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On (γ,eD)-Number of Edge Deleted Graphs

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ABSTRACT

In this paper, it is found for the (γ,eD)-number of an edge deleted graphs for a complete graph.

Keywords: Edge Detour, Edge detour domination, Edge detour domination of edge deleted graphs.

1. INTRODUCTION

The concept of domination was introduced by Ore and Berge[6]. Let G be a finite, undirected connected graph with neither loops nor multiple edges. A subset D of V(G) is a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number γ(G) of G. We consider connected graphs with atleast two vertices. For basic definitions and terminologies, we refer Harary[1]. For vertices u and v in a connected graph G, the detour distance D(u,v) is the length of longest u-v path in G. A u-v path of length D(u,v) is called a u-v detour. A subset S of V(G) is called a detour set if every vertex in G lie on a detour joining a pair of vertices of S. The detour number dn(G) of G is the minimum order of a detour set and any detour set of order dn(G) is called a detour basis of G. These concepts were studied by Chartrand[3]. A subset S of V(G) is called an edge detour set of G if every edge in G lie on a detour joining a pair of vertices of S. The edge detour number $dn_1(G)$ of G is the minimum order of its edge detour sets and any edge detour set of order dn_1 is an edge detour basis. A graph G is called an edge detour graph if it has as edge detour set. Edge detour graph were introduced and studied by Santhkumaran and Athisayanathan[8].

Notations 1.1:
1. [p] denote the integral part of p.
2. $\lceil p \rceil$ denote the smallest positive integer greater than or equal to p.
3. $\lfloor p \rfloor$ denote the smallest positive integer smaller than or equal to p.

The following results are from[4].

Theorem 1.2: The domination numbers of some standard graph are given as follows.
1. $\gamma(P_p) = \left\lceil \frac{p}{3} \right\rceil$, $p \geq 3$
2. $\gamma(C_p) = \left\lceil \frac{p}{3} \right\rceil$, $p \geq 3$
3. $\gamma(K_p) = \gamma(W_p) = \gamma(K_{1,n}) = 1$.
4. $\gamma(K_{m,n}) = 2$ if $m, n \geq 2$.

Theorem 1.3: A dominating set D of G is a minimal dominating set of G if and only if for every $v \in D$, there exists at least one vertex $w \in V-(D-\{v\})$ such that $N[w] \cap D = \{v\}$.
The following theorems were proved by A.P.Santhakumaran and S. Athisayanathan[8].

Theorem 1.4: For any edge detour graph G of order $p \geq 2$, $2 \leq dn_1(G) \leq p$.

Theorem 1.5: If G is an edge detour graph of order $p \geq 3$ such that {u,v} is an edge detour basis of G, then u and v are not adjacent.

Theorem 1.6: If T is a tree with k end vertices, then $dn_1(T) = k$.

Remark 1.7: $\left\lceil \frac{n-4}{3} \right\rceil + 2 = \begin{cases} \left\lceil \frac{n}{3} \right\rceil \text{if } n \equiv 1 (\text{mod } 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 \text{if otherwise} \end{cases}$

The following are from Mahalakshmi.A, Palani.K and Somasundaram. S[5]

Remark 1.8: For any edge detour dominating graph of order $p \geq 2$, $2 \leq \gamma_{eD}(G) \leq p$.

Theorem 1.9: K_P is an edge detour dominating graph and $\gamma_{eD}(K_P) = 3$ for $p \geq 3$.

Theorem 1.10: $\gamma_{eD}(K_{1,n}) = n$.

Theorem 1.11: $\gamma_{eD}(P_n) = \begin{cases} \left\lceil \frac{n-4}{3} \right\rceil + 2 \text{ if } n \geq 5 \\ 2 \text{ if } n = 2, 3 \text{ or } 4 \end{cases}$

Theorem 1.12: For n>5, $\gamma_{eD}(C_n) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 1.13: $\gamma_{eD}(K_{m,n}) = \begin{cases} m \text{ if } n = 1 \text{ and } m > 1 \\ n \text{ if } m = 1 \text{ and } n > 1 \\ 2 \text{ if } m = n = 1 \text{ or } m = 2, n \geq 2 \text{ or } m \geq 2 \text{ and } n = 2 \\ 3 \text{ if } m, n \geq 3. \end{cases}$

Remark 1.14:
1. $\gamma_{eD}(G) \geq dn_1(G)$ and $\gamma_{eD}(G) \geq \gamma(G)$.
2. If the set of all pendant vertices of a graph G forms an edge detour dominating set G, then S is the unique minimum edge detour dominating set of G.
3. Every super set of an edge detour dominating set of G is an edge detour dominating set of G.

2. (γ,eD)-NUMBER OF EDGE DELETED GRAPHS

Theorem 2.1: For a complete graph K_p, $\gamma_{eD}(K_p - \{e\}) = 2$ for every edge e in K_p.

Proof: Let $e = uv \in E(K_p)$. Let S = {u, v}. Then, every edge of $V(K_p - \{e\})$ lie on an edge detour joining the vertices of S. Also, u and v dominate all the vertices of $V(K_p - \{e\}) - S$. Hence, S is an edge detour dominating set of $K_p - \{e\}$. Further, By Remark 1.8, |S| = 2, S is a minimum edge detour dominating set of K_p-{e}. Therefore, $\gamma_{eD}(K_p - \{e\}) = 2$.

Theorem 2.2: Let p≥4. Suppose $e_1, e_2 \in E(K_p)$ such that they are non-adjacent in K_p. If $G' = K_p - \{e_1, e_2\}$ then, $\gamma_{eD}(G') = 2$ if $p \geq 4$ and $p \neq 5$.

Proof: Let $e_1 = uv$; $e_2 = u'v'$ be two non-adjacent edges of G'.

Case 1: Let p = 4.
Let S = {u, v} or {u', v'}. Obviously, S is an edge detour joining the pair of vertices of S and u, v dominate all K_p. Therefore, S is a minimum edge detour dominating set of G'. Hence, $\gamma_{eD}(G')=2$.

Case 2: Let p = 5.

Let S ={ u, v, w}. In G' every edge of V(G')-S lie in some edge detour joining the vertices of S and S dominates all the vertices of G'. So, S is an edge detour dominating set of G'. Therefore, $2 \leq \gamma_{eD}(G') \leq 3$.
Claim: $\gamma_{eD}(G') \neq 2$.
Suppose S = {x, y} is an edge detour dominating set of G'. Then, S is an edge detour basis of G' by theorem 1.5, x and y are non-adjacent in G'. Hence, S is either {u, v} or {u', v'}. In both cases, there is an edge which does not lie in any edge detour. Therefore, no two point set of G' is an edge detour dominating set of G'. Hence, $\gamma_{eD}(G') = 3$.

Case 3: Let $p \geq 6$.
Let S = {u, v} or {u', v'}. Then D(u, v) = p-1 and every edge of G' lie on some edge detour joining the vertices of S and also dominated by the vertices of S. Therefore, S is a minimum edge detour dominating set of G'. Hence, $\gamma_{eD}(G') = 2$.

Theorem 2.3: Let $p \geq 4$. Suppose $e_1, e_2 \in E(K_p)$ such that they are adjacent in K_p. If $G'=K_p-\{e_1,e_2\}$ then, $\gamma_{eD}(G') = \begin{cases} 2 \text{ if } p \geq 5 \\ 3 \text{ if } p = 4. \end{cases}$

Proof: Let $e_1 = uv$; $e_2 = u'v'$ and they are adjacent. Hence, e_1 and e_2 have a common vertex say v=u'.

Case 1: Let $p \geq 5$.
Let S = {u, v} or {v, v'}. Then, every edge of G' lie on some edge detour joining the vertices of S and also dominated by the vertices of S. Therefore, S is a minimum edge detour dominating set of G'. Hence, $\gamma_{eD}(G') = 2$.

Case 2: Let p = 4.
Let S = {u, v, v'}. Proceeding as in Case 2 of Theorem 2.2, no two element subset of V(G') is an edge detour dominating set of G'. Hence, $\gamma_{eD}(G') = 3$.

Theorem 2.4: Let p>3. Suppose $e_1, e_2, e_3 \in E(K_p)$ such that they form a path in K_p. If $G' = K_p-\{e_1, e_2, e_3\}$, then $\gamma_{eD}(G') = 2$.

Proof: Let p=(u, v, w, x) where $e_1 = uv$; $e_2 = vw$ and $e_3 = wx$.

Case 1: If p = 4 or 5.
Let S ={v,w}, then every edge of $V(K_p-\{e_1, e_2, e_3\})$-S lie on the edge detour joining the vertices of S. Also, S dominates all the vertices of G'. Hence, S is an edge detour dominating set of $V(K_p-\{e_1, e_2, e_3\})$. Further, by Remark 1.8 |S| = 2 is a minimum edge detour dominating set of G'. Therefore, $\gamma_{eD}(G') = 2$.

Case 2: If $p \geq 6$.
Let S = {u,v} or {v,w} or {w,x}. Then, every edge $V(K_p-\{e_1, e_2, e_3\})$-S lie in some edge detour joining the vertices of S. Also, S dominates all the vertices of G'. Hence, S is an edge detour

dominating set of $V(K_p-\{e_1, e_2, e_3\})$. Further, by Remark 1.8, $|S| = 2$ is a minimum edge detour dominating set of G'. Therefore, $\gamma_{eD}(G') = 2$.

Theorem 2.5: Let $p>3$ and $G' = K_p-\{e_1, e_2, e_3\}$ where e_1, e_2, e_3 form a cycle in K_p. Then,

$$\gamma_{eD}(G') = \begin{cases} 3 \text{ if } p \neq 5 \\ 5 \text{ if } p = 5. \end{cases}$$

Proof: Let $C = (v_1, v_2, v_3, v_1)$ be a cycle where $e_1 = v_1v_2$; $e_2 = v_2v_3$; $e_3 = v_3v_1$.

Case 1: If $p \neq 5$.

Let $S = \{v_1, v_2\}$ or $\{v_2, v_3\}$ or $\{v_3, v_1\}$. Then, every edge of G' lie on the edge detour joining the vertices of S. It is noted that v_1 and v_2 are adjacent to every vertex of $V(G) – V(C)$ and every vertex in $V(G) – V(C)$ is a full degree vertex. Therefore, every edge incident with the vertices of $V(G) – V(C)$ lie in a detour joining the vertices v_1 and v_2. Also there is no edge in G-C adjacent to two vertices of C. Therefore, $\{v_1, v_2\}$ forms an edge detour set of G. Since $C = K_3$, dominate all the vertices of G-C except v_3. Hence, $S = \{v_1, v_2, v_3\}$ forms a minimum edge detour dominating set of G. Since no two element set forms an edge detour dominating set of G. Therefore, $\gamma_{eD}(G') = |S| = 3$.

Case 2: If $p = 5$.

It is noted that the edge detour lie in no edge detour joining the vertices of any proper subset of V(G). Therefore, no proper subset of V(G) is the unique edge detour dominating set of G. Hence, $\gamma_{eD}(G') = |V(G)| = 5$.

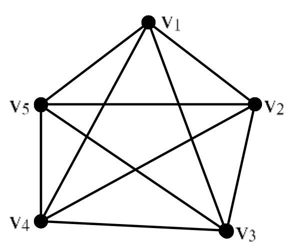

Figure 2.1

Theorem 2.6: Let $p > 3$ and $G' = K_p-\{e_1, e_2, e_3,e_4\}$ where e_1, e_2, e_3, e_4 forms a cycle in K_p.

$$\text{Then, } \gamma_{eD}(G') = \begin{cases} 2 \text{ if } p \geq 6 \\ 3 \text{ if } p = 5 \\ 4 \text{ if } p = 4. \end{cases}$$

Proof: Let $C = \{v_1, v_2, v_3, v_4, v_1\}$ be a cycle in K_p where $e_1 = v_1v_2$; $e_2 = v_2v_3$; $e_3 = v_3v_4$; $e_4 = v_4v_1$.

Case 1: If $p \geq 6$.

Proceeding as in previous Theorem, the sets $S_1 = \{v_1, v_2\}$; $S_2 = \{v_2, v_3\}$; $S_3 = \{v_3, v_4\}$ and $S_4 = \{v_4, v_1\}$ form an edge detour set of G – C. Also, S_i's dominate the vertices of V – C. Since, every vertex in V(G) – V(C) is a full degree vertex in G – C. Now, to check whether the vertices of C are dominated by S_i. Further, for all i, the vertices of S_i are full degree vertices in K_p and are non-adjacent to any one vertex of C in V(C) - S_i. Also, the vertices of C-S_i non-adjacent to the vertices of S_i are distinct. Hence, S_i dominates all the vertices of G and therefore, S_i is an edge detour dominating set. Hence, $\gamma_{eD}(G') = 2$.

Case 2: If p = 5.
Let S = $\{v_1, v_2, v_3\}$ forms an edge detour set of G – C. Also, no two element set forms an edge detour dominating set. Therefore, $\gamma_{eD}(G') = 3$.

Case 3: If p = 4.
Then, G – C ≅ $2P_2$. Therefore, $\gamma_{eD}(G') = 2\,(\gamma_{eD}(P_2)) = 2(2) = 4$.

Theorem 2.7: If G' is the graph obtained from K_p ($p \geq 4$) by removing the edges of a star with k-end vertices ($2 \leq k \leq p-2$) in K_p, then $\gamma_{eD}(G') = \begin{cases} 2 \text{ if } k \neq p-2 \\ 3 \text{ if } k = p-2. \end{cases}$

Proof: Let V(G) = $\{v, v_1, v_2, \ldots, v_{p-1}\}$. Let H be a star in G and V(H) = $\{v, v_1, v_2, \ldots, v_k\}$ where v is the vertex of degree k in H.

Case 1: If $k \neq p-2$.
Let S = $\{v, v_1\}$. Then, every edge of G' lie on the edge detour joining the vertices of G'. Also, S dominates all the vertices of G'. Therefore, S is a minimum edge detour dominating set of G'. Hence, $\gamma_{eD}(G') = 2$.

Case 2: If k = p-2.
Let S = $\{v, v_1\}$ or $\{v, v_2\}$ or … or $\{v, v_k\}$. Then, there is an edge v_1v_{p-1} or v_1v_{p-2} or …, $v_{p-2}v_{p-1}$ which does not lie on the edge detour joining the vertices of S. Therefore, no two point set of G' is a minimum edge detour. Now, let S' = $\{v, v_1, v_2\}$ and every edge of G' lie on edge detour joining the pair of vertices of S'. Also, S' dominates all the vertices of G'. Therefore, S' is a minimum edge detour dominating set of G'. Hence, $\gamma_{eD}(G') = |S'| = 3$.

Theorem 2.8: Let G' ba a graph obtained from G by removing the edge of a clique on m ($2 \leq m \leq p-1$) vertices in K_p ($p \neq 5$) then, $\gamma_{eD}(G') = m$.

Proof: Let V(G) = $\{v_1, v_2, \ldots, v_p\}$. Let H be a clique on m vertices in G. If S = V(H) = $\{v_1, v_2, \ldots, v_m\}$ then, every edge in G' lie on an edge detour joining the vertices of S. Also, S dominates the vertices of G'. Therefore, S is an edge detour dominating set of G'. Hence, $2 \leq \gamma_{eD}(G') \leq m$.

Claim: S is a minimum edge detour dominating set of G'.
Let S' be an edge detour dominating set of G' with t < m elements. If the vertices of S' forms a clique in G', then there is an edge which do not lie on the edge detour or it does not dominates

the vertices of G'. This is a contradiction to our assumption. Therefore, there is no edge detour dominating set of G' with less than m elements. Hence, $\gamma_{eD}(G') = m$.

Theorem 2.9: Let G' be a graph obtained from K_5 by removing the edges of a clique on $(2 \leq m \leq 4)$ vertices in G then, $\gamma_{eD}(G') = \begin{cases} m \text{ if } m \neq 3 \\ 5 \text{ if } m = 3. \end{cases}$

Proof: Let $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and H be a clique on m vertices in G.

Case 1: If $m \neq 3$.
Proceeding the proof as in previous Theorem we get $\gamma_{eD}(G') = m$.

Case 2: If $m = 3$.
By Theorem 2.5 Case 2, we get $\gamma_{eD}(G') = 5$.

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