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Groupoid Action on Anti-Fuzzy Lattice Ordered M-Group

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ABSTRACT

In this paper, the notion of anti-fuzzy lattice ordered m-group is introduced and investigated some of its basic properties. Also the homomorphic image, pre-image of anti fuzzy lattice ordered m-groups, arbitrary family of anti fuzzy lattice ordered m-groups and anti fuzzy lattice ordered m-groups using T-norm are studied. The notion of sensible anti fuzzy lattice ordered m-groups in group is introduced, and some related properties of lattices are discussed.

Keywords: lattice ordered M-group, anti-fuzzy group, groupoid action on fuzzy group, and anti-fuzzy lattice group.

INTRODUCTION

The notion of fuzzy sets was introduced by L.A. Zadeh [1965]. Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. Rosenfield [1971] introduced the concept of fuzzy subgroup. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the roader frame work of fuzzy settings. Ajmal and Thomas [1994] initiated such types of study. It was latter independently established by them that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. Biswas [1990] introduced the concept of anti- fuzzy subgroups of groups. Palaniappan and Muthuraj [2004] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy groups. Satya Saibaba [2008] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of

L-fuzzy sub l- groups. Goguen [1967] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. Solairaju and Nagarajan [2009] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. Marudai and Rajendran [2010] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu *et. al*. [1994] introduced concept of fuzzy groups with operator. Subramanian et.al. [2012] extended the concept to m fuzzy groups with operator. In this paper, a new algebraic structure of an anti-fuzzy lattice ordered m-group is defined, and studied some related properties.

SECTION 2- PRELIMINARIES

Definition 2.1: Let (G, .) be a group, and M be a non-empty set. Them M acts on the group G if there exists a map $*$ from $M \times G \rightarrow G$ such that $m * (x*y) = (m*x) *y = (x*m) *y$ for all x, y in G, and m in M.

Definition 2.2: An anti lattice ordered group is a system $(G, ., \leq)$ if (i) (G, .) is a group; (ii) $(G, \leq)$ is a lattice; (iii) $x \leq y$ implies $axb \leq ayb$ (compatibility) for a, b, x, y in G.

Definition 2.3: Let A; $X \rightarrow [0, 1]$ be a fuzzy set & G is a lattice ordered group, $G \in PF(X)$. A amp A on G is an anti fuzzy lattice ordered group if (i) $A(xy) \leq A(x) \wedge A(y)$; (ii) $A(x\text{-}1) \geq A(x)$ for all x, y in G.

Definition 2.4: Let G be a group, M be any set If (i). $mx \in G$, and (ii). $m(xy) = (mx)y = x(my)$ for all x, y in G m in M. Then is an M group.

Definition 2.5: Let A; $X \rightarrow [0, 1]$ be a fuzzy set & G be M-group, A fuzzy set on $G \in PF(X)$ is an anti fuzzy M group if (i).$A(m(xy)) \leq \max \{ A(mx), A(my) \}$ and (ii). $A(mx^{-1}) \leq A(mx)$ for all x, y in G and m in M.

Definition 2.6: Let A; $X \rightarrow [0, 1]$ be a fuzzy set & G in PF(X), $M \subseteq X$. A map A on G is an anti fuzzy lattice ordered M-group acted by a groupoid K if (i). K acts on an M-group (G, .); (ii) K acts on anti-lattice ordered group $(G, ., \leq)$; (iii).$A((k * m(xy))) \leq \max \{ A((k * (mx)), A((k * (my)) \}$ and (iv). $A((k * (mx)^{-1}) \leq A((k * (mx))$, (v). $A((k * (mx)) \vee (k * (my)) \leq \max \{ A((k *(mx)), A((k * my))\}$, (v). $A((k*(mx) \wedge (k*(my)) \leq \max \{A((k * (mx)), A((k*(my))\}$ for all x, y in G, m in M and k in K.

SECTION 3 - PROPERTIES OF ANTI-FUZZY LATTICE ORDERED M-GROUP

Theorem 3.1: Let G and G′ be two anti fuzzy lattice ordered M-groups both acted by a groupoid K and $\theta: G \rightarrow G'$ be a M-homomorphism with respect to K defined by $\theta(m(x * k)) = m\, \theta(x * k)$. If K acts on an anti fuzzy lattice ordered M-group B of G′, then K acts on an anti fuzzy lattice ordered M-group $A = \theta^{-1}(B)$ of G.

Proof: Assume B is an anti fuzzy lattice ordered M-group of G′ acted by K. Let x, y in G.

(i). $A(m((k * (xy))) = \theta^{-1}(B)(m(k * (xy)))$
$= B(\theta(k * (m(xy))))$
$= B(m\,\theta(k * (xy)))$
$= B(m(\theta(k * x)\,\theta(k * y)))$
$\leq \max\{B(k * (m\,\theta(x))),\ B((k * m(\theta(y))\}$
$= \max\{\theta^{-1}(B)(k * (mx)),\ \theta^{-1}(B)((k * (my))\}$
$= \max\{A((k * (mx)),\ A(k * (my))\}$

(ii). $A((k * (mx)^{-1})) = \theta^{-1}(B)((k * (mx)^{-1}))$
$= B(\theta((k * (mx)^{-1}))$
$= B(m\,\theta(k * x^{-1}))$
$\leq B((k * (m\,\theta(x)))$
$= B(\theta(k * (mx)))$
$= \theta^{-1}(B)(k * (mx))$
$= A(k * (m x))$.

(iii). $A(m(k * x) \vee m(k * y)) = \theta^{-1}(B)(m(k * x) \vee m(k * y))$
$= B(\theta(m(k * x) \vee m(k * y)))$
$\leq B(\theta(k * (mx)) \vee \theta(k * my))$
$\leq \max\{B(\theta(k * (mx))), B(\theta((k * (my)))\}$
$\leq \max[\theta^{-1}(B)(k * (mx)),\ \theta^{-1}(B)((k * (my))\}$.

(iv). $A(m(k * x) \wedge m(k * y)) = \theta^{-1}(B)(m(k * x) \wedge m(k * y))$
$= B(\theta(m(k * x) \wedge m(k * y)))$
$\leq \max\{B(\theta(k * (mx))), B(\theta((k * (my)))\}$
$\leq \max[\theta^{-1}(B)(k * (mx)),\ \theta^{-1}(B)((k * (my))\}$.

Therefore K acts on an anti fuzzy lattice ordered M-group $A = \theta^{-1}(B)$ of G.

Theorem 3.2: Let G and G′ be two anti fuzzy lattice ordered M-groups both acted by a groupoid K and $\theta: G \rightarrow G'$ be an M-epimorphism under K. Assume B is a fuzzy set in G′. If K acts on an anti fuzzy lattice ordered M-group $\theta^{-1}(B)$ of G, then K acts on an anti fuzzy lattice ordered M group B of G′.

Proof: Let x, y in G′, and $m \in M$.

There exist an element $a, b \in G$ with $\theta(a) = x$ and $\theta(y) = b$. Let k in K.

(i). $B(m((k * (xy))) = B(m(k * \theta(a)\,\theta(b)))$
$= B(m(k * (\theta(ab))))$
$= B(\theta(m(k * (ab))))$
$= \theta^{-1}(B)(m(k * (ab)))$

$\leq$ max { θ^{-1}(B) (k $*$ (ma)), θ^{-1}(B)(k $*$ (mb)) }
= max { B (θ(k $*$ (ma)), B(θ(k $*$ (mb))) }
= max { B (k $*$ (mθ(a)), B ((k $*$ (m θ(b)) }
= max { B (k $*$ (mx)), B (k $*$ (my)) }.

(ii). B (k $*$ (mx)$^{-1}$)) = B (k $*$ (m (θ(a))$^{-1}$))
= B (k $*$ (m (θ(a^{-1})))
= B (θ (k $*$ (ma)$^{-1}$))
= θ^{-1}(B) (k $*$ (ma)$^{-1}$)
$\leq \theta^{-1}$(B) (k $*$ (ma))
= B(θ (k $*$ (ma)))
= B(k $*$ mθ(a))
= B(k $*$ mx).

(iii).B(m(k $*$x) $\vee$ m(k $*$ y)) = B (m (k $*$ (θ(a)) $\vee$ m(k $*$ θ(b))))
= B (θ (k $*$ (ma)) $\vee$ θ(k $*$ (mb)))
= B (θ (k $*$ (ma)) $\vee$ (k $*$ (mb)))
= $\theta^{-`1}$B ((k $*$ (ma)) $\vee$ (k $*$ (mb))
$\leq$ max { $\theta^{-`1}$B (k $*$ (ma)), $\theta^{-`1}$B (k $*$ (mb)) }
= max { B (θ (k $*$ (ma)), B (θ (k $*$ (mb)) }
= max { B (k $*$ m (θ(x))), B(k $*$ m(θ(b))) }
= max { B(k $*$ (mx)), B (k $*$ (my)) }.

(iii).B(m(k $*$x) $\wedge$ m(k $*$ y)) = B (m (k $*$ (θ(a)) $\wedge$ m(k $*$ θ(b))))
= B (θ (k $*$ (ma)) $\wedge$ θ(k $*$ (mb)))
= B (θ (k $*$ (ma)) $\wedge$ (k $*$ (mb)))
= $\theta^{-`1}$B ((k $*$ (ma)) $\wedge$ (k $*$ (mb))
$\leq$ max { $\theta^{-`1}$B (k $*$ (ma)), $\theta^{-`1}$B (k $*$ (mb)) }
= max { B (θ (k $*$ (ma)), B (θ (k $*$ (mb)) }
= max { B (k $*$ m (θ(x))), B(k $*$ m(θ(b))) }
= max { B(k $*$ (mx)), B (k $*$ (my)) }.

So K acts on an anti fuzzy lattice ordered M group B of G.

Theorem 3.3: If a groupoid K acts on an anti fuzzy lattice ordered M-group A_i of G in a family $\{A_i\}_{i\in I}$, then K acts on an anti fuzzy lattice ordered M-group B = $\cup$ A_i of G where B = $\cup A_i$ = { (x, $\vee$ A_i(x): x $\in$G}.

Proof: Let x, y $\in$ G, m in M, and k in K.

(i). B(k $*$ m(xy)) = ($\cup_{i\in I}$ A_i) (k $*$ m(xy))
= $\vee_{i\in I}$ [A_i (k $*$ m(xy))]
$\leq \vee_{i\in I}$ [A_i (k $*$ (mx)) $\vee$ A_i(k $*$ (my))]
= [$\vee_{i\in I}$ (A_i (k $*$ (mx)))] $\vee$ [$\vee_{i\in I}$ (A_i(k $*$ (my))]

$= [(\cup_{i\in I} A_i) (k * (mx))] \vee [(\cup_{i\in I} A_i) (k * (my))]$
$= B (k * (mx)) \vee B(k * (my)).$

(ii). $B(k * (mx)^{-1}) = (\cup_{i\in I} A_i) (k * (mx)^{-1})$
$= \vee_{i\in I} [A_i k * (mx)^{-1})]$
$\leq \vee_{i\in I} [A_i (k * (mx))]$
$= (\cup_{i\in I} A_i) (k * (mx))$
$= B(k * (mx)).$

(iii).$B(m(k *x) \vee m(k * y)) = (\cup_{i\in I} A_i) (m(k *x) \vee m(k * y))$
$= \vee_{i\in I} [A_i (m(k *x) \vee m(k * y)]$
$\leq \max \{\vee_{i\in I} [A_i (k * (mx)), A_i(k * (my)) \}$
$= \max \{\vee_{i\in I} (A_i (k * (mx)))] , [\vee_{i\in I} (A_i(k * (my)))] \}$
$= \max \{ [(\cup_{i\in I} A_i) (k * (mx))], [(\cup_{i\in I} A_i) (k * (my))] \}$
$= \max \{ B (k * (mx)), B(k * (my) \}.$
(iii).$B(m(k *x) \wedge m(k * y)) = (\cup_{i\in I} A_i) (m(k *x) \wedge m(k * y))$
$= \vee_{i\in I} [A_i (m(k *x) \wedge m(k * y)]$
$\leq \max \{\vee_{i\in I} [A_i (k * (mx)), A_i(k * (my)) \}$
$= \max \{\vee_{i\in I} (A_i (k * (mx)))] , [\vee_{i\in I} (A_i(k * (my)))] \}$
$= \max \{ [(\cup_{i\in I} A_i) (k * (mx))], [(\cup_{i\in I} A_i) (k * (my))] \}$
$= \max \{ B (k * (mx)), B(k * (my) \}.$
Thus $B = \cup_{i\in I} A_i$ is an anti fuzzy lattice ordered M-group of G.

Theorem 3.4: If A is a fuzzy set in G such that all non-empty level subset L(A: t) is an anti fuzzy lattice ordered M-group of G acted by a groupoid K, then A is an anti fuzzy lattice ordered M-group of G acted by K.

Proof: Let $x, y \in L(A: t)$, m in M, and k in K.
It gives that $A((k *(mx)) \leq t$; $A(k * (my)) \leq t$ so that $A(k * m(xy)) \leq t$.
(i). $A(k * m(xy)) \leq t = t \vee t \leq A(k * (mx)) \vee A(k * (my))$
(ii). $A(k * (mx)^{-1}) \leq t = A(k * (mx)).$
(iiii).$A(k * ((mx) \vee (my)) \leq t = \max \{t, t\}$
$\leq A(k * (mx)) \vee A(k * (my)).$
(iv). $A(k * ((mx \wedge my)) \leq t = \max \{t, t\}$
$\leq A(k * (mx)) \vee A(k * (my)).$
Therefore K acts on an anti fuzzy lattice ordered M-group A of G..

Theorem 3.5: Let a groupoid K act an anti-fuzzy lattice ordered M-group A of G having identity e. Let A^+ be a fuzzy set in G defined by$A^+(k * x) = A(k * x) + 1 – A(k * e)$ for all x in G, and k in K. Then K acts on an anti fuzzy lattice ordered M-group A^+ of G containing A.

Proof: Let $x, y \in G$, m in M, and k in K.

(i). $A^+(k * m(xy)) = A(k * m(xy)) + 1 – A(k * e)$
$\leq \max \{A (k * (mx)), A(k * (my)).\} + 1 – A(k * e)$
$= \max \{A(k * (mx)) + 1 – A(k * e), A(k * (my)) + 1 – A(e) \}$
$= A^+(k * (mx)) \vee A^+(k * (my))$.

(ii). $A^+(k * (mx)^{-1}) = A(k * (mx)^{-1}) + 1 – A(k * e)$
$\leq A(k * (mx)) + 1 – A(k * e)$
$= A^+(k * (mx))$.

(iii). $A^+ (m(k *x) \vee m(k * y)) = A(m(k *x) \vee m(k * y)) + 1 – A(k * e)$
$\leq \max \{ A (k * (mx)), A(k * (my)) \} + 1 – A(k * e)$
$= \max \{A (k * (mx)) + 1 – A(k * e), [A(k * (my)).] + 1 – A(k * e)]$
$= \max \{A^+(k * (mx)), A^+(k * (my) \}$.

(iv). $A^+ (m(k *x) \wedge m(k * y)) = A(m(k *x) \wedge m(k * y)) + 1 – A(k * e)$
$\leq \max \{ A (k * (mx)), A(k * (my)) \} + 1 – A(k * e)$
$= \max \{A (k * (mx)) + 1 – A(k * e), [A(k * (my)).] + 1 – A(k * e)]$
$= \max \{A^+(k * (mx)), A^+(k * (my) \}$.

Also $A(k * x) \leq A^+(k * x)$ for all $x \in G$, and k in K. Thus K acts on a fuzzy lattice ordered M-group A^+ of G containing A.

SECTION 4 – HOMOMORPHIC PROPERTIES ON ANTI-FUZZY LATTICE ORDERED M-GROUP

Theorem 4.1: If K acts on an anti fuzzy lattice ordered M-group A of G and θ is an M-homomorphism of G under K, then the fuzzy set $A^\theta = \{ <mx, A^\theta(mx)> : x \text{ in } G \}$ is an anti fuzzy lattice ordered M-group acted by K.

Proof: Let $x, y \in G$, m in M, and k in K.

(i). $A^\theta (k * m(xy)) = A (\theta(k * m(xy)))$
$= A (m\, \theta(k * (xy)))$
$= A (m (k * (\theta(x)\, \theta(y)))$
$\leq \max \{ A(k * m(\theta(x)), A(k * m (\theta(y)) \}$
$= \max \{ A(\theta(k * (mx)), A((\theta(k * (my)) \}$
$= \max \{ A^\theta (k * (mx)), A^\theta (k * (my)) \}$.

(ii). $A^\theta (k * (mx)^{-1}) = A (\theta(k * (mx)^{-1}))$
$= A (m\, \theta(k * x)-^1)$
$\leq A (k * m\theta(x))$
$= A (k * \theta(mx))$
$= A^\theta(k * (mx))$.

(iii).$A^\theta (m(k *x) \vee m(k * y)) = A (\theta(m(k *x) \vee m(k * y))))$

$= \max \{A (\theta(k * (mx)), \theta(k * (my)) \}$
$= \max \{ A(\theta(k * (mx)), \quad A(\theta(k * (my)) \}$
$= A^{\theta}(k * (mx)) \vee A^{\theta}(k * (my))$.

(iv).A^{θ} ($m(k *x) \wedge m(k * y)$) = $A (\theta(m(k *x) \wedge m(k * y)))$)
$= \max \{A (\theta(k * (mx)), \theta(k * (my)) \}$
$= \max \{ A(\theta(k * (mx)), \quad A(\theta(k * (my)) \}$
$= A^{\theta}(k * (mx)) \vee A^{\theta}(k * (my))$.
Therefore K acts on an anti fuzzy lattice ordered M-group A^{θ} of G.

Theorem 4.2: Let T be a continuous t-norm and f be an M-homomorphism on G under K.If a groupoid K acts an anti fuzzy lattice ordered M-group A on G, then K acts on an anti fuzzy lattice ordered M-group A^f of f(G).
Proof: Let $S_1 = f^{-1}(k * (my_1))$; $S_2 = f^{-1}(k * (my_2))$; $S_{12} = f^{-1} (k * (my_1y_2))$.
Consider $S_1S_2 = \{k * (mx) \in G: k * (mx) = k * (mx_1)$ (k mx_2 for all $mx_1 \in S_1$, and $mx_2 \in S_2\}$.
If $k * (mx) \in S_1S_2$, then $k * (mx) = (k * mx_1) (k * mx_2)$ and $f(k * (mx)) = f((k * mx_1) (k * mx_2)) = f(k * (mx_1)) f(k * (mx_2)) = (k * my_1) (k * (my_2) = k * [m(y_1y_2)]$.

Then $k * (mx) \in f^{-1}(k * (my_1y_2))$) implies that $k * (mx) \in S_{12}$, and so $S_1S_2 \subseteq S_{12}$.
(i). $A^f (k * (m(y_1y_2)) \quad = \sup \{ A(k * (mx)) : k * (mx) \in f^{-1}(k * (m(y_1y_2))) \}$
$= \sup \{ A(k * (mx)) : k * (mx) \in S_{12} \}$
$\leq \sup \{ A(k * (mx)): k * (mx) \in S_!S_2 \}$
$= \sup \{ A(k * (mx_1) (k * (mx_2)): k * (mx_1) \in S_1; \; k * mx_2) \in S_2\}$
$\leq \sup \{ T \{A(k * (mx_1)), \; A(k * (mx_2)) \} : k * (mx_1) \in S_1; k * (mx_2) \in S_2\}$
$\leq T \{ \sup \{A(k * (mx_1)): k * (mx_1) \in S_!, \sup \{A(k * (mx_2)) : k * (mx_2) \in S_2 \}$
$\leq T\{\sup \{A(k * (mx_1)): k*(mx_1) \in f^{-1}(k * (my_1)), \; \sup \{A(k * (mx_2)): k * (mx_2) \in f^{-1}(k * (my_2))\}$
$\leq \; T \{A^f (k * (my_1)), \; A^f(k * (my_2)) \}$.

(ii). $A^f (k * (my)^{-1}) = \sup \{ A(k * (mx)^{-1}) : k * (mx)^{-1} \in f^{-1}(k * (my)^{-1}) \}$
$= \sup \{ A(k * (mx)^{-1}) : k * (mx) \in f^{-1}(k * (my)) \}$
$\leq \sup \{ A(k * (mx)) : k * (mx) \in f^{-1}(k * (my)) \}$
$= \; A^f (k * (my))$.

(iii). $A^f (k * (my_1) \vee (k * (my_2)) = \sup \{ A(k * (mx)): k * (mx) \in f^{-1} (k * (my_1) \vee (k * (my_2)) \}$
$= \sup \{ A(k * (mx)): k * (mx) \in S_{12}\}$
$\leq \sup \{ A(k * (mx)): k * (mx) \in S_1 \vee S_2\}$
$\leq \sup \{ A(k * (mx_1) \vee (k * (mx_2)): k * (mx_1) \in S_1 , k * (mx_2) \in S_2 \}$
$\leq \sup \{ T \{A(k * (mx_1), A(k * (mx_2)) \} : k * (mx_1) \in S_1; \; k * (mx_2) \in S_2\}$
$\leq T \{ \sup \{A(k * (mx_1)): k*(mx_1) \in S_!, \sup \{A(k * (mx_2)): k * (mx_2) \in S_2 \}$

$\leq T\{\sup \{A(k * (mx_1)): k * (mx_1) \in f^{-1}(k * (my_1)), \; \sup \{A(k * (mx_2)): k * (mx_2) \in f^{-1}(k * (my_2)\}\}$
$\leq \; T \{A^f (k * (my_1)), \; A^f(k * (my_2)) \}$.

(iv). $A^f (k * (my_1) \wedge (k * (my_2)) = \sup \{ A(k * (mx)): k * (mx) \in f^{-1} (k * (my_1) \wedge (k * (my_2)) \}$

$= \sup \{ A(k * (mx)): k * (mx) \in S_{12}\}$

$\leq \sup \{ A(k * (mx)): k * (mx) \in S_1 \wedge S_2\}$

$\leq \sup \{ A(k * (mx_1) \vee (k * (mx_2)): k * (mx_1) \in S_1 , k * (mx_2) \in S_2 \}$

$\leq \sup \{ T \{A(k * (mx_1), A(k * (mx_2)) \} : k * (mx_1) \in S_1; k * (mx_2) \in S_2\}$

$\leq T \{ \sup \{A(k * (mx_1)): k*(mx_1) \in S_!, \sup \{A(k * (mx_2)): k * (mx_2) \in S_2 \}$

$\leq T\{\sup \{A(k * (mx_1)): k * (mx_1) \in f^{-1}(k * (my_1)), \sup \{A(k * (mx_2)): k * (mx_2) \in f^{-1}(k * (my_2))\}$

$\leq T \{A^f (k * (my_1)), A^f(k * (my_2)) \}$.

Therefore K acts on an anti fuzzy lattice ordered M-group A^f of f(G).

Theorem 4.3: Let T be a t-norm. Then every sensible anti fuzzy lattice ordered M-group acted a groupoid K is an anti fuzzy lattice ordered M-group of G acted by K.

Proof: Given that K acts on sensible anti fuzzy lattice ordered M-group A of G.

Then $A (k * m(xy)). = T \{ A(k * (mx)), A(k * (my)) \}$

$\leq \max \{ T \{ A(k * (mx)), A(k * (my)) \}, T \{ A(k * (mx)), A(k * (my)) \} \}$

$= T\{\max \{A(k * (mx)), A(k * (my)) \}, \max \{A(k * (mx)), A(k * (my)) \} \}$

$= \max\{ A(k * (mx)), A(k * (my)) \}$

(ii). $A(k * (mx)^{-1}) \leq A(k * (mx))$.

(iii). $A(m(k *x) \vee m(k * y)) = \max\{ A(k * (mx)), A(k * (my)) \}$

$= T \{ \max \{ A(k * (mx)), A(k * (my)) \}, \max \{ A(k * (mx)), A(k * (my)) \} \}$

$\leq \max \{ T \{ A(k * (mx)), A(k * (my)) \}, T \{ A(k * (mx)), A(k * (my)) \} \}$

$= T \{ A(k * (mx)), A(k * (my)) \}$

(iv). $A(m(k *x) \wedge m(k * y)) = \min \{ A(k * (mx)), A(k * (my)) \}$

$= T \{ \min \{ A(k * (mx)), A(k * (my)) \}, \min \{ A(k * (mx)), A(k * (my)) \} \}$

$\leq \min \{ T \{ A(k * (mx)), A(k * (my)) \}, T \{ A(k * (mx)), A(k * (my)) \} \}$

$= T \{ A(k * (mx)), A(k * (my)) \}$

Therefore K acts on an anti fuzzy lattice ordered M-group A of G.

Theorem 4.4: An onto M-homomorphic image of an anti fuzzy lattice ordered M-group with **sup property** is an anti fuzzy lattice ordered M-group.

Proof: Let $f: G \to G'$ be a onto M-homomorphism of G and A be an anti fuzzy lattice ordered M-group of G with sup property. Let $k * (mx'), (k * (my') \in G'$.

Let $k * (mx_0) \in f^{-1}(k * (mx')); (k * (my_0) \in f^{-1}(k * (my'))$ with $A(k * (mx_0)) = \sup \{A(k * (mx)): (k * (mx)) \in f^{-1}(k * (mx')) \}$, and $A(k * (my_0)) = \sup \{A(k * (my)): k * (my) \in f^{-1}(k * (my')) \}$.

(i). $A^f (k * (m(x'y'))) = \sup \{ A(k * z): k * z \in f^{-1} (k * (m(x'y'))) \}$

$= \sup \{ A(k * z): k * z \in f^{-1} (k * (mx') .(k * my'))) \}$

$\leq \sup \{ A((k * (mx_0. my_0)): mx_0 \in f^{-1}(mx'); my_0 \in f^{-1}(my') \}$

$= \sup \{ A(k * (mx_0y_0)): k * (mx_0) \in f^{-1}(k * (mx')); k * (my_0) \in f^{-1}(k * (my')) \}$

$\leq \sup \{ \max \{A(k * (mx_0)), A(k * (my_0)) \}: k * (mx_0) \in f^{-1}(k * (mx')); k * (my_0) \in f^{-1}(k * (my')) \}$

$\leq \max \{\sup \{A(k*(mx_0)): k*(mx_0) \in f^{-1}(k*mx')) \}, \sup \{A(k*(my_0)): k*(my_0) \in f^{-1}(k*(my')) \}$

$= \max \{ A^f (k * (mx')), A^f (k * (my')) \}.$

(ii). $A^f (k * (mx')^{-1})) = \sup \{ A(k * (mx_0)^{-1}): (mx_0)^{-1} \in f^{-1} (k * (mx')^{-1}) \}$

$= \sup \{ A(k * (mx_0)^{-1}): k * (mx_0) \in f^{-1} (k * (mx')) \}$

$\leq \sup \{ A(k * (mx_0)): k * (mx_0) \in f^{-1} (k * (mx')) \}$

$= A^f (k * (mx')).$

(iii).$A^f (k * (mx')) \vee (k * (my'))) = \sup \{ A(k * z): k * z \in f^{-1}(k * (mx')) \vee (k * (my'))) \}$

$\leq \sup \{ A(k * z): k * z \in f^{-1} (k * (mx')) \vee f^{-1}(k * (my')) \}$

$\leq \sup \{ A(k * (mx_0)) \vee (k * (my_0)): k * (mx_0) \in f^{-1}(k * (mx')), k * (my_0) \in f^{-1}(k * (my'))$

$\leq \sup \{\max \{A(k*(mx_0)), A(k * (my_0)) \}: k * (mx_0) \in f^{-1}(k * (mx')); k * (my_0) \in f^{-1}(k * (my')) \}$

$\leq \max \{\sup \{A(k*(mx_0)): k*(mx_0) \in f^{-1}(k*(mx')) \}, \sup \{A(k*(my_0)): k*(my_0) \in f^{-1}(k*(my')) \}$

$= \max \{ A^f (k * (mx')), A^f (k * (my')) \}.$

(iv).$A^f (k * (mx')) \wedge (k * (my'))) = \sup \{ A(k * z): k * z \in f^{-1}(k * (mx')) \wedge (k * (my'))) \}$

$\leq \sup \{ A(k * z): k * z \in f^{-1} (k * (mx')) \wedge f^{-1}(k * (my')) \}$

$\leq \sup \{ A(k * (mx_0)) \wedge (k * (my_0)): k * (mx_0) \in f^{-1}(k * (mx')), k * (my_0) \in f^{-1}(k * (my'))$

$\leq \sup \{\max \{A(k*(mx_0)), A(k * (my_0)) \}: k * (mx_0) \in f^{-1}(k * (mx')); k * (my_0) \in f^{-1}(k * (my')) \}$

$\leq \max \{\sup \{A(k*(mx_0)): k*(mx_0) \in f^{-1}(k*(mx')) \}, \sup \{A(k*(my_0)): k*(my_0) \in f^{-1}(k*(my')) \}$

$= \max \{ A^f (k * (mx')), A^f (k * (my')) \}.$

Thus K acts on an anti fuzzy lattice ordered M-group A^f of G with sup property.

Theorem 4.5: Let f: G $\rightarrow$ G′ be a lattice group M-homomorphism under K and a groupoid K act an anti fuzzy lattice ordered M-group A of G′, then K acts an anti fuzzy lattice ordered M-group $f^{-1}(A)$ of G.

Proof: Let mx, my $\in$ G, and K acts on an anti fuzzy lattice ordered M-group A of G′.

(i). $f^{-1}(A) (k * (m(xy))) = A [f (k * (m(xy)))]$

$= A [f (k * ((mx). (my)))]$

$= A [f (k * (mx)) f(k * (my))]$

$= A [mf(k * x) mf(k * y)]$

$= A [m(f(k * x) f(k * y)]$

$\leq \max \{ A(k * (mf(x))), A(k * (mf(y))) \}$

$= \max \{ A(f(k * (mx))), A(f(k * (my))) \}$

$= \max \{ (f^{-1}A) (k * (mx)), (f^{-1}A) (k * (my)) \}.$

(ii). $f^{-1}(A) (k * (mx)^{-1}) = A [f (k * (mx)^{-1})]$

$= A [m f(k * x)^{-1})]$

$\leq$ A(k $*$ m f(x))
= A(f(k $*$ (mx)))
= f^{-1}(A) (k $*$ (mx))

(iii). f^{-1}(A) ((k $*$ (mx)) $\vee$ (k $*$ (my)))
= A [f ((k $*$ (mx)) $\vee$ (k $*$ (my)))]
= A [f (k $*$ (mx)) $\vee$ f(k $*$ (my))]
= A [mf(k $*$ x) $\vee$ mf(k $*$ y)]
$\leq$ max { A(k $*$ (mf(x))), A(k $*$ (mf(y))) }
= max { A(f(k $*$ (mx))), A(f(k $*$ (my))) }
= max { (f^{-1}A) (k $*$ (mx)), (f^{-1}A) (k $*$ (my)) }.

(iv). f^{-1}(A) ((k $*$ (mx)) $\wedge$ (k $*$ (my)))
= A [f ((k $*$ (mx)) $\wedge$ (k $*$ (my)))]
= A [f (k $*$ (mx)) $\wedge$ f(k $*$ (my))]
= A [mf(k $*$ x) $\wedge$ mf(k $*$ y)]
$\leq$ max { A(k $*$ (mf(x))), A(k $*$ (mf(y))) }
= max { A(f(k $*$ (mx))), A(f(k $*$ (my))) }
= max { (f^{-1}A) (k $*$ (mx)), (f^{-1}A) (k $*$ (my)) }.

Therefore K acts an anti fuzzy lattice ordered M-group f^{-1}(A) of G.

SECTION 5: DIRECT PRODUCT OF ANTI-FUZZY LATTICE ORDERED M-GROUP

Definition 5.1: Let A_i be an anti fuzzy lattice ordered M-group of G_i for i – 1, 2,2 …, n. Their product of Ai is the function $A_1 \times A_2 \times \ldots \times A_n : G_1 \times G_2 \times \ldots \times G_n \to L$ defined by $A_1 \times A_2 \times \ldots \times A_n$ (m ($x_1, x_2,\ldots, x_n$)) = max { $A_1(mx_1), A_2(mx_2),\ldots, A_n(mx_n)$}.

Theorem 5.2: The direct product of anti fuzzy lattice ordered M groups both acted by a groupois K is an anti fuzzy lattice ordered M-group acted by K.

Proof: Let x = ($x_1, x_2,\ldots, x_n$), y = ($y_1, y_2,\ldots, y_n$). be in $G_1 \times G_2 \times \ldots \times G_n$.
Assume that A = $A_1 \times A_2 \times \ldots \times A_n$.

(i). A (k $*$ (m(xy))) = $A_1 \times A_2 \times \ldots \times A_n$. (k $*$ (m ($x_1, x_2,\ldots, x_n$) ($y_1, y_2,\ldots, y_n$)))
= $A_1 \times A_2 \times \ldots \times A_n$. (k $*$ (m ($x_1y_1, x_2y_2,\ldots, x_ny_n$)))
= max { A_1(k $*$ (m(x_1y_1))), A_2(k $*$ (m(x_2y_2))), …, A_n(k $*$ (m(x_ny_n))) }
$\leq$ max {max{A_1(k$*$(mx_1)), A_1(k$*$(my_1))}, max {A_2(k$*$(mx_2)), A_2(k$*$my_2))}, …,
max {A_n(k$*$(mx_n)), A_n(k $*$ (my_n)) } }
= max { max {A_1(k $*$ (mx_1)), A_2(k $*$ (mx_2)) }, … , A_n(k $*$ (mx_n)) },
max {A_1(k $*$ (my_1)) }, A_2(k $*$ (my_2)),…, A_n(k $*$ (my_n)) } }
= max {$A_1 \times A_2 \times \ldots \times A_n$ (k$*$(m ($x_1, x_2,\ldots, x_n$))), $A_1 \times A_2 \times \ldots \times A_n$ (k$*$(m($y_1, y_2,\ldots, y_n$))) }
= max { $A_1 \times A_2 \times \ldots \times A_n$ (k $*$ (mx)), $A_1 \times A_2 \times \ldots \times A_n$ (k $*$ (my)) }
= max { A(k $*$ (mx)), A(k $*$ (my)) }.

(ii). A (k $*$ $(mx)^{-1}$) = $A_1 \times A_2 \times \ldots \times A_n$. (k $*$ $(m (x_1, x_2,\ldots, x_n))^{-1}$))
= $A_1 \times A_2 \times \ldots \times A_n$. (k $*$ (m (($x_1^{-1}, x_2^{-1},\ldots, x_n^{-1}$)))

$= \max \{ A_1(k * (mx_1)^{-1}), \; A_2(k * (mx_2)^{-1}), \ldots, A_n(k * (mx_n)^{-1}) \}$

$\leq \max \{ A_1(k * (mx_1)), \; A_2(k * mx_2)), \ldots, A_n(k * mx_n)) \}$

$= A_1 \times A_2 \times \ldots \times A_n. (k * (m ((x_1, x_2, \ldots, x_n)))$

$= A (k * (mx)).$

(iii). $A(m(k*x) \vee m (k*y)) = A_1 \times A_2 \times \ldots \times A_n. (m(k*(x_1, x_2,\ldots, x_n)) \vee m(k*(y_1, y_2,\ldots, y_n)))$

$= A_1 \times A_2 \times \ldots \times A_n. (m \; (k * (x_1 \vee y_1), k * (x_2 \vee y_2), \ldots, k * (x_n \vee y_n)))$

$= A_1\times A_2\times \ldots \times A_n. (\; (m(k * x_1) \vee (k * (my_1), m(k * x_2) \vee (k * (my_2) ,\ldots, m(k * x_n) \vee (k * (my_n))$

$= \max \{ A_1(m(k * x_1) \vee (k * (my_1), \; A_2 (m(k * x_2) \vee (k * (my_2), \ldots, A_n (m(k * x_n) \vee (k * (my_n) \}$

$\leq \max \{\max \{A_1(k*(mx_1)), A_1(k*(my_1))\}, \max \{A_2(k*(mx_2)), A_2(k*(my_2))\}, \ldots,$
$\max \{A_n(k*(mx_n)), A_n(k*(my_n)) \} \}$

$= \max \{ \max \{A_1(k*(mx_1)), A_2(k*(mx_2))\}, \ldots , A_n(k*(mx_n)) \} ,$
$\max \{A_1(k*(my_1))\}, \quad A_2(k*(my_2)),\ldots, A_n(k*my_n)) \} \}$

$= \max \{A_1 \times A_2 \times \ldots \times A_n \; (k * (m(x_1, x_2,\ldots, x_n))), \; A_1 \times A_2\times \ldots \times A_n \; (k*(m (y_1, y_2,\ldots, y_n)) \}$

$= \max \{ A_1 \times A_2 \times \ldots \times A_n \; (k *(mx)), \; A_1 \times A_2 \times \ldots \times A_n \; (k * (my)) \}$

$= \max \{ A(k * (mx)), A(k * (my)) \}.$

(iii). $A(m(k*x) \wedge m (k*y)) = A_1 \times A_2 \times \ldots \times A_n. (m(k*(x_1, x_2,\ldots, x_n)) \wedge m(k*(y_1, y_2,\ldots, y_n)))$

$= A_1 \times A_2 \times \ldots \times A_n. (m \; (k * (x_1 \wedge y_1), k * (x_2 \wedge y_2), \ldots, k * (x_n \wedge y_n)))$

$= A_1\times A_2\times \ldots \times A_n. (\; (m(k * x_1) \wedge (k * (my_1), m(k * x_2) \wedge (k * (my_2) ,\ldots, m(k * x_n) \wedge (k * (my_n))$

$= \max \{ A_1(m(k * x_1) \vee (k * (my_1), \; A_2 (m(k * x_2) \vee (k * (my_2), \ldots, A_n (m(k * x_n) \vee (k * (my_n) \}$

$\leq \max \{\max \{A_1(k*(mx_1)), A_1(k*(my_1))\}, \max \{A_2(k*(mx_2)), A_2(k*(my_2))\}, \ldots,$
$\max \{A_n(k*(mx_n)), A_n(k*(my_n)) \} \}$

$= \max \{ \max \{A_1(k*(mx_1)), A_2(k*(mx_2))\}, \ldots , A_n(k*(mx_n)) \} ,$
$\max \{A_1(k*(my_1))\}, \; A_2(k*(my_2)),\ldots, A_n(k*my_n)) \} \}$

$= \max \{A_1 \times A_2 \times \ldots \times A_n \; (k * (m(x_1, x_2,\ldots, x_n))), \; A_1 \times A_2\times \ldots \times A_n \; (k*(m (y_1, y_2,\ldots, y_n)) \}$

$= \max \{ A_1 \times A_2 \times \ldots \times A_n \; (k *(mx)), \; A_1 \times A_2 \times \ldots \times A_n \; (k * (my)) \}$

$= \max \{ A(k * (mx)), A(k * (my)) \}.$

Thus the direct product of anti fuzzy lattice ordered M groups both acted by a groupois K is an anti fuzzy lattice ordered M-group acted by K.

CONCLUSION

In this paper we studied the notion of an anti fuzzy lattice ordered m-groups and investigated some of its basic properties. We also studied the homomorphic image, pre-image of an anti fuzzy lattice ordered m-groups, arbitrary family of anti fuzzy lattice ordered m-groups and anti fuzzy lattice ordered m-groups using T-norms.

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