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Mathematical Model for Contesting Party Workers

[1]Nita H. Shah, Milan D. Shah, Foram A. Thakkar and Moksha H. Satia

[1]Department of Mathematics,
Gujarat University, Ahmedabad, 380009, Gujarat, INDIA.
Corresponding author: [1]Prof. Nita. H. Shah
email: nitahshah@gmail.com, milankumarshah162@gmail.com,
foramkhakhar3@gmail.com, mokshasatia.05@gmail.com.

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ABSTRACT

In this paper, a mathematical model for contesting party workers has been formulated as a system of non-linear ordinary differential equations during election. A basic reproduction number has been calculated which will decide the behavior of an individual in the system. Stability in terms of local and global is also discussed. Simulation analysis has been carried out which shows that the maximum number of individuals are trying to join ruling party.

Keywords: Mathematical model, System of non-linear ordinary differential equations, Basic Reproduction number, Stability, Simulation.

1. INTRODUCTION

India is a democratic country. Democracy means a system of government by the whole population or all eligible member of the state normally through elected representative. In today's world, democracy which is also referred as "rule of majority" is system of government in which citizen use power directly or elects' representative among themselves. In democracy elected representative at local or national should listen to the people and respond to their suggestions. In democracy voters can gives their votes secretly. Election have to occur at a regular intervals, as a prescribed by law. Those in power cannot extend their terms without asking for consent of the people again in election. According to Abraham Lincoln, democracy is the government of the people, for the people, by the people. In view of Mahatma Gandhi, democracy is something that gives weak the same chance as the strong[2,3,5,7].

The total population of India gets single and uniform citizenship by the constitution of India. Every citizen is equal in the eye of constitution of India. The constitution of India gives right to vote, whose age is above 18 years and who is native of India. In India, each election has single vote in which voter can gives only one vote to any one candidate, and among these candidates, one who receives maximum votes wins. In other word the source of power for the government is 'by the people' and 'for the people'[3,4,6].

In each election, number of parties fights for building their government. For winning the election many factors play effective role in their party. Party workers are one of the major factors who carry out duties for a candidate or party before an election. In this way, each party wanted that they have more number of party workers who can contact and attract more voters in the election time for their parties.

It may happen that the party workers contesting for the party may switch from their respective which they have joint initially. Shifting of individuals from one party to other has now become a common trend. The reason of switching from their party are either they do not get proper positions in their party or they are not satisfied with the change occurring in the party's ideology, due to which they leave their original party and join the other party[8].

Misra has done their research in titled "A simple mathematical model for the spread of two political parties" in which they have assumed that individual of voters class are susceptible to both the political parties[8].

In this paper, we will analyze contesting party workers during election. The notation and basic reproduction number are described and formulated in section 2. Section 3 consists of stability analysis in which local and global stability is included in subsection 3.1 and 3.2 respectively. Sensitivity Analysis, numerical simulation with their interpretation and conclusion are described in section 4, 5 and 6 respectively.

2. MATHEMATICAL MODEL

Here we have developed a mathematical model for contesting party workers using *SEIR* model.

The transmission diagram for contesting party workers is shown in the figure 1.

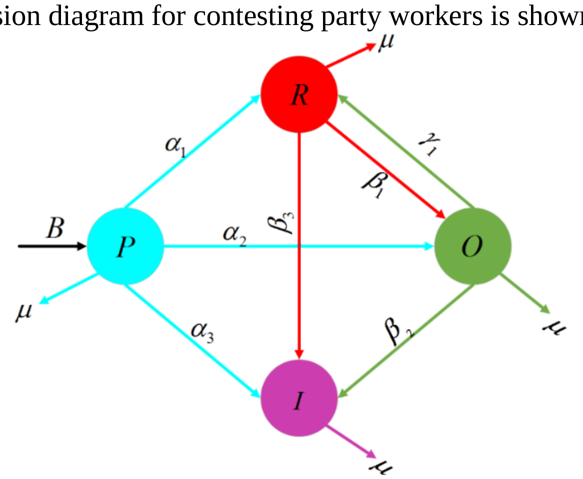

Figure 1: Transmission diagram for contesting party workers

Table 1. Notation and its parametric values

Notations	Description	Parametric values
$P(t)$	Number of party workers at any instant of time t	5000
$R(t)$	Number of ruling party workers at any instant of time t	2000
$O(t)$	Number of opposition party workers at any instant of time t	1500
$I(t)$	Number of independent party workers at any instant of time t	1000
$N(t)$	Sample Size at any instant of time t	10000
B	New recruitment rate	0.0003
μ	The rate of party workers who gives up from contesting election	0.0023
α_1	Rate of individual who contest for party R	0.02
α_2	Rate of individual who contest for party O	0.015
α_3	Rate of individual who contest independent I	0.0001
β_1	Rate of individual who switch to party O from party R	0.002
β_2	Rate of individual who switch to party I from party O	0.0015
β_3	Rate of individual who switch to party I from party R	0.0001
γ_1	Rate of individual who switch to party R from party O	0.0336

In the model, P is total number of party workers who are contesting in election. Here, α_1, α_2 and α_3 are the rate of party workers who wants to contest for the party R, O or I respectively. From the party R some of the individuals switches to party O and party I with the rate β_1 and β_3 respectively. Some individual switch from party O to party R with the rate of γ_1. Also, individual move to party I from party O with the rate β_2. μ represents the rate at which candidate who gives up from the election.

$$\begin{aligned}
\frac{dP}{dt} &= B - \alpha_1 PR - \alpha_2 PO - \alpha_3 PI - \mu P \\
\frac{dR}{dt} &= \alpha_1 PR - \beta_3 R - \beta_1 R + \gamma_1 O - \mu R \\
\frac{dO}{dt} &= \alpha_2 PO - \beta_2 O + \beta_1 R - \gamma_1 O - \mu O \\
\frac{dI}{dt} &= \alpha_3 PI + \beta_3 R + \beta_2 O - \mu I
\end{aligned} \qquad (1)$$

Now on taking addition of all the above equation, we get $\frac{d}{dt}(P+R+O+I)=B-\mu(P+R+O+I)$

This gives, $\limsup_{t\to\infty}(P+R+O+I)\le\frac{B}{\mu}$.

Therefore, the feasible region for (1) is $A=\left\{(P,R,O,I):P+R+O+I\le\frac{B}{\mu},P>0,R,O,I\ge 0\right\}$

Therefore, contestant free equilibrium point of the model is $E_0=\left(\frac{B}{\mu},0,0,0\right)$.

Now, we are interested in calculating the basic reproduction number R_0 using next generation matrix method. The next generation matrix is defined as FV^{-1} where F and V both are Jacobian matrices of f and v evaluated with respect to each compartment at equilibrium point.

Let $X=(R,O,I,P)$

Thus, $\frac{dX}{dt}=f(X)-v(X)$,

$$f=\begin{bmatrix}\alpha_1 PR\\ \alpha_2 PO\\ \alpha_3 PI\\ 0\end{bmatrix} \text{ and } v=\begin{bmatrix}\beta_2 R+\gamma_1 R-\beta_1 O+\mu R\\ \beta_2 O+\beta_1 O-\gamma_1 R+\mu O\\ -\beta_3 R-\beta_2 O+\mu I\\ -B+\alpha_1 PR+\alpha_2 PO+\alpha_3 PI+\mu P\end{bmatrix}$$

Now, we want to find F and V by the derivative of f and v at equilibrium point $E_0=\left(\frac{B}{\mu},0,0,0\right)$. Here F and V are 4×4 matrices defined as $F=\left[\frac{\partial f_i(E_0)}{dX_i}\right]$, $V=\left[\frac{\partial v_i(E_0)}{dX_i}\right]$ where $i,j=1,2,3,4$

$$F=\begin{bmatrix}\frac{\alpha_1 B}{\mu} & 0 & 0 & 0\\ 0 & \frac{\alpha_2 B}{\mu} & 0 & 0\\ 0 & 0 & \frac{\alpha_3 B}{\mu} & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \text{ and } V=\begin{bmatrix}\beta_3+\mu+\gamma_1 & -\beta_1 & 0 & 0\\ -\gamma_1 & \beta_1+\beta_2+\mu & 0 & 0\\ -\beta_3 & -\beta_2 & \mu & 0\\ \frac{\alpha_1 B}{\mu} & \frac{\alpha_2 B}{\mu} & \frac{\alpha_3 B}{\mu} & \mu\end{bmatrix}$$

Here the basic reproduction number R_0 is the spectral radius of matrix $F\cdot V^{-1}$ which is,

$$R_0=\frac{B(\alpha_1\mu a_1+\alpha_2\mu a_2+\alpha_3 a_3+\mu a_4)}{\mu^2(\mu a_5+\beta_3 a_6)}$$

where, $a_1=\beta_1+\beta_2+\mu$, $a_2=\beta_3+\gamma_1+\mu$, $a_3=\beta_1(\beta_3+\mu)+\beta_2(\beta_3+\gamma_1+\mu)$,

$a_4 = \beta_3 + \gamma_1 + \mu$, $a_5 = \beta_1 + \beta_2 + \beta_3 + \gamma_1 + \mu$, $a_6 = \beta_1 + \beta_2$.

On solving system (1), we get an endemic equilibrium points $E_1 = \left(\frac{\mu}{\alpha_3}, 0, 0, \frac{B\alpha_3 - \mu^2}{\mu\alpha_3} \right)$ and

$E^* = (P^*, R^*, O^*, I^*)$,

where, $P^* = A$

$$R^* = \frac{\begin{array}{l} A\alpha_2\mu\left(\alpha_3\beta_1^2 + B\alpha_1^2 + \alpha_3\beta_3^2 + \mu\left(2\beta_3 - \alpha_1 + 2\beta_1\beta_3 - \alpha_1(\beta_1 + \beta_3)\right)\right) + \mu\alpha_3\gamma_1(\alpha_1 B - \beta_1\beta_3) - \mu B\alpha_1^2\gamma_1 \\ +\mu^2\left(\alpha_1\beta_3(\gamma_1 + \mu + \beta_2) + \beta_2\alpha_1\beta_1 - \alpha_3\left(2\beta_2\beta_3 + \beta_1(\beta_2 + \gamma_1 + \beta_1) + 2\beta_3\gamma_1 - 1 + \beta_3^2 + \alpha_1 B\right)\right) \\ -\mu^4(\alpha_3 - \alpha_1) + \mu^3\left(\alpha_1(\gamma_1 + \beta_1 + \beta_2) - \alpha_3(\beta_2 + \gamma_1 + 2\beta_1 + 2\beta_3)\right) \\ +B\alpha_1\alpha_3\left(\beta_1\beta_2 + \beta_3(\mu + \gamma_1) + \mu(\beta_2 + \beta_1 + \mu) - A\alpha_2(\mu + \beta_1 + \beta_3)\right) \\ +\alpha_3\beta_2\beta_3(B\alpha_1 - 2\mu\beta_1) + \mu\alpha_3\left(\beta_3^2(\gamma_1 - \beta_2) - 2\mu\beta_1\beta_3 - \beta_1^2\beta_2\right) + \mu A\alpha_3\beta_1(2\alpha_2\beta_3 + \gamma_1\alpha_1) \end{array}}{\alpha_1\mu\left(\begin{array}{l} \mu\alpha_3(\beta_1 + \beta_3) + \beta_2\left(\beta_3(\alpha_1 + \alpha_3) - \mu\alpha_1\right) + A\alpha_2\left(\beta_1\alpha_3 + \mu(\alpha_1 - \alpha_3) - \left(\alpha_3(\beta_3 + \beta_1) - \alpha_1\beta_3\right)\right) \\ -\alpha_1\beta_1\beta_2 + \alpha_3\left(\mu(\gamma_1 + \mu) + \beta_1\beta_2 + \beta_3\gamma_1\right) - \alpha_1\left(\beta_3\gamma_1 + \mu(\mu + \gamma_1 + \beta_3 + \beta_1)\right) \end{array}\right)}$$

$$O^* = \frac{\beta_1\left(\begin{array}{l} \mu^2\alpha_3(\mu + \beta_1) - A\mu\alpha_2(\alpha_3\beta_1 - \mu\alpha_1) + \mu(\alpha_3\beta_3\gamma_1 - B\alpha_1\alpha_2 + \alpha_3\beta_2\beta_3) + \mu^2(\alpha_3\gamma_1 + \alpha_3\beta_3 + \beta_2\beta_3) \\ +AB\alpha_1\alpha_2\alpha_3 + \mu\alpha_3\left(\beta_1\beta_3 - A(\alpha_1\gamma_1 - \alpha_2\beta_3 - \alpha_1\beta_2)\right) - A\mu^2(\alpha_1\alpha_3 + \alpha_3\alpha_2) \end{array}\right)}{\mu\alpha_2\left(\begin{array}{l} -\alpha_1(\beta_3\gamma_1 + \beta_2\beta_1 + \beta_2\beta_3) + \alpha_3(\beta_2\beta_1 + \beta_2\beta_3 + \beta_3\gamma_1) + \mu\alpha_3(\beta_3 + \gamma_1 + \beta_2 + \beta_1 + \mu) \\ -\mu\alpha_1(\beta_3 + \beta_1 + \beta_2 + \mu + \gamma_1) + A\mu\alpha_2(\alpha_1 + \alpha_3) + A\alpha_1(\alpha_3\beta_1 + \alpha_2\beta_3) - A\alpha_2\alpha_3(\beta_3 + \beta_1) \end{array}\right)}$$

$$I^* = \frac{(A\mu + B)\left(\begin{array}{l} \alpha_1\alpha_2\left(\begin{array}{l} A\left(-\alpha_2\left(\beta_2(\mu + \beta_1 + \beta_3) + \beta_3\gamma_1\right) + \alpha_1\left(\mu\beta_3 + \beta_1\beta_2 + \beta_3(\beta_2 + \gamma_1)\right)\right) + \beta_1\beta_2 \\ (\gamma_1 + \mu - \beta_3 - \beta_1 - \gamma_1) + \beta_3\gamma_1(\gamma_1 + \mu + \beta_2) \end{array}\right) \\ -\alpha_1^2\left(\begin{array}{l} 2\mu\beta_3(\gamma_1 + \beta_2) + \beta_1\beta_2^2 + \mu^2\beta_3 + \beta_3(\beta_2^2 - \beta_3\beta_2 + \gamma_1^2) + \beta_1\beta_2(\mu - \gamma_1) \\ -\beta_3^2\gamma_1 + \beta_2\beta_3\gamma_1 \end{array}\right) \\ +\alpha_2^2\left(2\mu\beta_2(\beta_3 + \beta_1) + \mu^2(\beta_3 + \beta_2) + \mu\beta_3\gamma_1 + \beta_1\left(\beta_1\beta_2 + 2(\beta_2\beta_3 +_1 \beta_3\gamma_1)\right)\right) \end{array}\right)}{\left[\begin{array}{l} \mu(A\alpha_1\alpha_2\left((\alpha_1 - \alpha_2)\left(\mu(\beta_3 + \beta_2 + \gamma_1 + \beta_1 + \mu) + \beta_1\beta_2 + \beta_3(\beta_2 + \gamma_1)\right)\right) + \beta_1\beta_2(\alpha_1\alpha_3\gamma_1 - 2\alpha_3\alpha_2\beta_3) \\ +\alpha_1^2\left(\begin{array}{l} \mu\left(\beta_2(2\beta_1 - 2\gamma_1) - 2\beta_1\beta_3 - \gamma_1(\gamma_1 + \beta_1 + 2\beta_3) - \beta_2^2 + \mu(2\beta_3 - \mu - \gamma_1 - \beta_2 - \beta_1 - \beta_3)\right) \\ -\beta_1\beta_2(\beta_2 + \gamma_1) - \beta_2\beta_3(2\gamma_1 + \beta_2) - \beta_3\gamma_1^2 \end{array}\right) \\ +\alpha_2^2\left(\mu\left(\beta_1(\beta_1 + 2\beta_2) + 2\beta_3\gamma_1 + \mu(1 + 2\beta_1 + \gamma_1 + \beta_2)\right) + \beta_3^2(\beta_2 + \gamma_1 + 1) + \beta_1\left(\beta_2(1 + 2\beta_3) + \gamma_1(\beta_3 + 1)\right)\right) \\ +\gamma_1^2\left(\mu(\alpha_1\alpha_3 + \alpha_1\alpha_2 - \alpha_2\alpha_3) + \alpha_3\beta_3(\alpha_1 - \beta_3 + \alpha_1)\right) + \gamma_1(\alpha_1\alpha_3\beta_1\beta_3 - \alpha_2\alpha_3\beta_1\beta_2 + \alpha_1\alpha_2\beta_2\beta_3) \\ +\alpha_1\alpha_2\left(\mu\left(\gamma_1(\beta_2 + \beta_3) + \beta_1(\beta_3 - \mu - \beta - \beta_1)\right) + \beta_1\beta_3\gamma_1 + \beta_2\left(\beta_1(1 - \beta) + 2\beta_3\gamma_1 + \beta_1\gamma_1\right)\right) \\ -\alpha_2\alpha_3\left(\beta_3^2(\beta_2 + \gamma_1) + \beta_2\beta_3\gamma_1 + \beta_1^2\beta_2 + \beta_1\beta_3\gamma_1 + \mu\left(\beta_1\gamma_1 - 2\beta_2\beta_3 + \beta_1^2 + \beta_3^2 + 2\beta_1 + 3\beta_3\gamma_1 + \beta_2\gamma_1\right)\right) \\ +\alpha_1\alpha_3\left(\beta_2^2\beta_3^2 + \beta_1^2\beta_2 + \mu\left(\beta_2(2\mu + \beta_2) + \beta_1(2 + \beta_1 + 2\gamma_1 + 3\beta_2 + \beta_3) + 2\left(\beta_3(\beta_2 + \gamma_1) + \beta_2\gamma_1\right)\right)\right) \\ +\mu^2\left(\alpha_1\alpha_3(2\gamma_1 + \beta_3) - \alpha_2\alpha_3(2\gamma_1 + \beta_2 + 2\beta_3) + \alpha_1\alpha_2\gamma - \mu(\alpha_2\alpha_3 + \alpha_1\alpha_3)\right) - 2\mu\alpha_2\alpha_3(\beta_1\beta_2 + \beta_1\beta_3) \end{array}\right]}$$

where

$$A=\frac{1}{2\alpha_1\alpha_2}\left[\alpha_1\left(\mu+\gamma_1+\beta_2\right)+\alpha_2\left(\beta_3+\mu+\beta_1\right)\pm\begin{pmatrix}-2\alpha_1\alpha_2\left(\beta_1\beta_2+\mu\beta_3+\mu_1\beta+\mu^2\right)\\+\alpha_2^{\,2}\begin{pmatrix}\mu^2+2\mu\beta_3+\mu\beta_1\\+2\beta_1\beta_3+\beta_3^{\,2}+\beta_1^{\,2}\end{pmatrix}\\+2\alpha_1\alpha_2\left(\beta_1\gamma_1-\beta_2\beta_3-\mu\beta_2-\beta_3\gamma_1-\mu\gamma_1\right)\\+\alpha_1^{\,2}\left(2\gamma_1\beta_2+\gamma_1^{\,2}+\beta_1^{\,2}+2\mu\beta_2+\mu\gamma_1+\mu^2\right)\end{pmatrix}^{\frac{1}{2}}\right]$$

3. STABILITY ANALYSIS

In this section, we will discuss the stability of the model in terms of local and global stability.

3.1 Local Stability

The contestant free equilibrium point of the system is locally stable, if all the eigenvalues of Jacobian matrix of system (1) have negative real part. At point, $E_0=\left(\frac{B}{\mu},0,0,0\right)$ the Jacobian of the system is of the form

$$J=\begin{bmatrix}-\mu & \frac{-\alpha_1 B}{\mu} & \frac{-\alpha_2 B}{\mu} & \frac{-\alpha_3 B}{\mu}\\ 0 & \frac{\alpha_1 B}{\mu}-\mu & \beta_1 & 0\\ 0 & \gamma_1 & \frac{\alpha_2 B}{\mu}-\mu & 0\\ 0 & 0 & 0 & \frac{\alpha_3 B}{\mu}-\mu\end{bmatrix}$$

For this, $trace(J)=-4\mu+\left(\frac{(\alpha_1+\alpha_2+\alpha_3)B}{\mu}\right)<0$ if $\min\{\alpha_1 B,\alpha_2 B,\alpha_3 B\}<\mu^2$

So, it is locally stable.

3.2 Global Stability

If $\det\left(I-FV^{-1}\right)>0$ then the contestant free equilibrium point E_0 is globally stable.

$$\det\left(I-FV^{-1}\right)=1-R_0=1-0.6099=0.3901>0$$

So, the system is globally stable.

4. SENSITIVITY ANALYSIS

In this section, the sensitivity analyses for all model parameters are calculated for contesting the party workers in the system.

The sensitivity for all the parameters is computed by the formula, $V_{\theta}^{R_0} = \frac{\partial R_0}{\partial \theta} \cdot \frac{\theta}{R_0}$

where, θ denotes all the parameters of the model.

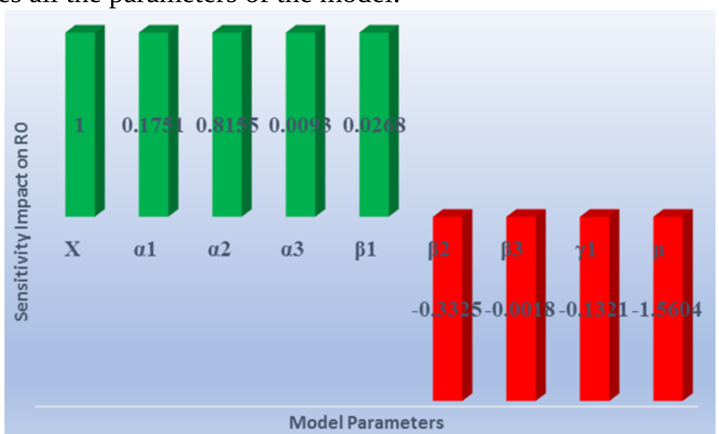

Figure 2: Impact of parameters for the contesting party workers

Here, the parameters which are above the axis denotes that it promotes more party workers for contesting in election, whereas the one which are below the axis denotes negative impact on them.

5. NUMERICAL SIMULATION

In this section, we observe the numerical results of party workers followed by different components.

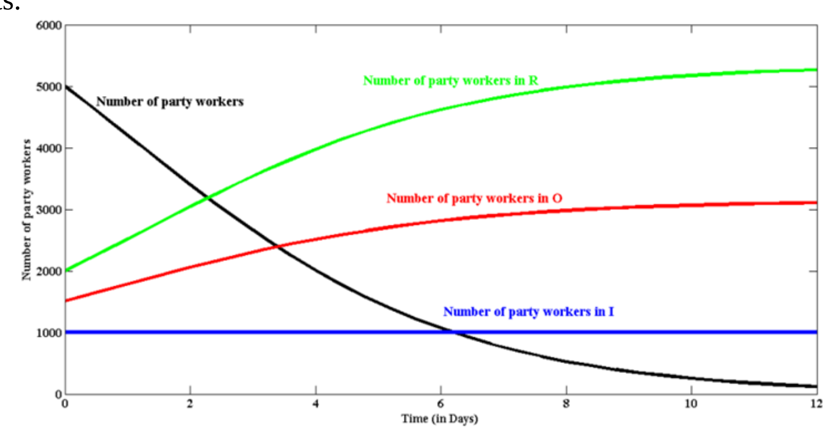

Figure 3: Movement of party workers in different parties

Figure 3 shows that the numbers of party workers are decreasing which means that this party worker wants to contest for either party R, O or I . Also one can observe from the figure that these party workers have joined their desired parties within a week.

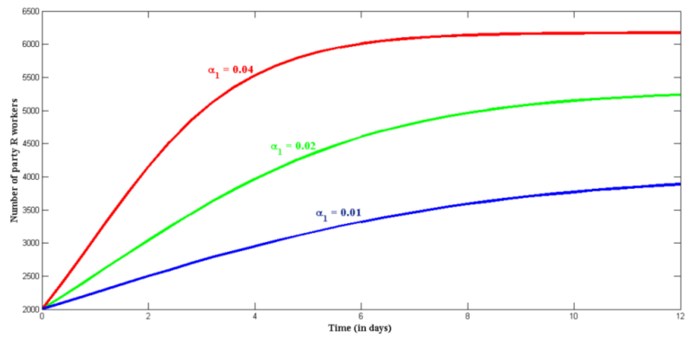

Figure 4: Effect of different value of α_1 on party R

Figure 4 shows that if the rate of party workers who are contesting for party R is increased from 1% to 4% then the number of party workers increases from approximately 2000 to 6000.

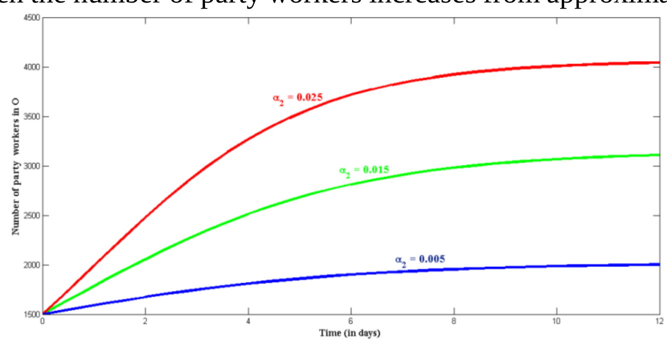

Figure 5: Effect of different value of α_2 on party O

Figure 5 describes that the number of party workers increased approximate 1500 to 4000 whenever we increase the rate of party workers from 0.5% to 2.5%.

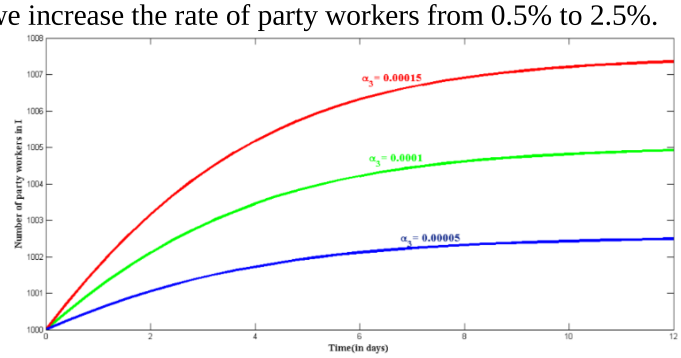

Figure 6: Effect of different value of α_3 on party I

Figure 6 shows that on increasing the rate of individuals joining to party I the number of party workers in party I also increases.

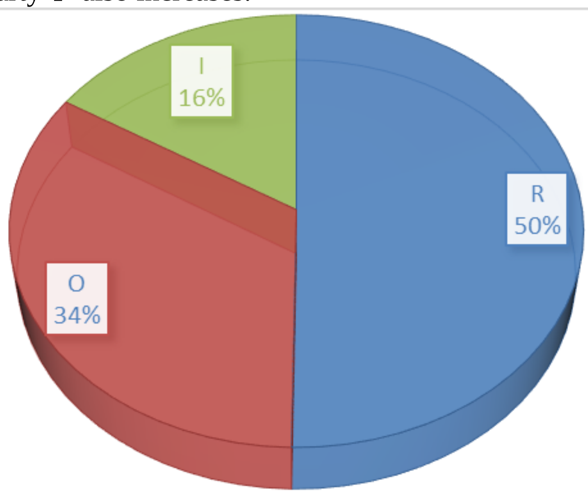

Figure 7: Percentage of party workers contesting for party R, O and I .

Figure 7 interprets that after a few days of declaration of election date 50% of individuals contest for party R whereas 34% and 16% individual contest for party O and I respectively. Also it is observed that party R has more advantage in the election as more number of party workers are contesting for party R .

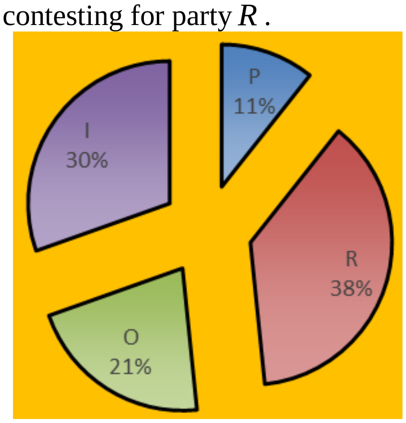

Figure 8: Percentage of contesting party workers at the equilibrium point E^*

Figure 8 describe the nature of individuals at the endemic equilibrium point E^*. One can observe that, at the equilibrium point E^*, 38% individual joint to party R whenever 30% and 21% individual are joining party I and O . At the equilibrium point E^* , party R has more advantage of party workers as compared to parties I and O .

Equilibrium point $E_1 = \left(\frac{\mu}{\alpha_3}, 0, 0, \frac{B\alpha_3 - \mu^2}{\mu\alpha_3} \right)$ is an infeasible solution as all the party workers contest as independent contestant.

6. CONCLUSION

Here, a mathematical model of the party workers contesting for the different parties has been formulated and analyzed. Basic reproduction number has been calculated which is found to be 0.6099 which shows that 60.99% of party workers are contesting during election. Contestant free equilibrium point has been proved to be locally and globally stable on satisfying the condition. Sensitivity analysis for the different model parameters has been studied. Simulation has been carried out which shows that majority of party workers are trying to join ruling party. It has been observed that 11% of party workers contest for ruling, opposition or independent contestant. 38% gets ticket from ruling party, 21% gets ticket from opposition party, 30% contest independently.

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