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Generalized Results on Mathematical Physics using special function and Integral Transforms

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ABSTRACT

In this paper we find some results on Mathematical physics using special functions, like Bessel's function of 1st kind and modified Bessel's function of 1st kind, Beta function, Gamma function, Meijer's G function, Hyper geometric function and some transform like Laplace transform, Mellin transform.

Keywords: Generalized hyper geometric function, Laplace transform, Meljer G-function, Mellin transform.

1.1. INTRODUCTION

The Pochhammer Symbol (or the shifed factorial)

$$\lambda_\nu = \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)} = \begin{Bmatrix} 1 & \nu = 0, \lambda \in C \\ \lambda(\lambda+1)\ldots\ldots(\lambda+n-1) & \nu = nty, \lambda \in C \end{Bmatrix} \qquad (1.1.1)$$

It is being understood conventionally that $\Theta_0 = 1$ and assumed tacitly that Gamma quotient exists.

The most rapidly growing research subject in mathematical physics and engineering, such as Jacobi and Laguerre polynomial, can be defined in the form of generalized Hyper geometric function or confluent hyper geometric function .

Recently, many mathematicians investigated some result on this type, such as Parmar has investigated some fundamental properties and characteristics of some generalized beta type function

$$\beta_p^{\alpha\,;\,\beta;\,m}\ (x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\ , \text{Fn}(\alpha\ ;\ \beta\ ;\ \frac{-p}{(t-1)^m\, t^m})\ dt \qquad (1..1.2)$$

R(P) > 0, min (R(x); R(y); R(α); R(β)) > 0

For α = β, then equation (1..1.2), reduces to the extended beta type function, due to Choudhry *et al*., defined as

β_p (x,y) = ($\beta_p^{(\alpha,\alpha)}(x,y)$)

β_p (x,y) = $\int_0^1 t^{(x-1)}\ (1-t)^{y-1}$ex P($\frac{-p}{t(1-t)}$) dt (1.1.3)

The classical Euler's beta function β(x,y) is defined by

β(x, y) = $\int_0^1 t^{(x-1)}\ (1-t)^{y-1}dt$ (1.1.4)

here, (R (x) > v , R(y) > 0)

It is easy to see the following relation:

β(x, y) = β_0(x, y) = $\beta_0^{(\alpha,\beta)}$(x, y) = $\beta_0^{(\alpha,\beta\ ;1)}$(x, y) (1.1.5)

by making use of β_p(x, y) , Chondhry *et al*. Extended the Gauss hyper geometric function as follows:

F_p(a; b; c; z) = $\sum_{n=0}^{\infty}(a)_n \frac{\beta_p}{\beta} \frac{(b+n,\ c-b)}{(b,\ c-b)} \frac{Z^n}{n!}$ here, (|Z| < 1)

Where R(c) > R(b) > 0 and R(p) > 0 and $(a)_n$ denotes the Pochhammer Symbol.

In special case when p = 0 , the function $F_p^{(\alpha,\beta)}$*(a; b; c; z)* reduces to the investigated Ganss hyper geometric function(1.1.2)

Then (1.1.2) is the special case of the well known generalized hyper geometric series $P^{Fiq}(.)$ defined as

$p\ ^{Fiq}\ [{}^{\alpha_1 \ldots\ldots\ldots\ldots\ldots\ldots}_{\beta_1 \ldots\ldots\ldots\ldots\ldots\ldots}\ {}^{\alpha_p\,;}_{p_q;}\ Z\ ]\ = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n}{(\beta_1)_n}\text{-------------}\frac{\alpha_{p_n}}{\beta_{q_n}} \frac{Z^n}{n!}$

= $p^{F_q}(\alpha_1 \ldots\ldots\ldots\ldots \alpha_p\ ;\ \beta_1 -\ \beta_q\ ; Z\ )$ (1.1.6)

1.2 MAIN THEOREMS

In this section, we shall prove the theorem of generalized Gauss hyper geometric function of equation (1.1.6)

Theorem 1: The Beta transform

B{$F_q^{(\alpha,\beta)}$*(l+m, b; c, yz; l, m }*

=β(l,m) $\sum_{n=0}^{\infty} ln\ \frac{\beta_p^{\alpha,\beta}}{\beta} \frac{(b+n,c-b)}{(b,\ c-b)} \frac{y^n}{n!}$

=β(l, m) $F_p^{(\alpha,\beta)}$(l,b,c;y)

=E(p,α_r, q; β_b ; yz)

=$G_{q+1,p}^{p,1}$ ($Y^z\ \frac{1,\beta_1,\ldots\ldots\ldots\ldots\ldots}{\alpha_1 \ldots\ldots\ldots\ldots}\text{----}\frac{\beta_q}{\alpha_q}$)

Proof

$\beta_p^{\alpha,\beta}$(x,y) = $\int_0^1 t^{(x-1)}\ (1-t)^{(y-1)}$, F_{11}(α; β; $\frac{-p}{t(1-t)}$) dt (1.1.7)

Then,

$\beta\{ F(z); a,b \} = \int_0^1 z^{(a-1)} (1-z)^{y-1}$, F(z) dz (1.1.8)

On Using the equation (1.1.6) and applying (1.1.8) we get

$\int_0^1 z^{(l-1)} (1-z)^{(m-1)} F_p^{(\alpha,\beta)} (l+m, b; c, yz)dz$

$= \sum_0^1 (l+m)_n \frac{\beta_p^{(\alpha,\beta)}}{\beta} \frac{(b+n, c-b)}{b,\ c-b} \frac{\Gamma(l+m)\Gamma(m)}{\Gamma(l+m+n)} \frac{y^n}{n!}$

$= \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)} F_p (\alpha,\beta) (l,b,c; y)$

$= \beta(l,m) E(p, dr; q; \beta_c ; yz)$

$= \beta(l,m) G_{q+1}^{p,1} (yz| \frac{1,\quad \beta_1 \ldots\ldots\ldots \beta_q}{\alpha 1 \ldots\ldots\ldots \alpha_q} |)$

Theorem 2: If $R(s) > 0$, $R(p) > 0$ and $|\frac{y}{x}| < 1$

Then, $L\{Z^{p-1} F_p^{(\alpha,\beta)} (a,b;c;yz)\} = \frac{\Gamma^l}{S^l}, F_q^{(\alpha,\beta)} (a;l;b;c_s^y)$

Where L indicates the laplase transform and is defined as

$L\{F(z)\} = \int_0^\infty e^{-sz} f(z)\, dz$

$= = \frac{\Gamma(l+n)}{S^{l+n}} \frac{y^n}{n!} G_{q+1}^1 (yz |\frac{1,\quad \beta_1 \ldots\ldots\ldots \beta_q}{\alpha 1 \ldots\ldots\ldots \alpha_q})$

Proof

Since, $\int_0^\infty z^{(l-1)} e^{-sz} F_p^{(\alpha,\beta)} (a;b;c;yz)\, dz$

$= \int_0^\infty z^{(l-1)} e^{-sz} \sum_{n=0}^\infty a_n \frac{\beta_p^{(\alpha,\beta)}}{\beta} \frac{(b+n, c-b)}{(b,\ c-b)} \frac{yz^n}{n!} dz$

By Integrating the summation and Integration

$= \int_0^\infty z^{(l-1)} e^{-sz} F_p^{(\alpha,\beta)} (a;b;c;yz)\, dz$

$= \sum_{n=0}^\infty \frac{\beta_p^{(\alpha,\beta)}}{\beta} \frac{(b+n, c-b)}{(b,\ c-b)} \frac{\Gamma(l+n)}{S^{l+n}} \frac{y^n}{n!}$

$= \frac{\Gamma(l+n)}{S^{l+n}} \frac{y^n}{n!} G_{q+1}^1 (yz |\frac{1,\quad \beta_1 \ldots\ldots\ldots \beta_q}{\alpha 1 \ldots\ldots\ldots \alpha_q}).$

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