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Some Fixed Point Theorems in Menger Space Satisfying an Implicit Relation

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ABSTRACT

In this paper, we prove some common fixed point theorems for weakly compatible mappings and employing common property (E.A) using integral type inequality in Menger space satisfying an implicit relation. Our results improve and generalize several known fixed point theorems existing in the Menger as well as metric spaces.

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INTRODUCTION

In the year 1942 Menger[10] introduced the notion of a probabilistic metric space (PM-space) which was, in fact, a generalization of metric space. The idea behind this is to associate a distribution function with a pair of points, say (p,q), denoted by Fp,q(t) where $t > 0$ and interpret this function as the probability that distance between p and q is less than t, whereas in the metric space, the distance function is a single positive number. Sehgal[27] initiated the study of fixed points in probabilistic metric spaces. The study of these spaces was expanded rapidly with the pioneering works of Schweizer and Sklar[2]. Jungck[5] introduced the notion of compatible mappings and utilized the same to improve commutativity conditions in common fixed point theorems. This concept has been frequently employed to prove existence theorems on common fixed points. However, the study of common fixed points of non-compatible mappings was initiated by Pant[22]. Recently, Aamri and Moutawakil[13] and Liu *et al*.[28] respectively defined the property (E.A) and the common property (E.A) and proved interesting common fixed point theorems in metric spaces. Most recently, Kubiaczyk and Sharma[7] adopted the property (E.A) in PM spaces and used it to prove results on common fixed points. Recently, Imdad *et al*.[14] adopted the common property (E.A) in PM spaces and proved some coincidence and common fixed point results in Menger spaces. Our results substantially improve the corresponding theorems contained in[21,24,26,28] along with some other relevant results in Menger as well as metric spaces.

2. PRELIMINARIES

Before going to our main result we require some definitions:

Definition 1.1 (see [2]) : Let X be a non empty set and L denote the set of all distribution functions. A probabilistic metric space is an ordered pair (X,F) where $F : X * X \rightarrow L$. we shall denote the distribution function by F (p, q) or F p ,q ; p, q ∈ X and F(p, q, x) will represent the value of F (p, q) at x ∈ R. the function F p, q is assumed to satisfy the following conditions :

1. F p, q (t) = 1, ∀ t > 0 if and if p = q
2. F p, q (0) = 0 for every p, q ∈ X
3. F p, q (t) = F q, p (t) for every p, q ∈ X
4. If F p, q (t) = 1 and F q, r (s) = 1 it follows that F q r (t +s) =1 ∀ p, q, r ∈ X and t ,s ≥ 0.

In metric space (X ,d) , the metric d induces a mapping $F : X * X \rightarrow L$ such that F p, q (t) = H(t-d(p, q)) for all p, q ∈ X and t ∈ R , where H is the distribution function defined as

$$H(x) = \begin{cases} 0, \text{if } x \leq 0 \\ 1, \text{if } x > 0 \end{cases}$$

Definition 1.2 (see [2]) : A mapping $\Delta: [0, 1] * [0,1] \rightarrow [0,1]$ is called t- norm if the following conditions are satisfied

(1) Δ (a, 1) = a for all a ∈ [0, 1], Δ (0,0) = 0,

(2) Δ (a ,b) = Δ (b, a)

(3) Δ (c, d) ≤ Δ (a, b) for c ≥ a, d ≥ b, and

(4) Δ (Δ (c, d),c) = Δ (a,Δ (b, c)) for all a, b, c ∈ [0,1]

Example 1. **(see [2])** The following are the four basic t-norms:

(i) The minimum t-norm: $\Delta_M(a, b) = \min\{a,b\}$.

(ii) The product t-norm: $\Delta_P(a,b) = a.b$

(iii) The Lukasiewicz t-norm: $\Delta_L(a, b) = \max\{a + b - 1, 0\}$.

(iv) The weakest t-norm, the drastic product:

$$\Delta_D(a, b) = \begin{cases} \min\{a, b\}, & if \ \max\{a, b\} = 1 \\ 0, & otherwise \end{cases}$$

In respect of above mentioned t-norms, we have the following ordering:

$\Delta_D < \Delta_L < \Delta_P < \Delta_M$.

Throughout this paper, Δ stands for an arbitrary continuous t-norm.

Definition 1.3 (see [2]): A Menger probabilistic space is a triplet (X, F, Δ) where (X, F) is a PM-space and Δ is a t- norm with the following condition

$F_{p,r}(t+s) \geq \Delta(F_{p,r}(t), F_{p,r}(s))$ for all p, q, r ∈ X and t, s ≥ 0.

The above inequality is called Menger's triangle inequality.

Definition 1.4 (see [2]) : A sequence $\{x_n\}$ in (X, F, Δ) is said to be a convergent to a point x ∈ X if for every ε > 0 and λ > 0, there exists an integer N=N (ε ,λ) such that $F_{x_n, x}(\varepsilon) \to 1-\lambda\ \forall\ n \geq N(\varepsilon, \lambda)$.

Definition 1.5 (see [2]) : A sequence $\{x_n\}$ in (X, F, Δ) is said to be a Cauchy sequence if for every ε > 0 and λ >0 , there exists an integer N=N (ε ,λ) such that $F_{x_n, x_m}(\varepsilon) \to 1-\lambda\ \forall\ n, m \geq N(\varepsilon, \lambda)$.

Definition 1.6 (see [2]) : A Menger Space (X, F, Δ) with the continuous t- norm is said to be complete if every Cauchy sequence in X converges to a point in X.

Definition 1.7. (see [25]) : Let (X, F, Δ) be a Menger PM Space . A pair (f, g) of self mapping on X is said to be weakly commuting if and only if $F_{fgx, gfx}(t) \geq F_{fx, gx}(t)$ for each x ∈ X and t > 0.

Definition 1.8 (see [25]) : Let (X, F, Δ) be a Menger PM Space . A pair (f, g) of self mapping on X is said to be compatible if and only if $F_{fgx_n, gfx_n}(t) \to 1$ for all t > 0 whenever $\{x_n\}$ in X such that $fx_n, gx_n \to z$ for some z ∈ X as $n \to \infty$.

Clearly, a weakly commuting pair is compatible but every compatible pair need not be weakly commuting.

Definition 1.9 (see [4]): Let (X, F, Δ) be a Menger PM Space . A pair (f, g) of self mapping on X is said to be non–compatible if and only if there exist at least one sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some z ∈ X, implies that

$\lim_{n\to\infty} F_{fgx_n, gfx_n}(t_0)$ (for some $t_0 > 0$) is either less than 1 or non-existent.

Definition 1.10 (see [19]): Let (X, F, Δ) be a Menger PM Space . A pair (f, g) of self mapping on X is said to satisfy the property (E.A) if there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z, \text{ for some } z \in X.$$

Clearly, a pair of compatible mappings as well as non- Compatible mappings satisfies the property (E.A).

Inspired by Liu *et al.*[28], Imdad *et al.*[18] defined the following:

Definition 1.11 (see [19]). Two pairs (f, g) and (p, q) of self mappings of a Menger PM space (X,F, Δ) are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$, $\{y_n\}$ in X and some t in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} px_n = \lim_{n\to\infty} qx_n = z$$

Definition 1.12. (see [14]) A pair (f, g) of self mappings of a nonempty set X is said to be weakly compatible if the pair commutes on the set of their coincidence points i.e. fx = gx (for some $x \in X$) implies fgx = gfx.

Definition 1.13. (see [16]) : Two finite families of self mappings $\{A_i\}$ and $\{B_j\}$ are said to be pair wise commuting if:

(i) $A_iA_j = A_jA_i$, $i, j \in \{1, 2...m\}$,

(ii) $B_iB_j = B_jB_i$, $i, j \in \{1, 2...n\}$,

(iii) $A_iB_j = B_jA_i$, $i \in \{1, 2...m\}$, $j \in \{1, 2...n\}$.

2. IMPLICIT RELATION

Let F_6 be the set of all continuous functions $\Phi(t_1, t_2, t_3, t_4, t_5, t_6) : [0, 1]^6 \to R$ satisfying the following conditions :

(Φ_1) $\Phi(u, 1, u, 1,1, u) < 0$, for all $u \in (0,1)$,

(Φ_2) $\Phi(u, 1,1, u, 1,1) < 0$, for all $u \in (0,1)$,

(Φ_3) $\Phi(u, 1, u, 1,1,1) < 0$, for all $u \in (0,1)$,

Example 2.1. **(see [18])** Define $\Phi(t_1, t_2, t_3, t_4, t_5, t_6) : [0, 1]^6 \to R$ as

$$\Phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \Psi(\min\{t_2, t_3, t_4, t_5, t_6\}), \tag{2.1}$$

Where Ψ: $[0,1] \to [0,1]$ is increasing and continuous function such that $\Psi(t) > t$ for all $t \in (0,1)$. Observe that

(Φ_1) $\Phi(u, 1, u, 1,1, u) = u - \Psi(u) < 0$, for all $u \in (0,1)$,

(Φ_2) $\Phi(u, 1,1, u, 1,1) < 0 = u - \Psi(u) < 0$, for all $u \in (0,1)$,

(Φ_3) $\Phi(u, 1, u, 1,1,1) < 0 = u - \Psi(u) < 0$, for all $u \in (0,1)$,

Example 2.2. **(see [18]) :** Define $\Phi(t_1, t_2, t_3, t_4, t_5, t_6) : [0, 1]^6 \to R$ as

$$\Phi(t_1, t_2, t_3, t_4, t_5, t_6) = \int_0^{t_1} \Phi(t)dt - \Psi\left(\int_0^{min\{t_2, t_3, t_4, t_5, t_6\}}\right)\Phi(t)dt, \tag{2.2}$$

Where Ψ: $[0,1] \to [0,1]$ is increasing and continuous function such that $\Psi(t) > t$ for all $t \in (0,1)$ and $\Phi : R \to R$, is a Lebesque integrable function which is summable and satisfies

$$0 < \int_0^{\epsilon} \emptyset(s)ds < 1, \text{ for all } 0 < \epsilon < 1, \quad \int_0^1 \Phi(s)ds = 1 \tag{2.3}$$

Observe that

(Φ_1) $\Phi(u, 1, u, 1,1, u) = \int_0^u \Phi(t)dt - \Psi(\int_0^u \Phi(t)dt) < 0$, for all $u \in (0,1)$,

(Φ_2) $\Phi(u, 1,1, u, 1,1) = \int_0^u \Phi(t)dt - \Psi(\int_0^u \Phi(t)dt) < 0$, for all $u \in (0,1)$,

(Φ_3) $\Phi(u, 1, u, 1,1,1) = \int_0^u \Phi(t)dt - \Psi(\int_0^u \Phi(t)dt) < 0$, for all $u \in (0,1)$,,

3. MAIN RESULT

Lemma 3.1 : Let (X.F, Δ) be a complete Menger Space and f, g, p and q are self mapping of X satisfying the conditions :

(i) The pairs (f, g) (or (p, q) enjoys the property E.A.

(ii) For all x,y ∈X, $\Phi \in F_6$ and for all t > 0,

$$\Phi\left(F_{fx,py}(t), F_{gx,qy}(t),\ F_{fx,gx}(t), F_{py,qy}(t),\ \frac{F_{gx,qy}(t).F_{py,qy}(t)}{F_{gx,py}(t)}, \frac{F_{gx,qy}(t),.F_{fx,gx}(t)}{F_{qy,fx}(t)}\right) \geq 0 \quad (3.1.1)$$

(iii) f(X) ⊂ q(X) (or p(X) ⊂ g(X))for all x,y ∈X, $\Phi \in F_6$ and for all t > 0,

Then the pair (f,g) and (p,q) share the common property (E.A).

Proof : Suppose that the pair (f, g) owns the property (E.A), then there exists a sequences $\{x_n\}$ in X

such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n =$ u, for some u ∈ X.

Since f(X) ⊂ q(X), hence for each $\{x_n\}$ there exists $\{y_n\}$ ∈ X, such that $fx_n = qy_n$, therefore

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} qy_n =$ u

Thus in all, we have $fx_n \to u,\ gx_n \to u\ and\ qx_n \to u \quad \lim_{n\to\infty} qy_n$

Now, we assert that $py_n \to u$.

If it is not so, on applying inequality (3.1.1), we get

$$\Phi\left(F_{fx_n,py_n}(t), F_{gx_n,qy_n}(t), F_{gx_n,fx_n}(t), F_{qy_n,py_n}(t), \frac{F_{gx_n,qy_n}(t).F_{qy_n,py_n}(t)}{F_{gx_n,py_n}(t)}, \frac{F_{gx_n,qy_n}(t).\ F_{fx_n,gx_n}(t)}{F_{qy_n,fx_n}(t)}\right) \geq 0$$

Which on making n → ∞, reduces to

$$\Phi\left(F_{u,\lim_{n\to\infty} py_n}(t), F_{u,u}(t),\ F_{u,u}(t), F_{u,\lim_{n\to\infty} py_n}(t),\ \frac{F_{u,u}(t).F_{u,\lim_{n\to\infty} py_n}(t)}{F_{u,\lim_{n\to\infty} py_n}(t)}, \frac{F_{u,u}(t).F_{u,u}(t)}{F_{u,u}(t)}\right) \geq 0$$

or,

$$\Phi\left(F_{u,\lim_{n\to\infty} py_n}(t), 1, 1, F_{u,\lim_{n\to\infty} py_n}(t), 1,1\right) \geq 0$$

Which is a contradiction to (Φ_2), and therefore $py_n \to u$.

Hence the pair (f,g) and (p,q) share the common property (E.A).

Theorem 3.2: Let (X.F, Δ) be a complete Menger Space and f, g, p and q are self mapping of X satisfying inequality (3.1.1). Suppose that

(i) The pairs {f, g} (or (p, q) enjoys the property E.A.

(ii) f(X) ⊂ q(X) (or p(X) ⊂ g(X))

(iii) g(X) (or q(X)) is a closed subset of X.

Then the pair (f,g) and (p,q) have a point of coincidence each. Moreover, f, g, p and q have a unique common fixed point provided that both the pairs (f,g) and (p,q) are weakly compatible.

Proof : In view of Lemma 3.1 the pair (f, g) and (p, q) share the common property (E.A), that is, there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} qy_n =$ u, for some u ∈ X

Suppose that g(X) is a closed subset of X then u = gz for some z ∈ X.

Then on applying inequality (3.1.1) we obtain

$$\Phi\left(F_{fz,py_n}(t), F_{gz,qy_n}(t), F_{gz,fz}(t), F_{qy_n,py_n}(t),\ \frac{F_{gz,\,qy_n}(t).F_{qy_n,py_n}(t)}{F_{gz,\,py_n}(t)}, \frac{F_{gz,qy_n}(t).F_{gz,fz}(t)}{F_{qy_n,fz}(t)}\right) \geq 0$$

Which on making n $\rightarrow\ \infty$, reduces to

$\Phi\left(F_{fz,u}(t), 1, F_{u,fz}(t), 1,\ 1, 1\right) \geq\ 0$

Which is a contradiction to (Φ_3). Hence fz = gz = u.

Since f(X) $\subset$ q(X), there exists a w$\in$ X such that u = fz = qw.

If u $\neq$ pw, then using inequality (3.1.1),

$$\Phi\left(F_{fz,pw}(t), F_{gz,qw}(t),\ F_{gz,fz}(t), F_{qw,pw}(t),\ \frac{F_{gz,qw}(t).F_{qw,pw}(t)}{F_{qw,pw}(t)}, \frac{F_{gz,qw}(t).F_{gz,fz}(t)}{F_{qw,fz}(t)}\right) \geq\ 0$$

or

$$\Phi\left(F_{u,pw}(t), 1, 1, F_{u,pw}(t), 1, 1\right) \geq\ 0$$

Which is a contradiction to (Φ_2), and therefore fz = gz = u = pw = qw

Since the pair (f, g) and (p, q) are weakly compatible and fz = gz, pw = qw

Therefore

fu = fgz = gfz = gu, and pu = pqw = qpw = qu

If fu $\neq$ u, then using inequality (3.1.1), we have

$$\Phi\left(F_{fu,pw}(t), F_{gu,qw}(t),\ F_{gu,fu}(t), F_{qw,pw}(t),\ \frac{F_{gu,qw}(t).F_{qw,pw}(t)}{F_{gu,pw}(t)}, \frac{F_{gu,qw}(t).\ F_{gu,fu}(t)}{F_{qw,fu}(t)}\right) \geq\ 0$$

or,

$$\Phi\left(F_{fu,u}(t), 1, F_{fu,u}(t), 1, 1, 1\right) \geq\ 0$$

Which is a contradiction to (Φ_3), and therefore fu = gu = u. Similarly one can prove that pu = qu = u.

Hence u = pu = qu =fu = gu. Thus u is a common fixed point of f, g, p and q. The uniqueness of common fixed point follows easily from (3.1.1). This completes the proof.

Lemma 3.3 : Let (X.F, Δ) be a complete Menger Space and f, g, p and q are self mapping of X satisfying the conditions :

(i) $\Psi : [0,1] \rightarrow [0,1]$ be a lower semicontinuous function such that $\Psi(t) > t$ for all t $\in$ (0,1) along with $\Psi(0) = 0$ and $\Psi(1) = 1$.

(ii) The pairs (f, g) enjoys the property E.A.

(ii) for all x,y $\in$X, $\Phi\in F_6$ and for all t > 0,

$$\int_0^{F_{fx,py}(t)} \emptyset(u)du\ \geq \Psi\left(\int_0^{m(x,y)} \emptyset(u)du\right)$$

Where, m(x,y) =

$$\min\left\{F_{gx,qy}(t),\ F_{gx,fx}(t), F_{qy,py}(t),, \frac{F_{gx,qy}(t).F_{qy,py}(t)}{F_{gx,py}(t)}, \frac{F_{gx,qy}(t).F_{fx,gx}(t)}{F_{qy,fx}(t)}\right\} \qquad (3.3.1)$$

Then the pair (f,g) and (p,q) have point of coincidence each. Moreover, f,g,p and q have unique common fixed point provided that both the pairs (f, g) and (p, q) are weakly compatible.

Proof : As both the pairs (f, g) and (p, q)share the common property (E.A), there exists two sequences

$\{x_n\}$,$\{y_n\}$in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} qy_n =$ u, for some u $\in$ X.

If g(X) is a closed subset of X, then (3.3.1).
Therefore, there exists a point z ∈ X such that gz = u.
Now, we assert that fz = gz.
To prove this, on using (3.3.1) with x = z, y = y_n, we get

$$\int_0^{F_{fz,py_n}(t)} \emptyset(u)du \geq \Psi\left(\int_0^{\min\left\{F_{gz,qy_n}(t),\, F_{gz,fz}(t),\, F_{qy_n,py_n}(t),\, \frac{F_{gz,qy_n}y(t).F_{py_n,qy_n}(t)}{F_{gz,py_n}(t)},\, \frac{F_{gz,qy_n}(t).F_{fz,gz}(t)}{F_{qy_n,fz}(t)}\right\}} \emptyset(u)du\right)$$

Which on making n → ∞, reduces to

$$\int_0^{F_{fz,u}(t)} \emptyset(u)du \geq \Psi\left(\int_0^{\min\left\{F_{u,u}(t),\, F_{fz,u}(t),\, F_{u,u}(t),\, \frac{F_{u,u}(t).\,F_{u,u}(t)}{F_{u,u}(t)},\, \frac{F_{u,u}(t).F_{fz,u}(t)}{F_{u,fz}(t)}\right\}} \emptyset(u)du\right)$$

Or $\int_0^{F_{fz,u}(t)} \emptyset(u)du \geq \Psi\left(\int_0^{F_{fz,u}(t)} \emptyset(u)du\right) > \int_0^{F_{fz,u}(t)} \emptyset(u)du$

A contradiction. Therefore, fz = u, and hence fz = gz which shows that the pair (f, g) has a point of coincidence.
If q(X) is a closed subset of X, then (3.3.1). Hence there exists a point w ∈ X such that qw = u.
Now we show that pw = qw.
To prove this, on using (3.3.1) with x = x_n, y = w, we get

$$\int_0^{F_{f\mathrm{xn},\mathrm{pw}}(t)} \emptyset(u)du \geq$$

$$\Psi\left(\int_0^{\min\left\{F_{gx_n,q\mathrm{w}}(t),\, F_{gx_n,fx_n}(t),\, F_{q\mathrm{w},p\mathrm{w}}(t),\, \frac{F_{gx_n,q\mathrm{w}}(t).F_{p\mathrm{w},q\mathrm{w}}(t)}{F_{gx_n,p\mathrm{w}}(t)},\, \frac{F_{gx_n,q\mathrm{w}}(t).F_{fx_n,gx_n}(t)}{F_{q\mathrm{w},fx_n}(t)}\right\}} \emptyset(u)du\right)$$

Which on making n → ∞, reduces to

$$\int_0^{F_{u,\mathrm{pw}}(t)} \emptyset(u)du \geq \Psi\left(\int_0^{\min\left\{F_{u,u}(t), F_{u,u}(t), F_{p\mathrm{w},u}(t), \frac{F_{u,u}(t).\,F_{pw,u}(t)}{F_{pw,u}(t)}, \frac{F_{u,u}(t).\,F_{u,u}(t)}{F_{u,u}(t)}\right\}} \emptyset(u)du\right)$$

Or $\int_0^{F_{u,\mathrm{pw}}(t)} \emptyset(u)du \geq \Psi\left(\int_0^{F_{u,\mathrm{pw}}(t)} \emptyset(u)du\right) > \int_0^{F_{u,pw}(t)} \emptyset(u)du$

A contradiction. Therefore, pw = u, and hence qw = pw which shows that the pair (p, q) has a point of coincidence.

Since the pair (f,g) and (p,q) are weakly compatible and both the pairs have point of coincidence z and w, respectively. Following the lines of the proof of theorem 3.3, one can easily prove the existence of unique common fixed point of mappings f, g, p and q. This completes the proof.

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