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Coloring of Strong Intuitionistic Fuzzy Petersen Graph $P_{\hat{G}}$

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ABSTRACT

Intuitionistic fuzzy graph theory has variety of applications in modern science and technology, especially in the fields of neural networks artificial intelligence and decision making. An Intuitionistic fuzzy set is a generalization of the concept of fuzzy set.

In this paper, the concept of Intuitionistic Fuzzy Petersen graph, Strong Intuitionistic Fuzzy Petersen graph and its coloring are introduced, with an illustrative example.

Keywords: Vertex coloring of IFG, Edge coloring of IFG, Intuitionistic fuzzy Petersen graph, Strong Intuitionistic Fuzzy Petersen graph.

1. INTRODUCTION

Graph coloring was introduced in 1852, when Francis Guthrie come up with the four color conjecture[1]. A graph coloring is the assignment of a color to each of the vertices or edges or both in such a way that no two adjacent vertices and incident edges give the same color[7].

The first definition of fuzzy graph was introduced by Kaufmann (1973), based on Zadeh's fuzzy relations (1971). The intuitionistic fuzzy graph and its properties was discussed by R. Parvathi *et al*.[4,5]. The concept of chromatic number of fuzzy graph was introduced by Munoz *et al*.[7]. S. Lavanya and R. Sattanathan discussed total fuzzy coloring[2]. In recent years, research in intuitionistic fuzzy graph theory and its applications have been increased.

In this paper, we consider only the intuitionistic fuzzy Petersen graph $P_{\hat{G}}$ and also we define vertex coloring and edge coloring of $P_{\hat{G}}$.

2. PRELIMINARIES

Definition: 2.1 (Fuzzy Set)

Let X be a non-empty set. Then a fuzzy set A in X. (i.e., a fuzzy subset A of X) is characterized by a function of the form $«_A : X \to [0,1]$, such a function $«_A$ is called the membership of x in the fuzzy set A. In other words $A = \{(x, «_A(x)) / x \in X\}$.

Definition: 2.2 (Fuzzy Graph)

A fuzzy graph $G = (±, «)$ is a pair of functions $± : V \to [0,1]$ and $« : V \times V \to [0,1]$, where for all $u, v \in V$ we have $«(u,v) \le ±(u) \wedge ±(v)$.

Definition: 2.3 (Intuitionistic Fuzzy Set)

An intuitionistic fuzzy set A in a set X is defined as an object of the form $A = \{\langle x, «_A(x), ¬_A(x)\rangle / x \in X\}$ where $«_A : X \to [0,1]$ is degree of membership and $¬_A : X \to [0,1]$ is degree of non-membership of the elements of $x \in X$. For every $x \in X$, $0 \le «_A(x) + ¬_A(x) \le 1$.

Definition: 2.4 (Intuitionistic Fuzzy Graph)

Intuitionistic Fuzzy Graph (IFG) is of the form $G = \langle V, E\rangle$ where

(i) $V = \{v_1, v_2, ..., v_n\}$ such that $«_1 : V \to [0,1]$ and $¬_1 : V \to [0,1]$ are degree of membership and non- membership of $v_i \in V$. For every $v_i \in V$, $0 \le «_1(v_i) + ¬_1(v_i) \le 1$; i= 1,2,...,n.

(ii) $E \subset V \times V$ where $«_2 : V \times V \to [0,1]$ and $¬_2 : V \times V \to [0,1]$ are such that

$«_2(v_i, v_j) \le \min[«_1(v_i), «_1(v_j)]$

$¬_2(v_i, v_j) \le \max[¬_1(v_i), ¬_1(v_j)]$ and $0 \le «_2(v_i, v_j) + ¬_2(v_i, v_j) \le 1$, $(v_i, v_j) \in E$.

Definition: 2.5 (k-Fuzzy Coloring)

A family $\Gamma = \{¥_1, ..., ¥_k\}$ of fuzzy sets on V is called a k- fuzzy coloring of $\hat{G} = (V, ±, «)$ if

(a) $\vee \Gamma = 0$

(b) $¥_i \wedge ¥_j = 0$

(c) For every strong edge xy of G, $\wedge \{¥_i(x), ¥_j(y)\} = 0$, $(1 \le i \le k)$.

Definition: 2.6 (k- Vertex Coloring of IFG)

A family $\Gamma = \{¥_1, ..., ¥_k\}$ of intuitionistic fuzzy sets on V is called a k- vertex coloring of $\hat{G} = (V, E)$ if

(a) $\vee ¥_i(x) = V$ for all $x \in V$

(b) $¥_i \wedge ¥_j = 0$

(c) For every strong edge xy of $\hat{G}$, $\min\{¥_i(«_1(x)), ¥_i(«_1(y))\} = 0$; $\max\{¥_i(¬_1(x)), ¥_i(¬_1(y))\} = 1$, $(1 \le i \le k)$.

The least value of k for which $\hat{G}$ has a k- vertex coloring denoted by ¡ $(\hat{G})$, is called the chromatic number of the IFG($\hat{G}$).

Definition: 2.7 (k- Edge Coloring of IFG)

A family $\Gamma = \{¥_1, ..., ¥_k\}$ of intuitionistic fuzzy sets on V is called a k- edge coloring of $\hat{G} = (V, E)$ if

(a) $\vee ¥_i(xy) = E$ for all $xy \in E$

(b) $¥_i \wedge ¥_j = 0$

(c) For every incident edges xy of E, $\min\{¥_i(«_2(xy))\} = 0$; $\max\{¥_i(¬_2(xy))\} = 1$, $(1 \le i \le k)$.

The least value of k for which $\hat{G}$ has a k- edge coloring denoted by ¡ $'(\hat{G})$, is called the edge chromatic number of the IFG ($\hat{G}$).

Definition: 2.8 (The Petersen Graph)

Let G is a simple graph with 10 vertices and 15 edges. The Petersen graph is most commonly drawn as a pentagon inside with five spokes.

Definition: 2.9 (Fuzzy Petersen Graph)

A fuzzy Petersen graph $P_G = (±, «)$ is a pair of functions $± : V_s \to [0,1]$ and $« : V_S \times V_S \to [0,1]$, S=1,2,..10.where for all $u, v \in V_S$ we have $«(u,v) \le ±(u) \wedge ±(v)$ for $(u,v) \in E_T$, T=1,2,…15.

3. INTUITIONISTIC FUZZY PETERSEN GRAPH

Intuitionistic fuzzy Petersen graph is of the form $P_G = (V_S, E_T)$ where S=1,2,…,10; T=1,2,…,15.

(i) $V_S = \{v_1, v_2, ..., v_{10}\}$ such that $«_1 : V_S \to [0,1]$ and $¬_1 : V_S \to [0,1]$ are degree of membership and non-membership of $v_i \in V_S$. For every $v_i \in V$,

$0 \le «_1(v_i) + ¬_1(v_i) \le 1$ and $v_i \in V_S$.

(ii) $E_T \subset V_S \times V_S$ where $«_2 : V_S \times V_S \to [0,1]$ and $¬_2 : V_S \times V_S \to [0,1]$ are such that $«_2(v_i, v_j) \le \min\lfloor «_1(v_i), «_1(v_j)\rfloor$ $¬_2(v_i, v_j) \le \max[¬_1(v_i), ¬_1(v_j)]$; $0 \le «_2(v_i, v_j) + ¬_2(v_i, v_j) \le 1$, $(v_i, v_j) \in E_T$.

4. STRONG INTUITIONISTIC FUZZY PETERSEN GRAPH (P_G)

An Intuitionistic fuzzy Petersen graph, $P_G = \langle V_S, E_T \rangle$ is called a Strong Intuitionistic fuzzy Petersen graph if $«_{2ij} = \min(«_{1i}, «_{1j})$ and $¬_{2ij} = \max(¬_{1i}, ¬_{1j})$ for all $(v_i, v_j) \in E_T$; $v_i, v_j \in V_S$.

4.1 Vertex Coloring in Strong Intuitionistic fuzzy Petersen graph ($P_{\hat{G}}$):

A family $\Gamma = \{¥_1, ..., ¥_k\}$ of intuitionistic fuzzy sets on V_S is called k-vertex coloring of $P_{\hat{G}} = (V_S, E_T)$ if

(a) $\vee ¥_i(x) = V_S$ for all $x \in V_S$

(b) $¥_i \wedge ¥_j = 0$

(c) For every strong edge xy of $P_{\hat{G}}$, $\min\{¥_i(«_1(x)), ¥_i(«_1(y))\} = 0$; $\max\{¥_i(¬_1(x)), ¥_i(¬_1(y))\} = 1$, $(1 \le i \le k)$.

The least value of k for which $P_{\hat{G}}$ has a k- vertex coloring denoted by $¡(P_{\hat{G}})$, is called the chromatic number of P_G.

4.2 Edge Coloring in Strong Intuitionistic fuzzy Petersen graph ($P_{\hat{G}}$):

A family $\Gamma = \{¥_1, ..., ¥_k\}$ of intuitionistic fuzzy sets on E_T is called a m- edge coloring of $P_{\hat{G}} = (V_S, E_T)$ if

(a) $\vee ¥_i(xy) = E_T$ for all $xy \in E_T$

(b) $¥_i \wedge ¥_j = 0$

(c) For every incident edges xy of E_T, $\min\{¥_i(«_2(xy))\} = 0$; $\max\{¥_i(¬_2(xy))\} = 1$, $(1 \le i \le k)$.

The least value of k for which $P_{\hat{G}}$ has a m- edge coloring denoted by $¡'(P_{\hat{G}})$, is called the edge chromatic number of P_G.

5. EXAMPLE

Strong Intuitionistic fuzzy Petersen graph vertex coloring:

Consider the strong intuitionistic fuzzy Petersen graph $P_{\hat{G}} = (V_S, E_T)$ in the figure with ten vertices and fifteen edges. $V_S = \{v_1, v_2, ..., v_{10})$ and $E_T = \{e_1, e_2, ..., e_{15})$.

Let $\Gamma = \{¥_1, ¥_2, ¥_3\}$ be a family of intuitionistic fuzzy sets defined on V_S as follows;

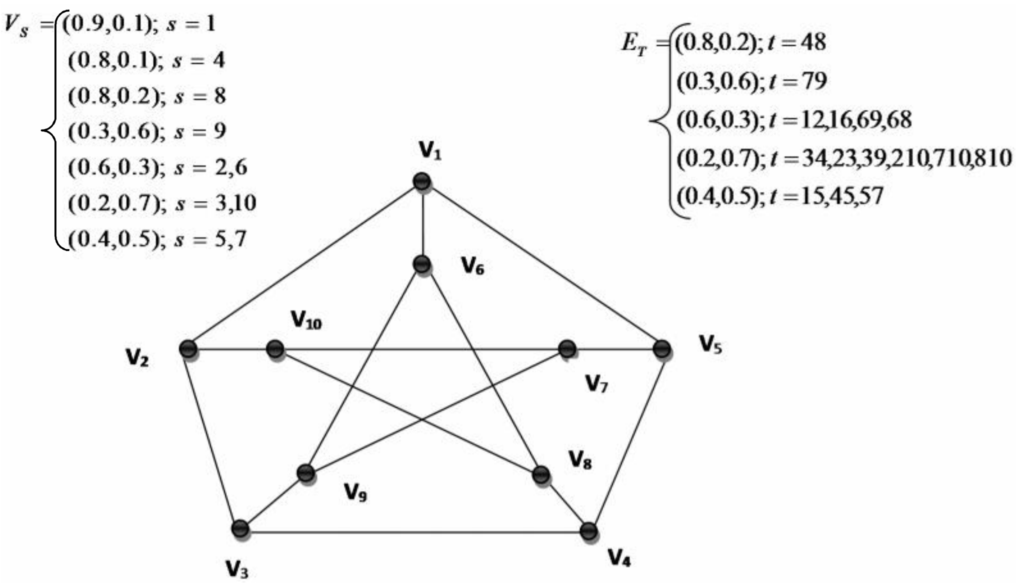

Figure 5.1 Strong Intuitionistic Fuzzy Petersen Graph (P_G)

$$¥_1(V_S) = \begin{cases} (0.9,0.1);\ s=1 \\ (0.2,0.7);\ s=3 \\ (0.4,0.5);\ s=7 \\ (0.8,0.2);\ s=8 \\ (0,1);\ otherwise \end{cases} \quad ¥_2(V_S) = \begin{cases} (0.6,0.3);\ s=2,6 \\ (0.4,0.5);\ s=5 \\ (0,1);\ otherwise \end{cases} \quad ¥_3(V_S) = \begin{cases} (0.8,0.1);\ s=4 \\ (0.3,0.6);\ s=9 \\ (0.2,0.7);\ s=10 \\ (0,1);\ otherwise \end{cases}$$

Here the family $\Gamma = \{¥_1, ¥_2, ¥_3\}$ satisfied our definition of vertex coloring of strong intuitionistic fuzzy Petersen graph. Any family of intuitionistic fuzzy sets having less than three members could not satisfy our definition.

In this case the chromatic number ¡ $(P_{\hat{G}})$ is 3.

Strong Intuitionistic fuzzy Petersen graph edge coloring:

Consider the strong intuitionistic fuzzy Petersen graph $P_{\hat{G}} = (V_S, E_T)$ in the above figure. Let $\Gamma = \{¥_1, ¥_2, ¥_3, ¥_4\}$ be a family of intuitionistic fuzzy sets defined on E_T as follows;

$$¥_1(V_iV_j)=\begin{cases}(0.2,0.7);\ ij=34,210\\(0.6,0.3);\ ij=16\\(0.4,0.5);\ ij=57\\(0,1);\ otherwise\end{cases}\qquad ¥_2(V_iV_j)=\begin{cases}(0.4,0.5);\ ij=15\\(0.2,0.7);\ ij=23,710\\(0.8,0.2);\ ij=48\\(0.6,0.3);\ ij=69\\(0,1);\ otherwise\end{cases}$$

$$¥_3(V_S)=\begin{cases}(0.3,0.6);\ ij=79\\(0.2,0.7);\ ij=810\\(0,1);\ otherwise\end{cases}\qquad ¥_4(V_iV_j)=\begin{cases}(0.2,0.7);\ ij=39\\(0.6,0.3);\ ij=12,68\\(0.4,0.5);\ ij=45\\(0,1);\ otherwise\end{cases}$$

Here the family $\Gamma=\{¥_1,¥_2,¥_3,¥_4\}$ satisfied our definition of edge coloring of strong intuitionistic fuzzy Petersen graph. Any family of intuitionistic fuzzy sets having less than three members could not satisfy our definition.

In this case the chromatic number $¡'(P_{\hat{G}})$ is 4.

6. CONCLUSION

In this paper, we define an intuitionistic fuzzy Petersen graph, Strong intuitionistic fuzzy Petersen graph and also find a vertex and edge coloring for strong intuitionistic fuzzy Petersen graph with an illustrative example. Finally we get the chromatic number as a crisp number.

7. REFERENCES

1. Gray Chartrand and Ping Zhang, " Chromatic Graph Theory", CRC Press, A Chapman & Hall book (2009).
2. S.Lavanya, R.Sattanathan, "Fuzzy total coloring of fuzzy graphs", *International Journal of Information Technology and Knowledge Management*, Vol.2, No.1.pp- 37-39 (2009).
3. G.Nirmala and P.Sinthamani, "Characteristics of Fuzzy Petersen graph with fuzzy rule", *International Journal of Science and Research (IJSR),* ISSN: 2319-7064, Vol 3, Issue 11, November (2014).
4. R.Parvathi, M.G.Karuambigai, "Intuitionistic fuzzy graphs", *Computational Intelligence, Theory and Applications,* part: 6, pp: 139-150 (2006).
5. R.Parvathi, M.G.Karuambigai, and Krassimir T.Atanassov, "Operations on Intuitionistic fuzzy graphs", *FUZZ-IEEE* 2009, Korea, pp: 20-24 (2009).
6. A.Prasanna, M.A.Rifayathali and S.Ismail Mohideen, "Strong Intuitionistic fuzzy graph coloring", *International Journal of Latest Engineering Research and Applications (IJLERA)* ISSN: 2455-7137, Vol 2, Issue-8, pp: 163-169.
7. Susana Munoz, M. Teresaortuno, Javier Yanez, "Coloring fuzzy graphs", *Elsevier*, pp: 211-221.