JOURNAL OF COMPUTER AND MATHEMATICAL SCIENCES
An International Open Free Access, Peer Reviewed Research Journal
www.compmath-journal.org

ISSN 0976-5727 (Print)
ISSN 2319-8133 (Online)
Abbr:J.Comp.&Math.Sci.
2014, Vol.5(2): Pg.223-227

Some Common Fixed Point Menger PM Spaces in Intuitionistic Fuzzy Metric Spaces

A. D. Singh and Ram Milan Singh

Motilal Vigyan Mahavidyalaya, Bhopal, M. P., INDIA.

(Received on: April 15, 2014)

ABSTRACT

The purpose of this paper is to obtain a new common fixed point theorem by using a new contractive condition and properties in intuitionistic fuzzy metric spaces.

Keyword: Triangular norm, Triangular co-norm, intuitionistic fuzzy metric space, fuzzy metric space, fixed point.

INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh[7]. Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek[3], and George and Veeramani[2] modified the notion of fuzzy metric spaces with the help of continuous T-norms. Recently, many authors have proved fixed point theorems involving fuzzy sets. As a generalization of fuzzy sets, Atanassov[1] introduced and studied the concept of intuitionistic fuzzy sets. Recently, using the idea of intuitionistic fuzzy sets, Park[5] introduced the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous *t*-conorms as a generalization of fuzzy metric spaces due to George and Veeramani[2], and showed that every metric induces an intuitionistic fuzzy metric, every fuzzy metric space is an intuitionistic fuzzy metricspace and found a necessary and sufficient condition for an intuitionistic fuzzy metric space to be complete.

The purpose of this paper is to obtain a new common fixed point theorem by using a new contractive condition in intuitionistic fuzzy metric spaces.

1. PRELIMINARIES

Definition 1.1 A binary operation $*: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norm if it satisfies the following conditions:

(a) $*$ is commutative and associative:
(b) $*$ is continuous:
(c) $a * 1 = a$ for all $a \in [0,1]$
(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Definition 1.2. A binary operation ♦: [0, 1] × [0, 1] → [0, 1] is a continuous t-conorm if it satisfies the following conditions:

(a) ♦ is commutative and associative:

(b) ♦ is continuous:

(c) a ♦0 = a for all a ∈ [0,1]:

(d) a ♦b ≤ c ♦d whenever a ≤ c and b ≤ d, for each a, b, c, b ∈ [0, 1]:

Definition 1.3. (X, M,*) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and Mis a fuzzy set on X^2 x (o, ∞) satisfying the following condition:

(a) M (x, y, t) > 0

(b) M (x, y, t) = 1, ⇔ x = y

(c) M (x, y, t) = M (y, x, t)

(d) M (x, y, t) ∗ M(y, z, s) ≤ M (x, z, t + s)

(e) M (x, y,.) : (0, ∞) → [0, 1] is continuous for all x, y, z ∈ X and s > 0.

Definition 1.4. A 5-tuple (X, M, N, ∗, ♦) is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, ∗ is a continuous t-norm, ♦ is a continuous t-conorm and M, N are fuzzy sets on X^2 x [0, ∞] satisfying the following conditions: for all x, y, z ∈ X and s, t > 0:

(a) M (x, y, t) + N (x, y, t) ≤ 1;

(b) M (x, y, t)> 0;

(c) M (x, y, t)= 1 if and only if x = y;

(d) M (x, y, t) = M (y, x, t)

(e) M (x, y, t) ∗ M(y, z, s) ≤ M (x, z, t + s);

(f) M (x, y, .) : [0, ∞]→[0, 1] is left continuous

(g) $\lim_{t\to\infty}$ M (x, y, t) = 1 for all x, y ∈ X and t > 0;

(h) N (x, y, t) >0

(i) N (x, y, t) = N (y, x, t);

(j) N (x, y, t) ♦N (y, z, s) ≥ N (x, z, t + s)

(k) N (x, y, .) : (0, ∞) → [0,1] is right continuous

(l) $\lim_{t\to\infty}$ N (x, y, t) = 0 for all x, y in X.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x, y, t) and N (x, y, t) denote the degree of nearness and degree of non nearness between x and y with respect to t, respectively.

Definition 1. 5: Let (X, M, N, ∗, ♦) be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ is said to be convergent x in X if for each ∈>0 and t>0, there exist n_0∈N such that $M(x_n, x, t) > 1-\in$ and $N(x_n, x, t) < \in$ for all $n \geq n_0$.

(b) a sequence $\{x_n\}$ is said to be Cauchy if for each ∈>0 and t>0, there exist n_0∈N such that $M(x_n, x_m, t) > 1-\in$ and $N(x_n, x_m, t) < \in$ for all $n,m \geq n_0$.

(c) An intuitionistic fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 1. 6: A sequence {Si} of self maps on a complete intuitionistic fuzzy metric space (X, M, N, ∗, ♦) is said to be intuitionistic mutually contractive if for t>0 and i∈N.

$M(S_ix, S_jy, t) \geq M(x, y, t/p)$ and $N(S_ix, S_jy, t) \geq N(x, y, t/p)$. where x,y ∈ X, p∈(0,1), i≠ j and x ≠ y

Lemma 1.1. If (X, M, N, ∗, ♦) be an intuitionistic fuzzy metric space and M(p,q,t) = C, N(p,q,t) = D for all t>0, then C = H(t) and D = I(t) and p = q.

Lemma 1.2. If (X, M, N, ∗, ♦) be an intuitionistic fuzzy metric space. Suppose that $\{p_n\} \subset X$ is such that $M(p_n,p_{n+1},ø^n(t)) \geq M(p_0,p_1,t)$ and $N(p_n,p_{n+1},ø^n(t)) \leq N(p_0,p_1,t)$ for all $t > 0$, where the function $ø:[0,\infty) \to [0,\infty)$ is onto, strictly increasing and satisfy condition (*). Also assume $E_M(p_0,p_1) = \sup\{t < 0: \max\{E(\gamma,_M(p_0,p_1): \gamma\in(0,1)\} < \infty$, and $E_N(p_0,p_1) = \inf\{t > 0: \min\{E(\gamma,_N(p_0,p_1): \gamma\in(0,1)\} > \infty$. Then $\{p_n\}$ is a cauchy sequence.

2. MAIN RESULTS

Lemma-2.1- Let $(X, M, N, *, \blacklozenge)$ be an intuitionistic fuzzy metric space. Let A,B,S and T be a mappings from X into itself such that

(a) The pair (A,S) (or(B,T)) satisfies the property (S.B.);

(b) for any $p,q \in X$, $ø\in F_6$, and for all $x > 0$

$ø\{M(A_p,B_q,t),M(S_p,T_q,t),M(S_p,A_p,t), M(T_q,B_q,t),M(Sp,B_q,t),M(T_q,A_p,t)\} \geq 0$, and

$ø\{N(A_p,B_q,t),N(S_p,T_q,t),N(S_p,A_p,t), N(T_q,B_q,t), N(Sp,B_q,t),N(T_q,A_p,t)\} \leq 0,$ (2.1)

(c) $A(X) \subset T(X)$ (or $B(X) \subset S(X)$)

Then the pairs (A,S) and (B,T) share the common property (S.B).

Proof- Suppose that the pair (A,S), then there exist a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} = t$
for some $t\in X$ (2.2)

since $A(X) \subset T(X)$, hence for each $\{x_n\}$ there exists $\{y_n\} \in X$ such that

$Ax_n = Ty_n$. Therefore, $\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Ax_n = t$ (2.3)

Thus in all, we have $Ax_n \to t$, $Sx_n \to t$ and $Ty_n \to t$. Now we assert that $By_n \to t$. Suppose that $By_n \to t$, then applying inequality (3.1), we obtain.

$ø\{M(Ax_n,By_n,x), M(Sx_n,Ty_n,x), M(Sx_n,Ax_n,x), M(Ty_n,By_n,x), M(Sx_n,By_n,x), M(Ty_n,Ax_n,x)\} \geq 0$, and

$ø\{N(Ax_n,By_n,x), N(Sx_n,Ty_n,x), N(Sx_n,Ax_n,x), N(Ty_n,By_n,x), N(Sx_n,By_n,x), N(Ty_n,Ax_n,x)\} \leq 0$, (2.4)

which on making $n\to\infty$ reduces to

$ø\{M(t,\lim_{n\to\infty} By_n,x),M(t,t,x),M(t,t,x), M(t,\lim_{n\to\infty} By_n,x), M(t,\lim_{n\to\infty} By_n,x), M(t,t,x) \geq 0$ and

$ø\{N(t,\lim_{n\to\infty} By_n,x),N(t,t,x),N(t,t,x), N(t,\lim_{n\to\infty} By_n,x), N(t,\lim_{n\to\infty} By_n,x), N(t,t,x) \leq 0$. (2.5)

or

$ø\{M(t,\lim_{n\to\infty} By_n,x),1,1,M(t,\lim_{n\to\infty} By_n,x), M(t,\lim_{n\to\infty} By_n,x), 1 \geq 0$, and

$ø\{N(t,\lim_{n\to\infty} By_n,x),1,1,N(t,\lim_{n\to\infty} By_n,x), N(t,\lim_{n\to\infty} By_n,x),1 \leq 0$, (2.6)

which is the contradiction ($ø_2$), and therefore $By_n \to t$.Hence the pairs (A,S) and (B,T) share the common property (S.B.)

Theorem-2.1. Let $(X, M, N, *, \blacklozenge)$ be an intuitionistic fuzzy metric space. Let A,B,S and T be a mappings from X into itself such that

(a) The pair (A,S) (or(B,T)) have the property(S.B.),

(b) $A(X) \subset T(X)$ (or $B(X) \subset S(X)$)

(c) S(X) (or T(X)) is a closed suset of X.Then the pair (A,S) and (B,T) have a point of coincidence each. Moreover A,B,S and T, have a unique common fixed pointprovided that both the pairs (A,S) and (B,T) are weakly compatible.

Proof- In view of lemma 2.1 the pairs (A,S) and (B,T) share the common property (S.B.), that is, there exist two sequence $\{x_n\}$ and $\{y_n\}$ inX such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t$,

for some $t \in X$ (2.7)

suppose that S(X) (or T(X)) is a closed suset of X, then t = Su, for some u ∈ X. If $t \neq Au$, then applying inequality (3.1) we obtain

$ø\{M(A_u,B_v,t), M(S_u,T_v,t), M(S_u,v_u,t), M(T_v,B_v,t), M(S_u,B_v,t), M(T_v,A_u,t)\} \geq 0$, and
$ø\{N(A_u,B_v,t), N(S_u,T_v,t), N(S_u,v_u,t), N(T_v,B_v,t), N(S_u,B_v,t), N(T_v,A_u,t)\} \leq 0$ (2.8)

which on making $n \to \infty$ reduces to

$\{M(A_u,x,t), 1, M(x,A_u,t), 1, 1, M(x,A_u,t)\} \geq 0$, and
$ø\{N(A_u,x,t),1,N(x,A_u,t),1,1, N(x,A_u,t)\} \leq 0$, (2.9)

which is a ontraduction to($ø_1$).Hence Au = Su = t. Since $A(X) \subset T(X)$,there exist $v \in X$, such that t = Au = Su.

If $t \neq B_v$ then using inequality (3.1) we have,

$ø\{M(A_u,B_v,t), M(S_u,T_v,t), M(S_u,A_u,t), M(T_v,B_v,t), M(S_u,B_v,t), M(T_v,A_u,t)\} \geq 0$, and
$ø\{N(A_u,B_v,t), N(S_u,T_v,t), N(S_u,A_u,t), N(T_v,B_v,t), N(S_u,B_v,t), N(T_v,A_u,t)\} \leq 0$, (2.10)

or

$ø\{M(p,B_v,t), 1, 1, M(p,B_v,t), M(p,B_v,t)), 1\} \geq 0$ and $ø\{N(p,B_v,t), 1, 1, N(p,B_v,t), N(p,B_v,t)), 1\} \leq 0$, (2.11)

Which is a contradiction to $ø_2$ and therefore Au = Su = p = Bv = Tv.

Since the pair (A,S) and (B,T) are weakly compatible and Au = Su, Bv = Tv, therefore

Ap = ASu = SAu = Sp, Bp = BTv = TBv = Tp, (2.12)

If Ap≠p, then using inequality (2.1) we have,

$ø\{M(A_p,B_v,t), M(S_p,T_v,t), M(S_p,A_u,t), M(S_v,B_v,t), M(T_v,A_p,t)\} \geq 0$, and
$ø\{N(A_p,B_v,t), N(S_p,T_v,t), N(S_p,A_u,t), N(S_v,B_v,t), N(T_v,A_p,t)\} \leq 0$, (2.13)

or

$ø\{M(A_p,p,t), 1, M(A_p,p,t), 1, M(A_p,p,t_0), M(p,A_p,t_0)\} \geq 0$, and
$ø\{N(A_p,p,t), 1, N(A_p,p,t), 1, N(A_p,p,t_0), N(p,A_p,t_0)\} \leq 0$, (2.14)

which is a contradiction to ($ø_3$), and therefore Ap = Sp = p.

Similarly, one can prove that Bp = Sp = p.Hence p = Bp = Tp = Ap = Sp, and p is a common fixed point of A,B,S and T.The uniqueness of common fixed point is an easy consequence of inequality (3.1)

By choosing A,B,S and T suitably, one can drive corollaries involving two or three mappings. As a sample, we deduce the following natural result for a pair of self–mapping by setting B = A and T = S.

REFERENCES

1. K.Atanassov, *intuititonistic fuzzy Sets and Systems* 20:86, 96 (1986).
2. A.Georgeand veeramani. On some results in fuzzy metric spaces, *Fuzzy Sets and system* 64, (1994).
3. O.Kramosil and J.Michalck, Fuzzy metric and statistical metric space, *Kybernetika* 11, 326-334 (1975).
4. S.Kutukcu. D.Torkoglu and C. Yildiz, Common fixed points of compatible maps of type (β) on fuzzy metric spaces, *Commun. Korean Math. Soc*. 21 No. 1.89-100 (2006).
5. J.H. Park, Intuitionistic fuzzy metric spaces. *Chhaos, Solitions and Fractals* 22,1039- 1046 (2004).

6. S. Sharma and J.K. Tiwari. Common fixed point in fuzzy metric spaces, *J. Korean Soc. Math Educ: Ser. B: Pure Appl. Math.* 12 No. 1, 17-31 (2005).
7. L.A. Zadeh, *Fuzzy sets Inform and Control* 8, 338-353 (1965).