_{1,n}$, $< F_n^1 \Delta F_n^2 >$, open star of Swastik graphs $S(t.Sw_n)$ when t is odd, are sum divisor cordial graphs.

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Keywords: Divisor cordial labeling, sum divisor cordial labeling.

1. INTRODUCTION

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory, we refer to Harary[2]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc.

Varatharajan *et al*.[9] introduced the concept of divisor cordial labeling. For dynamic survey of various graph labeling, we refer to Gallian[1]. Lourdusamy and Patrick[4] introduced the concept of sum divisor cordial labeling. Sugumaran and Rajesh[6] proved that Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph

Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph are sum divisor cordial graphs. Sugumaran and Rajesh[7] proved that the Theta graph and some graph operations in Theta graph are sum divisor cordial graphs. Sugumaran and Rajesh[8] proved that the Herschel graph and some graph operations in Herschel graph are sum divisor cordial graphs. In this paper we investigate the sum divisor cordial labeling on the graphs such as H_n (n is odd), $C_3 @ K_{1,n}$, $< F_n^1 \Delta F_n^2 >$, open star of Swastik graph $S(t.Sw_n)$, when t is odd.

Definition 1.1:[9] Let $G = (V(G), E(G))$ be a simple graph and let $f : V(G) \rightarrow \{1, 2, ..., |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u) | f(v)$ or $f(v) | f(v)$ and the label 0 otherwise. The function f is called a *divisor cordial labeling* if $|e_f(0) - e_f(1)| \le 1$. A graph which admits a divisor cordial labeling is called a *divisor cordial graph*.

Definition 1.2:[4] Let $G = (V(G), E(G))$ be a simple graph and let $f : V(G) \rightarrow \{1, 2, ..., |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $2 | (f(u) + f(v))$ and the label 0 otherwise. The function f is called a *sum divisor cordial labeling* if $|e_f(0) - e_f(1)| \le 1$. A graph which admits a sum divisor cordial labeling is called a *sum divisor cordial graph*.

Definition 1.4: The $H - graph$ is a graph obtained from two copies of the path P_n with vertices $\{u_1, u_2, ..., u_n\}$ and $\{v_1, v_2, ..., v_n\}$ by joining the vertices $u_{\frac{(n+1)}{2}}$ and $v_{\frac{(n+1)}{2}}$ if n is odd, otherwise $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even.

Definition 1.5: The *crown* graph $C_n @ K_{1,n}$ is the graph obtained by taking n copies of $K_{1,n}$ and each i^{th} vertex of C_n is attached with the apex vertex of $K_{1,n}$, for $i = 1, 2, ..., n$.

Definition 1.6:[3] A graph obtained by replacing each vertex of $K_{1,n}$ except the apex vertex by the graphs $G_1, G_2, ..., G_n$ is known as open star of graphs. We shall denote it by $S(G_1, G_2, ..., G_n)$. Suppose that $G_1 = G_2 = \cdots = G_n = G$, such *open star* of graph is denoted by $S(n.G)$.

Definition 1.7: Let G_1 and G_2 be any two graphs having apex vertices. The graph $G = < G_1 \Delta G_2 >$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex v.

2. MAIN RESULTS

Theorem 2.1: The graph H_n is a sum divisor cordial graph, when n is odd.

Proof: Let $G = H_n$. Let $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and

$E(G) = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : i = \frac{(n+1)}{2}\}$. Then G has $2n$ vertices and $2n-1$ edges. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(u_i) = 2i-1,\ i = 1, 2, ..., \frac{(n+1)}{2},$

$f(u_i) = n+i,\ i = \frac{(n+1)}{2} + 1, ..., n,$

$f(v_i) = 2i,\ i = 1, 2, ..., \frac{(n+1)}{2},$

$f(v_i) = i + \frac{(n+1)}{2},\ i = \frac{(n+1)}{2} + 1, ..., n.$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \le 1$.

Hence G is a sum divisor cordial graph.

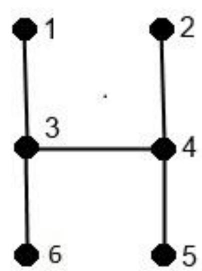

Figure 1: Sum divisor cordial labeling of graph H_3

Theorem 2.2: The crown graph $C_3 @ K_{1,n}$, $n \ge 1$ is a sum divisor cordial graph.

Proof: Let $G = C_3 @ K_{1,n}$. Let $V(G) = \{u, v, w\} \cup \{u_i, v_i, w_i : 1 \le i \le n\}$, where $\{u, v, w\}$ be the vertex set of C_3 and $\{u_i, v_i, w_i : 1 \le i \le n\}$ be the vertex set of $K_{1,n}$. Then G has $3n+3$ vertices and $3n+3$ edges. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(u) = 1,\ \ f(v) = 3,\ \ f(w) = 2,$

$f(u_i) = i+3,\ 1 \le i \le n,$

$f(v_i) = n+3+i,\ 1 \le i \le n,$

$f(w_i)=2n+3+i,\ 1\le i\le n.$

From the above labeling pattern, we have $|e_f(0)-e_f(1)|\le 1$.

Hence G is a sum divisor cordial graph.

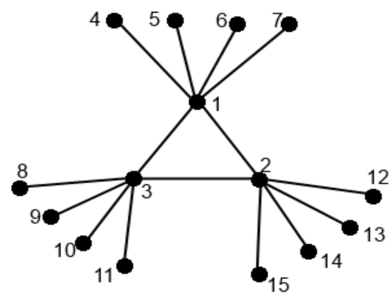

Figure 2: Sum divisor cordial labeling of $C_3 @ K_{1,4}$

Theorem 2.3: The graph $< F_n^1 \Delta F_n^2 >$ is a sum divisor cordial graph, when n is odd.

Proof: Let $p_1, p_2, ..., p_n$ be the vertices of F_n^1 and let $q_1, q_2, ..., q_n$ be the vertices of F_n^2. Let p and q be the apex vertices of F_n^1 and F_n^2 respectively. Let $G = < F_n^1 \Delta F_n^2 >$. Then G is of order $2n+3$ and its size is $4n+1$. In G, let v be a new vertex adjacent to both p and q. we define the vertex labeling $f: V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v)=1,\ f(p)=2,\ f(q)=3,$

$f(p_i)=i+3;\ i=1,2,...,n,$

$$f(q_i)=n+3+2i-1;\ i=1,2,...,\frac{(n+1)}{2},$$

$$f(q_i)=2(i+4)-6;\ i=\frac{(n+1)}{2}+1,...,n.$$

From the above labeling pattern, we have $|e_f(0)-e_f(1)|\le 1$.

Hence G is a sum divisor cordial graph.

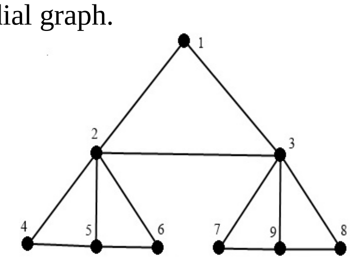

Figure 3: Sum divisor cordial labeling of $< F_3^1 \Delta F_3^2 >$

Theorem 2.4: Open star of Swastik graph $S(t.Sw_n)$ is a sum divisor cordial graph, where t is odd.

Proof: Let $G = S(t.Sw_n)$ be a graph obtained by replacing each vertices of $K_{1,t}$ except the apex vertex by the Swastik graph Sw_n (t is odd). Let v be the apex vertex of $K_{1,t}$. Then v is a central vertex of the graph G. We denote r^{th} copy of Sw_n by $Sw_n{}^r$ $(1 \le r \le t)$. Let $V(Sw_n{}^r) = \{v_{i,j}^r : 1 \le i \le 4, 1 \le j \le 4n\}$ where $1 \le r \le t$. Note that each copy of Sw_n has $p = 16n-4$ vertices and $q = 16n$ edges. Join the vertex $v_{4,2n+1}^r$ with the vertex v by an edge to form the open star of graph G for $r = 1, 2, ..., t$. Finally, we observe that G has $t(16n-4)+1$ vertices and $t(1+16n)$ edges. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v) = t(16n-4)+1,$

$f(v_{1,1}^r) = (r-1)p+1 = f(v_{4,4n}^r),$

$f(v_{2,1}^r) = (r-1)p+2 = f(v_{1,4n}^r),$

$f(v_{3,1}^r) = (r-1)p+4 = f(v_{2,4n}^r),$

$f(v_{4,1}^r) = (r-1)p+3 = f(v_{3,4n}^r),$

$f(v_{1,j}^r) = (r-1)p+1+j+2, \quad j = 2, 3, ..., 4n-1,$

$f(v_{2,j}^r) = (r-1)p+4n+1+j, \quad j = 2, 3, ..., 4n-1,$

$f(v_{3,j}^r) = (r-1)p+2(4n+j)-2, \quad j = 2, 3, ..., 2n,$

$f(v_{3,j}^r) = (r-1)p+2(2n+j)+1, \quad j = 2n+1, ..., 4n-1.$

The vertices of $v_{4,j}^r$ $(1 \le r \le t; 2 \le j \le 4n-1)$ are labeled from the following two cases:

Case 1. When $r \equiv 1 (\bmod 4)$ (or) $r \equiv 2 (\bmod 4)$.

$f(v_{4,j}^r) = (r-1)p+2(6n+j)-4, \quad j = 2, 3, ..., 2n,$

$f(v_{4,j}^r) = (r-1)p+2(4n+j)-3, \quad j = 2n+1, ..., 4n-1.$

Case 2. When $r \equiv 0 (\bmod 4)$ (or) $r \equiv 3 (\bmod 4)$.

$f(v_{4,j}^r) = (r-1)p+2(4n+j)+3, \quad j = 2, 3, ..., 2n,$

$f(v_{4,j}^r) = (r-1)p+2(3n+j)+2, \quad j = 2n+1, ..., 4n-1.$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \le 1$.

Hence G is a sum divisor cordial graph.

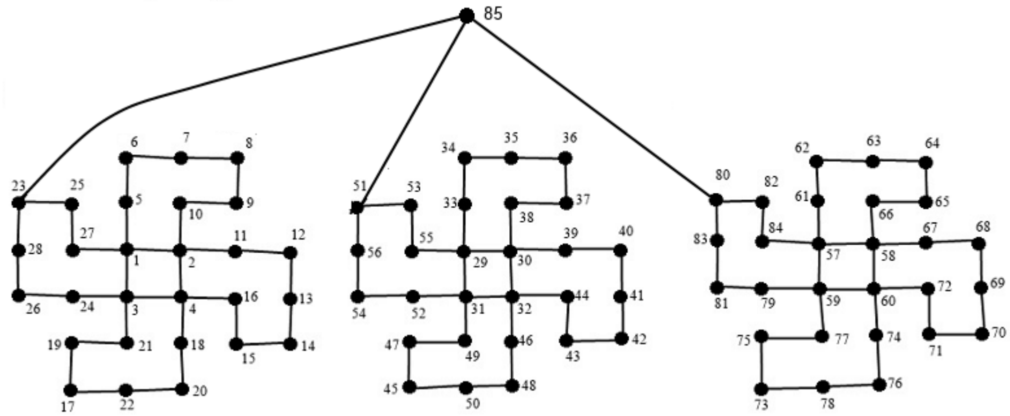

Figure 4: Sum divisor cordial labeling of $S(3.Sw_2)$

3. CONCLUSION

In this paper, we have proved that the graphs H_n (n is odd), $C_3 @ K_{1,n}$, $< F_n^1 \Delta F_n^2 >$, open star of swastik graph $S(t.Sw_n)$, where t is odd, are sum divisor cordial graphs.

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